Project correlation in portfolio theory

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The use of portfolio theory allows the consideration of correlation between projects and may rationalize the selection of the projects and capital budgeting. Positive coefficient of correlation increases the risk for a portfolio, while negative coefficient decreases the risk. On the other hand, the expected return of a portfolio may increase or decrease due to project correlation. The correlation between projects is due to several casual factors which are of different importance and contribution to project correlation. The importance and the level of contribution of each factor can be estimated based on experience and judgement. Experience and judgement may easily be expressed in semantic measures rather than mathematical terms. Classical portfolio theory fails to incorporate subjective information. The semantic measures can be translated into mathematical values using the fuzzy set theory. A method by which project correlation may be estimated based on experience and judgement is proposed. The method utilizes the fuzzy set theory to estimate the coefficient of correlation and the judgement uncertainty. Then, the total risk of a portfolio can be estimated.

Keywords: project correlation, portfolio theory, project evaluation, fuzzy sets

Notation

A, B	fuzzy sets
C	level of correlation due to factors
COV	coefficient of variation
E	expected (mean) value
I	importance factor
m	number of elements in a set
n	number of projects or securities
P	proportion of investment in a security
R	return on a project or security
S	security
Var	total variance
x	element of a set
α	significance of judgement
μ	membership value of an element of a set
$\bar{\rho}$	mean correlation coefficient
ρ	correlation coefficient
σ	standard deviation
Σ	summation
$\cap$	intersection
$\otimes$	cartesian product
$\mid$	delimiter

Introduction

Most corporations are concerned with the efficient use of their funds under budgetary constraints, and various objectives of financial investments and preferences. The alternatives of capital-investment projects are not mutually exclusive, i.e., the selection of a particular project does not preclude the selection of any other project. The corporation's problem become the determination, from within its own investment opportunity set, of the investments that best satisfy the corporation's objectives and also of the proportions in which the funds should be distributed among the chosen investment opportunities. The field of capital budgeting, or called portfolio analysis and selection, is well developed and researched[1-7]. The selection of the portfolio depends largely on the correlation between the projects or the investments as discussed in a later section. The degree of linear relationship or correlation between the cost of two projects can be measured by the coefficient of correlation. This coefficient can take values from -1 to $+1$. A zero coefficient indicates that there is no linear relationship between the cost (or return) of projects.

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Problem description

In construction engineering, some of the factors that may cause linear relationship or correlation between two projects are[8]: (1) Geographical factors; (2) type of project; (3) supervision; (4) weather; (5) project schedule; (6) owner; (7) economy; (8) subcontractors; (9) political factors; (10) construction methods; (11) resources (money, material, equipment and labour); (12) specifications; and (13) cost estimates. Many of these factors are not and cannot be precisely defined. This creates uncertainty for the decision-maker. As a result, experience and judgement are used to supplement scientific knowledge. The combination of 'objective' information and 'subjective' judgement can be performed methodically by the use of the theory of fuzzy sets and systems[9-16].

The major problem in estimating the correlation between projects lies in the causal factors that are expressed in linguistic, rather than mathematical terms. Good or bad weather, or similar or different construction methods, etc., fall into this category. Even the importance of

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problem, therefore, three kinds of uncertainty are encountered in practice: random uncertainty, statistical uncertainty and judgement uncertainty[11,17–20]. Random uncertainty arises from the randomness of precisely defined events or propositions and can be dealt with using the theory of probability. Statistical uncertainty arises from estimating some of the factors using limited data (or a finite sample size) and can be dealt with using the theory of statistics. Judgement uncertainty derives from the lack of precision or the lack of understanding of an event, a proposition, or a system. The theory of fuzzy sets and systems has proved to be an effective tool in handling the judgement uncertainty[21–23].

In this paper, a method of estimating the correlation between projects based on the judgement of experts is proposed. The method is based on the concepts of fuzzy sets and systems. The proposed method can be easily programmed on any business computer. The proposed method is presented and used in the context of construction projects. However, the same approach can be used to estimate correlation between any financial investment projects, such as bonds and stocks. The correlation between projects in the construction industry usually takes the form of determining the correlation between a new project and an existing portfolio. By desegregating correlation into the various causal factors, correlation may be estimated. The coefficients of correlation should be estimated at the starting time of a new project, and should consider the status of each existing project at that time.

