Fuzzy Sets and Systems 48 (1992) 1–14
North-Holland

Invited Review

Multi-criteria ranking of components according to their priority for inspection

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Received July 1991
Revised October 1991

Abstract: Inspection can play a significant role in reducing the likelihood of unexpected structural failures. However, for many critical components and systems that are required to maintain pressure boundary integrity or that are subjected to severe service conditions, inspection requirements for these vital components are either established based upon prior experience and engineering judgment or are non-existent. Most inspection requirements or guidelines, if they exist, are usually established with only an implicit consideration of risk. Recent catastrophic structural failures over the past decade highlight the societal need to relate more explicitly risk-based methods and uncertainty with inspection programs. In this study, fuzzy multi-criteria risk-based ranking methodology with uncertainty evaluation and propagation was developed for the purpose of developing inspection strategies. The methodology results in establishing priority ranking lists for components where actions need to be taken accordingly. The ranking priority list for inspection purposes was based on the assessments of the probabilities of failure, resulting consequences, expected human and economic risks and the uncertainties associated with these assessments. The fuzzy-based multi-criteria decision making method was utilized for prioritizing the components of a system for inspection purposes. Interval analysis and logic diagram techniques were utilized to propagate uncertainties for the process of assessing the magnitude of failure probabilities, consequences and risk due to failure.

Keywords: Fuzzy sets; risk-based inspection; consequences; interval analysis; probability; uncertainty.

1. Introduction

The purpose of this study is to utilize fuzzy-based multi-criteria decision making methods for prioritizing the components of a system for inspection purposes. The priority of an element for inspection can be based on several criteria that include, for example, its probability of failure, magnitude of human fatality and property damage due to failure, and the economic and human risks considering the underlying uncertainty associated with their estimates. Other criteria can be included, such as the availability of inspection crews and equipment, management of inspection tasks, availability of sites, production schedules, and target dates specified by codes and contracts. The primary reasons for the usefulness of using the fuzzy sets in handling multi-criteria decision making are (1) the ability to represent the underlying criteria; (2) the availability of convenient forms for combining the criteria, where the criteria can be vaguely defined, i.e., fuzzy, as well as precisely defined; and (3) the realistic means of including different degrees of importance for the criteria.

Ibrahim and Ayyub [1, 2] discussed several criteria for inspection priority ranking of a set of components. However, the ranking procedure did not include all the criteria in its process at the same time. It utilizes only one criterion at a time. Combining the resulting rankings that correspond to all the criteria can be a highly subjective and difficult task. The aim herein is to develop a method for ranking components such that a higher rank for a component represents a higher combinational level of its probability of failure, magnitude of fatality and damage, human and economic risks, and the uncertainties in the estimates.

2. Methodology

Assume that $X = \{X_j: j = 1, \ldots, n\}$ represents a set of n alternatives or candidates and $C = \{C_i: i = 1, \ldots, m\}$ represents a fuzzy set of m criteria, such that $C_i(X_j) \in [0, 1]$ indicates the degree to which an alternative X_j satisfies a criterion C_i [3, 4]. The objective can now be stated as among these alternatives the one which best satisfies all the criteria needs to be selected. If for each alternative a number to indicate how well it satisfies the criteria as a group is assigned, then the alternative which has the highest value would be selected as the candidate. Let R be the decision function where $R[X_j]$ is the degree to which X_j satisfies the set of criteria C and X_j with the highest $R[X_j]$ is considered to be the best candidate. Assigning a fuzzy set for each C_i that indicates how well the set of alternatives X satisfies this criterion and using the linguistic connection 'and' [5], the decision function R can be determined as

$$R = C_1 \cap C_2 \cap \cdots \cap C_m \tag{1}$$

where $\cap$ is the intersection operator between fuzzy sets. The intersection operator on two fuzzy sets C_1 and C_2 results in a fuzzy set with membership grade values that are determined as the minimum membership grade value of the corresponding elements of the two fuzzy sets C_1 and C_2 [6]. Alternatively, for example, the algebraic product of the membership grade value of the corresponding elements of the two fuzzy sets C_1 and C_2 can be used to represent a softer 'and' operation. One of the main advantages of using the minimum approach rather than the product operation is the ability of having inter-alternative comparisons of the membership values of the fuzzy-based criteria. Therefore, in this study, the minimum approach was adopted.