Portfolio theory

A portfolio represents a collection of securities (or projects). Some combinations of securities provide better portfolios than others. The objective of portfolio theory is not to identify the best portfolio for the investor, but rather to identify, from all possible portfolios, what is called the 'efficient set' of portfolios. The investor then will choose from the efficient set of portfolios the portfolio that best satisfies his objectives. Most research in portfolio theory has been made in portfolios composed of security type of investment[4,5,8]. Little has been done with portfolios composed of capital assets and investments other than securities. The objective of an investor is a portfolio with his/her desired return and risk. Most investors desire high returns; and returns that are dependable, stable and not subject to large uncertainty. Investors generally are risk aversive, i.e., for any given level of risk, investors prefer higher returns to lower returns. The investors' problem in portfolio analysis can be divided into three steps; namely, (1) security analysis, (2) portfolio analysis, and (3) portfolio selection.

Security analysis

The objective of this step is to estimate the rate of return of each security in terms of the mean value of return $E(S_i)$; standard deviation of return $\sigma(S_i)$; and correlation between the return of security i and other securities within the portfolio. The correlation between securities i and j is measured in terms of coefficient of correlation ρ_{ij} which falls in the range $[-1, 1]$. If the cost (or return) of two securities move up or down together, they are said to tions, they are negatively correlated.

Portfolio analysis

Most corporations have capital investment projects or securities including alternatives that are not mutually exclusive. A budgetary constraint restricts investing in all possible projects. An investor can generate a large number of different portfolios by varying the proportions invested in each security and the securities themselves. For n securities, the budgetary constraint can be represented as

$$\sum_{i=1}^{n} P_i = 1 \tag{1}$$

where P_i = the proportion invested in security i. The expected return, $E(R)$, and the variance of return, $\sigma^2(R)$, of a portfolio of n securities is given by

$$E(R) = \sum_{i=1}^{n} P_i E(S_i) \tag{2}$$

$$\sigma^2(R) = \sum_{i=1}^{n} P_i^2 \sigma^2(S_i) + \sum_{i=1}^{n} \sum_{j=1}^{n} P_i P_j \rho_{ij} \sigma(S_i) \sigma(S_j) \quad i \neq j \tag{3}$$

For a group of securities, the set of all possible portfolios can be generated as shown in *Figure 1*. Associated with each portfolio are an expected return, $E(R)$, and a risk measured by the variance of return, $\sigma^2(R)$. Since investors are risk aversive, only portfolios which fall on curve ABC are considered by the investors. Curve ABC is called efficiency frontier of all possible portfolios. The shape of the efficiency frontier is concave and smooth.

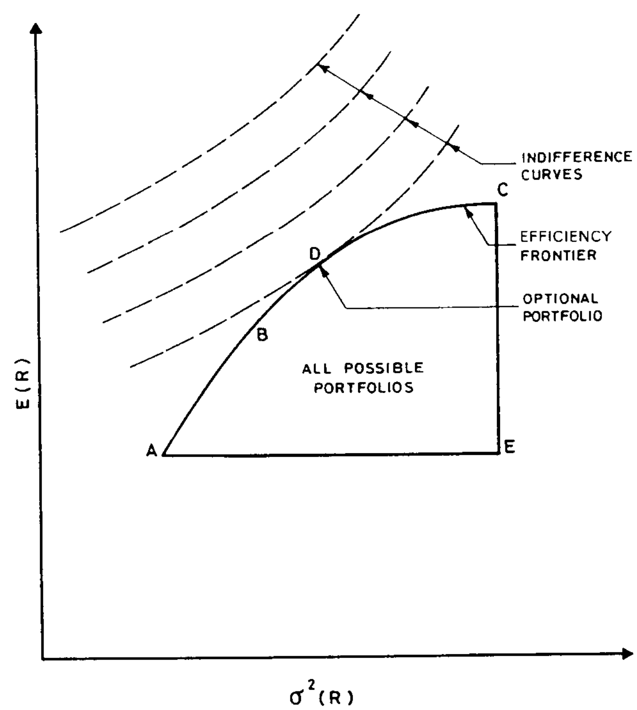

Fig 1 Portfolio analysis and selection

The optimal portfolio for an investor depends on personal preferences of expected value of return and risk. The personal preference is subjectively measured by utility values. A utility value is a measure of the personal preference of an investor towards an outcome of expected return and risk. Expected return and risk combinations of the same utility value form indifference curves as shown in *Figure 1*. The optimal portfolio is represented by the point of intersection between the efficiency frontier and the lowest possible indifference curve, i.e., point D in *Figure 1*. More details about portfolio theory are available elsewhere[1–8,24,25].