In order to select one candidate from the set of candidates, the maximum operation over R should be performed, i.e., the selected alternative is the one which maximizes the minimums over all the criteria. This computational procedure can be represented by

$$\max_{X_j} (R) = \max(C_1 \cap C_2 \cap \cdots \cap C_m) \tag{2}$$

in which $j = 1, 2, \ldots, n$. In order to illustrate the methodology of comparing and choosing from a set of alternatives, the following example is presented. Consider four employment candidates for a job: X_1, X_2, X_3 and X_4, and the following three criteria that need to be satisfied in the selected candidate: (1) the candidate should be technically and educationally qualified, (2) the candidate should be experienced and (3) the candidate should be able to communicate well. Assuming that the candidates were evaluated on a subjective basis with respect to the three criteria as follows:

$$C_1 = \left\{\frac{0.5}{X_1}, \frac{0.7}{X_2}, \frac{0.3}{X_3}, \frac{0.6}{X_4}\right\},$$

$$C_2 = \left\{\frac{0.5}{X_1}, \frac{0.4}{X_2}, \frac{0.8}{X_3}, \frac{0.4}{X_4}\right\},$$

$$C_3 = \left\{\frac{0.2}{X_1}, \frac{0.01}{X_2}, \frac{0.6}{X_3}, \frac{0.9}{X_4}\right\},$$

where the notation means that the degree of belief that candidate X_1 satisfies criterion C_1 is 0.5, i.e., the membership value of element X_1 in the fuzzy criterion set C_1 is 0.5, etc. Using equation (1), the ranking decision set R can be determined as

$$R = \left\{\frac{0.2}{X_1}, \frac{0.01}{X_2}, \frac{0.3}{X_3}, \frac{0.4}{X_4}\right\}.$$

The best candidate for the job is X_4 since X_4 satisfies the criteria with the highest value of 0.4. The next candidate in rank is X_3 with a score 0.3 and so forth. Therefore, the method can be summarized by selecting for each alternative X_j its smallest membership value in the criteria that results into the ranking decision set R, and then the best alternative can be selected as the one with the highest membership in the ranking decision set.

The model R of the ranking decision set as given by Equation (1) does not account for a varying degree of importance of each criterion. If a particular criterion is of great importance, it would be unlikely to select any alternative that has a relatively small membership value in this criterion. Therefore, alternatives that are of low membership grade levels in some important criteria should have low membership values in R [7–10], consequently minimizing their chance of being selected as the best alternative. Since the membership value for each candidate in R is determined by its minimum membership value in all the criteria, the likelihood of selecting alternatives with low membership levels in some important criteria can be reduced by decreasing their membership levels in the ranking decision set R. This objective can be achieved by raising the fuzzy sets $C_1, C_2, \ldots, C_m$ to some powers. This means that by assigning to each criterion a scalar number $\alpha \geqslant 0$, indicative of its importance, the desired effect can be obtained. The more important the criteria the higher α. In this case equations (1) and (2) can be modified as follows:

$$R = C_1^{\alpha_1} \cap C_2^{\alpha_2} \cap \cdots \cap C_m^{\alpha_m}, \tag{3}$$

$$\max(R) = \max_{X_j} (C_1^{\alpha_1} \cap C_2^{\alpha_2} \cap \cdots \cap C_m^{\alpha_m}), \tag{4}$$

where $\alpha_i \geqslant 0$, $i = 1, 2, \ldots, m$. Since the membership values of the criterion are always within the range $[0, 1]$, therefore, as α gets bigger C^α gets smaller, closer to zero, whereas as $\alpha \to 0$, $C^\alpha \to 1$, gets bigger. The effect of the approach can be summarized as follows: The membership values of all the criteria having little importance, e.g., $\alpha < 1$, become larger, and those in criteria having more importance, e.g., $\alpha > 1$, become smaller. Therefore, the membership values of the decision set R, which is the minimum membership value of each X_j over all the criteria, are determined by the important criteria, as it should be. This trend is shown in Figure 1. Furthermore, this operation makes alternatives that are weak in some important criteria become even less appealing as potential solutions.

In order to establish these importance factors, a hierarchal system should be formed such that each candidate is first rated or ranked on its ability to satisfy each of the criteria and then the criteria are rated as to their importance. In order to obtain the factors $\alpha_i, i = 1, 2, \ldots, m$, by which the importance of each criterion is measured, a methodology developed by Saaty [11] was used and is briefly discussed in this section.

Saaty [11] has developed a procedure for obtaining a ratio scale for a group of elements based upon a paired comparison of the elements. For m criteria $C_1, C_2, \ldots, C_m$, the objective of this analysis is to construct a scale rating of these criteria as to their importance with respect to each other, as seen or judged by a decision-maker. The quantified judgments on pairs of criteria C_i and C_j, are represented by an m-by-m matrix,

$$A = \{a_{ij}: i = 1, 2, \ldots, m; j = 1, 2, \ldots, m\}.$$

The entries of A are defined by the following rules:

Rule 1 If $a_{ij} = \beta$ then $a_{ji} = 1/\beta$, where $\beta \neq 0$.

Rule 2. If C_i is judged to be of equal relative importance as C_j, then $a_{ij} = a_{ji}$. All diagonal elements $a_{ii} = 1$ for $i = 1, 2, \ldots, m$.