Correlation between projects

As discussed earlier, several factors cause correlation between projects. To estimate the coefficient of correlation for a pair of projects, relevant factors must be considered by one or more expert. The strength of correlation due to each factor and the importance of each factor need to be estimated. Moreover, the significance of the judgement of each expert need to be evaluated.

The coefficient of correlation can be estimated by: (1) identifying the relevant factors, (2) determining the correlation caused by each factor and the weight that each factor should be given, and (3) determining a weighted average coefficient of correlation between the two projects[8]. The main shortcoming of the weighted average method is that it does not consider the different types of uncertainty; especially the judgement uncertainty. The judgement uncertainty adds up to the total uncertainty in the return of a portfolio, and the risk of a portfolio. In addition, the weighted average method does not consider the significance of the judgement of experts.

To illustrate the significance of project correlation in portfolio analysis, consider an investor with $X to invest. Two securities are available to him. The percentages of X the investor would invest in the two securities are P_1 and P_2. The budgetary constraint is $P_1 + P_2 = 1.0$. The expected value and the variance of the rate of return of the portfolio can be evaluated using equations 2 and 3 as follows.

$$E(R) = P_1 E(S_1) + P_2 E(S_2) \tag{4}$$

$$\sigma^2(R) = P_1^2 \sigma^2(S_1) + (1 - P_1)^2 \sigma^2(S_2) + 2P_1(1 - P_1)\rho_{12}\sigma(S_1)\sigma(S_2) \tag{5}$$

Assuming that $E(S_1) = 10\%$, $E(S_2) = 15\%$, $\sigma(S_1) = 3\%$ and $\sigma(S_2) = 5\%$, the values of $E(R)$ and $\sigma(R)$ can be determined for different values of P_1 and P_2, and $\rho_{12} = 0$, $+0.5$, $+1$, -0.5 and -1. The calculations are shown in *Table 1* and the results are plotted in *Figure 2*. From *Table 1* and *Figure 2*, the following can be observed: (1) positive coefficient of correlation increased the risk; (2) negative coefficient of correlation reduces the risk; (3) a portfolio which consists of 62% and 38% of the $X invested in projects 1 and 2, respectively, and