Thus the matrix A has the form

$$A = \begin{bmatrix} 1 & a_{12} & \cdots & a_{1m} \\ \frac{1}{a_{12}} & 1 & & a_{2m} \\ \vdots & & \ddots & \vdots \\ \frac{1}{a_{1m}} & \frac{1}{a_{2m}} & \cdots & 1 \end{bmatrix}. \tag{5}$$

The values for β can be taken from Table 1 which was constructed by Saaty. The upper value of comparison was limited to 9 due to the human ability to make effective quantitative distinctions to five attributes: equal ($\beta = 1$), weak ($\beta = 3$), strong ($\beta = 5$), very strong ($\beta = 7$) and absolute ($\beta = 9$). Compromises between adjacent attributes ($\beta = 2, 4, 6$ and 8) can be used where a greater precision is needed. Saaty [11] has shown that the eigenvector W corresponding to the maximum eigenvalue $\lambda_{\max}$ of A is a cardinal ratio scale for the compared criteria.

It should be noted that the matrix A is reciprocal as $a_{ij} = 1/a_{ji}$, and only $m - 1$ pairwise comparison judgments are needed to form a consistent matrix. For example, in the case of the three criteria C_1, C_2 and C_3 where C_1 is 3 times more important than C_2 and C_1 is 6 times more important than C_3, then $C_1 = 3C_2$ and $C_1 = 6C_3$. It should follow that $3C_2 = 6C_3$ or $C_2 = 2C_3$ and $C_3 = 0.5C_2$. In this case the matrix A is given by

$$A = \begin{bmatrix} 1 & 3 & 6 \\ \frac{1}{3} & 1 & 2 \\ \frac{1}{6} & \frac{1}{2} & 1 \end{bmatrix}.$$

There are 3 eigenvalues or roots to this matrix, $\{0, 0, 3\}$. In general for any m-by-m

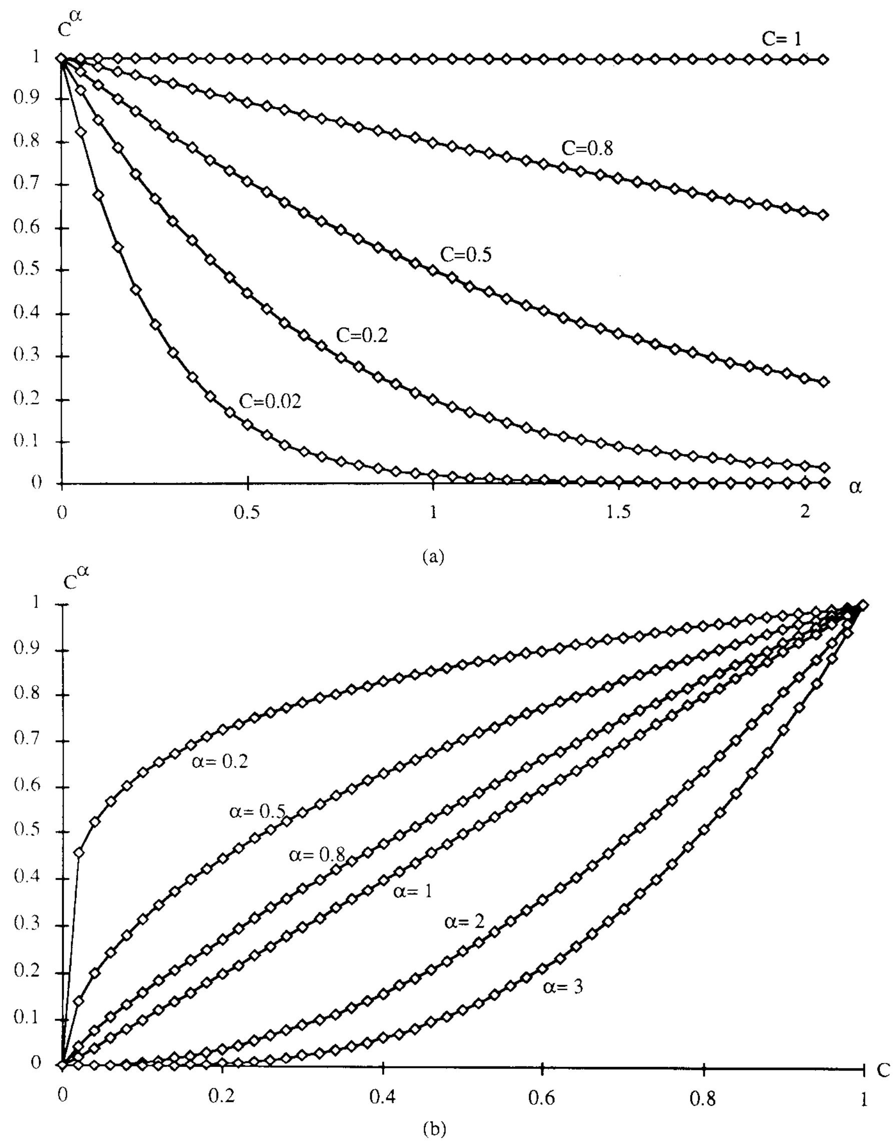

Fig. 1. Effect of importance factor on criteria.