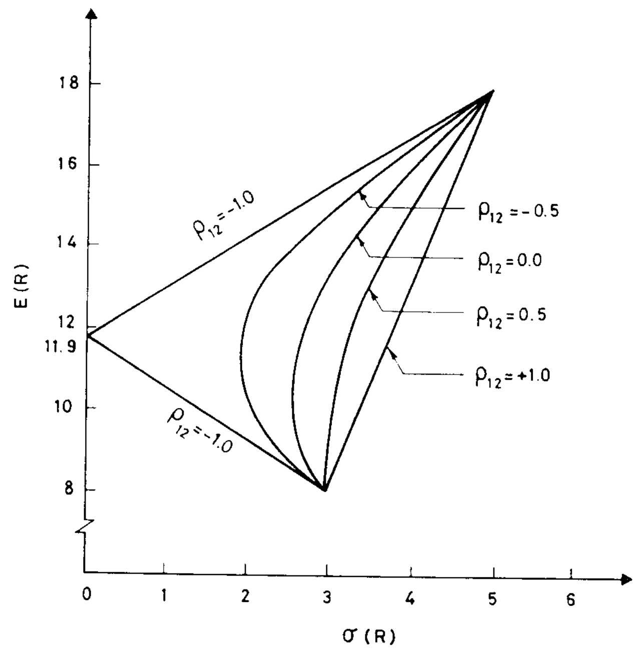

Fig 2 Significance of project correlation

Table 1 Portfolio analysis of two projects

Proportion Invested in Project No. 1, p_1	0.0	0.2	0.4	0.5	0.6	0.8	1.0
Proportion Invested in Project No. 2, p_2	1.0	0.8	0.6	0.5	0.4	0.2	0.0
$E(R)$, % For all values of ρ_{12}	15.0	14.0	13.0	12.5	12.0	11.0	10.0
$\sigma(R)$, % For $\rho_{12} = 0$	5.00	4.04	3.23	2.92	2.69	2.60	3.00
$\sigma(R)$, % For $\rho_{12} = 0.5$	5.00	4.33	3.75	3.50	3.29	3.03	3.00
$\sigma(R)$, % For $\rho_{12} = +1$	5.00	4.60	4.20	4.00	3.80	3.40	3.00
$\sigma(R)$, % For $\rho_{12} = -0.5$	5.00	3.73	2.61	2.19	1.91	2.09	3.00
$\sigma(R)$, % For $\rho_{12} = -1$	5.00	3.39	1.80	1.01	0.19	1.40	3.00

Table 2 The judgement of the experts

		Expert no. 1	Expert no. 2
Significance of Judgement, α Assigned by the Analyst		1.0	1.5
Factor no. 1 (Geographical effects)	Importance of the factor, I_1	high = {0.8|0.5 + 0.9|0.9 + 1|1}	high = {0.8|0.5 + 0.9|0.9 + 1|1}
	Level of correlation due to the factor, C_1	very high = {0.8|0.25 + 0.9|0.81 + 1|1}	high = {0.8|0.4 + 0.9|0.9 + 1|1}
Factor no. 2 (Type of project)	Importance of the factor, I_2	low = {0|1 + 0.1|0.8 + 0.2|0.4}	very low = {0|1 + 0.1|0.81 + 0.2|0.25}
	Level of correlation due to the factor, C_2	low = {0|1 + 0.1|0.8 + 0.2|0.4}	very low = {0|1 + 0.1|0.81 + 0.2|0.25}

However, this does not mean that the real risk taken by the investor is zero because of the presence of judgement uncertainty in estimating the coefficient of correlation.

Project correlation increases or reduces the expected return of a portfolio. Therefore, an accurate estimate of the coefficient of correlation, and a proper consideration of judgement uncertainty are essential in portfolio analysis. The proposed method of estimating project correlation and judgement uncertainty can be illustrated with the help of the following example.

Example

Consider the following two construction projects: (1) a concrete office building and (2) a concrete highway bridge; to be constructed in the same city. The coefficient of correlation for the two projects is estimated based on the judgement of two experts and considering only two causal factors for correlation; for example, (a) geographical factors and (b) type of project. This example is to illustrate the proposed method. The number of experts and factors in the example are limited to two each in order to reduce the manual calculations to a manageable level, show all the calculations and achieve the objectives of the example. Assume that the importance of each causal factor and level of project correlation due to each factor based on the judgement of the two experts as shown in *Table 2*. The judgement of the experts is expressed in linguistic terms. The linguistic terms are translated into mathematical measures (fuzzy sets) as shown in *Table 2*. The concepts of translating the linguistic terms into mathematical measures were proposed by Ayyub and Haldar[9,10], Blockley[26] and Yao[27]. These translations are quite logical from a practical point of view.

The information in *Table 2* can be combined according to the following proposed method:

1. The judgement of the experts are modified using the degree of significance of judgement α and the following equation which represents a general form for a fuzzy set A. The modified judgements are shown in *Table 3*.

$$A^{\alpha} = x_1|(\mu_A(x_1))^{\alpha}x_1 + x_2|(\mu_A(x_2))^{\alpha}x_2 + \cdots + x_m|(\mu_A(x_m))^{\alpha}x_m\} \quad (6)$$

where x_i, $(i = 1, 2, \ldots, m)$ = elements or grade levels of the underlying variable of set A, | = delimiter, and $\mu_A(x_i)$ = the membership level of element x_i to set A.

2. The judgements of the two experts are combined using the intersection operation as defined for any two fuzzy sets A and B, by the following equation. The combined judgements are shown in *Table 3*.

$$\mu_{A \cap B}(x) = \min[\mu_A(x), \mu_B(x)] \quad (7)$$

3. The correlation between the two projects, ρ_{12}, is defined as

$$\rho_{12} = (I_1 \otimes C_1) \cap (I_2 \otimes C_2) \quad (8)$$

where I_1 and I_2 = the importance of the two factors, C_1 and C_2 = the level of correlation due to the two factors, $\otimes$ = the cartesian product, and $\cap$ = the intersection as defined by equation 7. Evaluating the operations of equation 8, the correlation between the

	Importance										
	0.	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.	0	0	0	0	0	0	0	0	0	0	0
0.1	0	0.07	0.03	0	0	0	0	0	0	0	0
0.2	0	0.03	0.03	0	0	0	0	0	0	0	0
0.3	0	0	0	0	0	0	0	0	0	0	0
0.4	0	0	0	0	0	0	0	0	0	0	0
ρ_{12} 0.5	0	0	0	0	0	0	0	0	0	0	0
0.6	0	0	0	0	0	0	0	0	0	0	0
0.7	0	0	0	0	0	0	0	0	0	0	0
0.8	0	0	0	0	0	0	0	0	0.2	0.2	0.2
0.9	0	0	0	0	0	0	0	0	0.28	0.65	0.65
1.0	0	0	0	0	0	0	0	0	0.28	0.77	1.0
ΣCol.	0	0.1	0.06	0	0	0	0	0	0.76	1.62	1.85

4. Choose from equation 9 a column which maximizes the product of the column summation given in equation 9 and the corresponding importance level[10]. The last column of equation 9 gives the maximum value of this product for the problem under consideration. Therefore, the following fuzzy subset of the project correlation, ρ_{112}, is chosen:

$$\rho_{12} = \{0.8|0.2 + 0.9|0.65 + 1.0|1.0\} \tag{10}$$

According to Zadeh[28] the probability mass function of the project correlation can be calculated as follows:

$$\text{Prob}(\rho_{12} = 0.8) = (0.2)/(0.2 + 0.65 + 1.0) = 0.108$$

$$\text{Prob}(\rho_{12} = 0.9) = 0.351 \text{ and;}$$

$$\text{Prob}(\rho_{12} = 1.0) = 0.5405 \tag{11}$$

Therefore, the mean value, $\bar{\rho}_{12}$, and the standard deviation $\sigma(\rho_{12})$ of the project correlation can be calculated as follows:

$$\bar{\rho}_{12} = 0.8(0.1081) + 0.9(0.3514) + 1.0(0.5405)$$
$$= 0.9433$$

$$\sigma^2(\rho_{12}) = 0.8^2(0.1081) + 0.9^2(0.3514) + 1^2 \times (0.5405) - (0.9433)^2 = 0.0045$$

$$\sigma(\rho_{12}) = 0.0671, \quad \text{and } \text{COV}(\rho_{12}) = 0.711 \tag{12}$$

where $\text{COV}(\rho_{12})$ = the coefficient of variation of the project correlation, that is defined as the ratio of the standard deviation to the mean value. The variance given in equation 12 is the judgement uncertainty. The judgement uncertainty can be combined with the portfolio risk, which is given by equation 5 as follows:

$$\text{Var}(R) = P_1^2\sigma^2(S_1) + (1 - P_1)^2\sigma^2(S_2) + 2P_1(1 - P_1)\rho_{12}\sigma(S_1)\sigma(S_2) + [2P_1(1 - P_1)\sigma(S_1)\sigma(S_2)]^2\,\sigma^2(\rho_{12}) \tag{13}$$

where $\text{Var}(R)$ = the total variance of the portfolio. The success in estimating project correlation depends on the assumptions used in translating the linguistic variables into fuzzy sets, i.e. *Table 2*, and the estimates of the significance level of the judgement of experts. The more this method is used and compared with actual performance of portfolios, the higher the level of success will be in choosing the proper membership values in the definition of the linguistic variable and the judgement significance levels.

In order to evaluate the sensitivity of the proposed method to membership values in the definition of the linguistic variables, the example is solved again using different membership values in defining the judgement of the experts, i.e. *Table 2*. If the values in *Table 2* are changed to the values shown in *Table 4*; then, according to the proposed method, the coefficient of correlation would be as follows:

$$\rho_{12} = \{0.8|0.13 + 0.9|0.53 + 1|1\} \tag{14}$$

$$\bar{\rho}_{12} = 0.9524, \quad \text{and } \text{COV}(\rho_{12}) = 0.0664 \tag{15}$$

Comparing these results with equations 10 and 12, it is clear that the proposed method is not sensitive to small variations in the membership values. A sensitivity analysis of the results to variations in the judgement significance levels of the experts is shown in *Table 5*, and *Figures 3* and *4*. In general, the mean value of the coefficient of correlation is sensitive to the judgement significance levels if one of the levels is larger than 1.0 and the other level is smaller than 1. The standard deviation of the coefficient of correlation is sensitive to the judgement significance levels.

Summary and Conclusions

The use of portfolio theory allows the consideration of correlation between projects and may rationalize the selection of the projects and capital budgeting. The correlation between projects is a very important parameter in portfolio selection. Positive coefficient of correlation increases the risk for a portfolio, while negative coefficient decreases the risk. On the other hand, the expected return of a portfolio may increase or decrease due to project correlation. Therefore, an accurate estimate of the coefficient of correlation and a proper consideration of judgement uncertainty are essential in portfolio analysis.

The correlation between projects is due to several causal factors. These factors are of different importance and contribution that can be estimated based on experience and judgement. Therefore, the resulting additional

Table 3 The modified judgement of the experts

		Expert no 1	Expert no. 2	Combined judgement intersection
Factor no. 1 (Geographical factors)	Importance of the factor I_1	high = {0.8|0.4 + 0.9| + 0.81 + 1|1}	high = {0.8|0.28 + 0.9|0.77 + 1|1}	{0.8|0.28 + 0.9|0.77 + 1|1}
	Level of correlation due to the factor, C_1	very high = {0.8|0.2 + 0.9|0.73}	high = {0.8|0.28 + 0.9|0.65}	{0.8|0.2 + 0.9|0.65 + 1|1}
Factor no. 2 (Type of project)	Importance of the factor, I_2	low = {0|0 + 0.1|0.08}	very low = 0|0 + 0.1|0.07}	{0|0 + 0.1|0.07 + 0.2|0.03}
	Level of correlation due to the factor C_2	low = {0|0 + 0.1|0.08 + 0.2|0.08}	very low = {0|0 + 0.1|0.07 + 0.2|0.03	{0|0 + 0.1|0.07 + 0.2|0.03}

Table 4 Sensitivity analysis—The judgement of the experts

		Expert no. 1	Expert no. 2
Significance of judgement, α (Assigned by the analyst)		1.0	1.5
Factor no. 1 (Geographical factors)	Importance of the factor, I_1	high = {0.8|0.6 + 0.9|0.8 + 1|1}	high = {0.8|0.6 + 0.9|0.8 + 1|1}
	Level of correlation due to the factor, C_1	very high = {0.8|0.36 + 0.9|0.64 + 1|1}	high = {0.8|0.3 + 0.9|0.7 + 1|1}
Factor no. 2 (Type of project)	Importance of the factor, I_2	low = {0|1 + 0.1|0.7 + 0.2|0.3}	very low = {0|1 + 0.1|0.7 + 0.2|0.2}
	Level of correlation due to the factor, C_2	low = {0|1 + 0.1|0.7 + 0.2|0.3}	very low = {0|1 + 0.1|0.7 + 0.2|0.2}

Judgement Significance of Expert no. 1, α_1		Judgement of Significance of Expert no. 2, 0.5	1.0	1.5	2.0
0.5	ρ_{12}	0.9272	0.9335	0.9433	0.9509
	$\sigma(\rho_{12})$	0.0742	0.0721	0.0671	0.0632
	$COV(\rho_{12})$	0.08	0.0772	0.0711	0.0665
1.0	$\bar{\rho}_{12}$	0.9414	0.9417	0.9433	0.9509
	$\sigma(\rho_{12})$	0.0671	0.0671	0.0671	0.0632
	$COV(\rho_{12})$	0.0713	0.0712	0.0711	0.0665
1.5	$\bar{\rho}_{12}$	0.9511	0.9511	0.9514	0.9535
	$\sigma(\rho_{12})$	0.0616	0.0616	0.0592	0.0608
	$COV(\rho_{12})$	0.0648	0.0648	0.0622	0.0638
2.0	$\bar{\rho}_{12}$	0.9573	0.9575	0.9580	0.9583
	$\sigma(\rho_{12})$	0.0608	0.0566	0.0539	0.0548
	$COV(\rho_{12})$	0.0635	0.0591	0.0562	0.0572

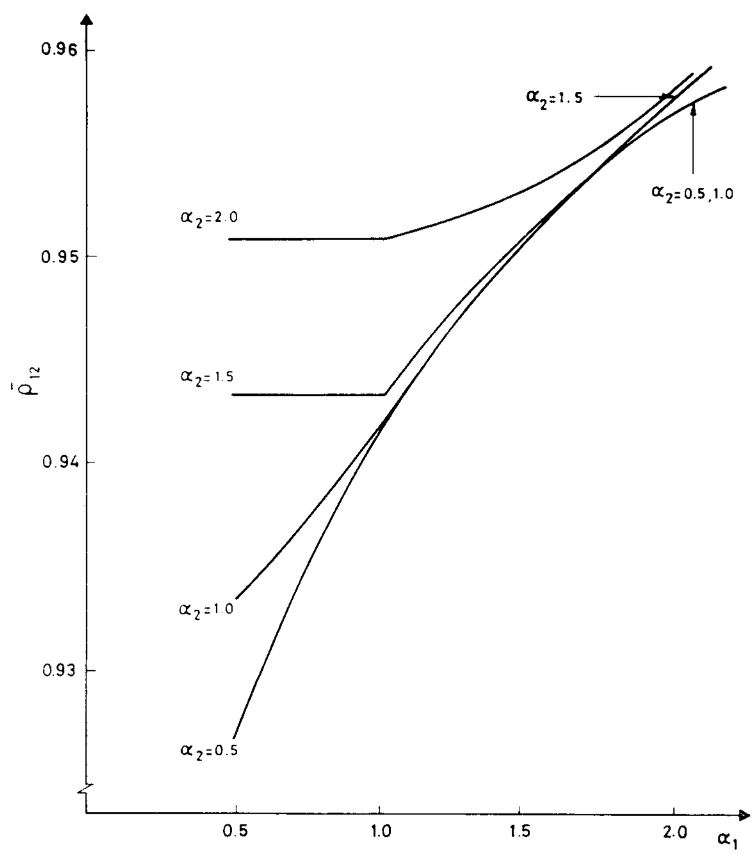

Fig 3 Sensitivity analysis—Mean coefficient of correlation

uncertainty due to judgement should be considered in the decision-making process. Experience and judgement of an expert can be easily expressed in linguistic rather than mathematical terms. Classical portfolio theory fails to incorporate linguistic information. The linguistic terms can be translated into mathematical measures using fuzzy set theory. A method by which the project correlation may be estimated based on experience and judgement is proposed. The method utilizes fuzzy set theory to estimate the coefficient of correlation and the judgement uncertainty. Then, the total risk of a portfolio can be estimated.

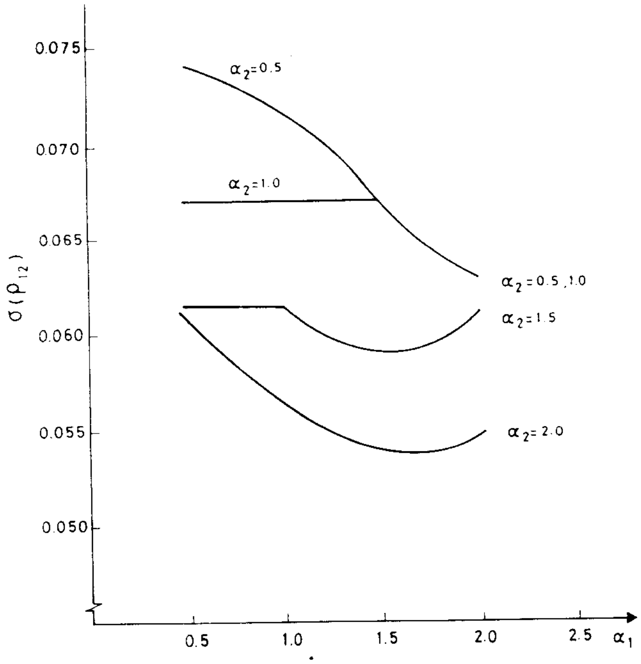

Fig 4 Sensitivity analysis—Standard deviation of coefficient of correlation

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