Table 1. Importance β values

Intensity of importance	Definition
1	Equal importance
3	Weak importance of one over another
5	Essential or strong importance
7	Very strong or demonstrated importance
9	Absolute importance
2, 4, 6, 8	Intermediate values between two adjacent scale judgments

positive reciprocal consistent matrix, there are m eigenvalues where $m-1$ of them are equal to 0 and the last one is equal to m, and therefore $\lambda_{max} = m$. On the other hand, for any m-by-m non-consistent matrix, $\lambda_{max} \geq m$. A measure of consistency is given by the consistency index CI as follows:

$$\mathrm{CI} = \frac{\lambda_{max} - m}{m-1}. \tag{6}$$

In order to convert the relative paired comparison of the criteria, i.e., matrix A, to the scale rating factor α for each criterion, the following procedure should be followed. First the maximum eigenvalue λ_{max} of the A matrix should be obtained. Secondly, the eigenvector W corresponding to λ_{max} should be obtained such that

$$AW = \lambda_{max} W \tag{7}$$

where $W = \{w_1, w_2, \ldots, w_m\}$. Thirdly, the eigenvector W should be normalized to obtain $\hat{W} = \{\hat{w}_1, \hat{w}_2, \ldots, \hat{w}_m\}$, such that $\sum_{i=1}^{m} \hat{w}_i = 1$. The normalization process is performed as follows:

$$\hat{w}_i = w_i \Big/ \sum_{i=1}^{m} w_i \tag{8}$$

where $\hat{w}_i$ is the normalized eigenvector value, w_i is the eigenvector value, $i = 1, \ldots, m$, and m is the number of rows in W which represents the total number of compared criteria. Finally, the scale rating α for each criterion is obtained as the product of the normalized eigenvector $\hat{W}$ and the total number of the criteria m. The last two steps are performed in order to ensure that if all the criteria are equally important, i.e., $\beta = 1$, then the scale factors α for all the criteria are equal and each $\alpha_i = 1$, and therefore C^{α} is equal to C.

To numerically illustrate this procedure, the scale rating factor α was obtained for the three criteria C_1, C_2 and C_3. Recalling that their relative importance was given by the matrix A and the corresponding $\lambda_{max} = 3$. The eigenvector corresponding to $\lambda_{max} = 3$ was determined based on equation (7) such that $A\ W_3 = \lambda_{max}\ W_3$. The eigenvector W_3 was determined to be

$$W_3 = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \\ 1 \end{bmatrix}.$$

The normalized eigenvector $\hat{W}_3$ was obtained using equation (8) and is given by

$$\hat{W}_3 = \begin{bmatrix} \frac{6}{9} \\ \frac{2}{9} \\ \frac{1}{9} \end{bmatrix}.$$

Multiplying the normalized eigenvector $\hat{W}$ by m, i.e., 3 in this example, results in the following scale factor α:

$$\alpha = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} 2 \\ \frac{2}{3} \\ \frac{1}{3} \end{bmatrix}.$$

Substituting the α factor in equation (3) results in

$$C_1^2 = \left\{\frac{0.25}{X_1}, \frac{0.49}{X_2}, \frac{0.09}{X_3}, \frac{0.36}{X_4}\right\},$$

$$C_1^{2/3} = \left\{\frac{0.63}{X_1}, \frac{0.54}{X_2}, \frac{0.86}{X_3}, \frac{0.54}{X_4}\right\},$$

$$C_3^{1/3} = \left\{\frac{0.58}{X_1}, \frac{0.22}{X_2}, \frac{0.84}{X_3}, \frac{0.97}{X_4}\right\},$$

Then according to equation (3), the decision set R is

$$R = \left\{\frac{0.25}{X_1}, \frac{0.22}{X_2}, \frac{0.09}{X_3}, \frac{0.36}{X_4}\right\}.$$

Therefore, according to equation (4), the best candidate is X_4, the next in rank is X_1, the third in rank is X_2 and the last is X_3. A comparison summary between the ranking of the candidates is shown in Table 2. In this table, the differences between the ranks for the candidates based on criteria that are equally important and the case of unequally important criteria are shown. Therefore, utilizing different importance measures for the criteria results in altering the ranking of the candidates. For example, with equally important criteria the candidate X_3 has a rank of 2, while with unequally important criteria the candidate X_3 has a rank of 4.

Table 2. Summary of multi-criteria ranking

Candidates	Degree of satisfaction according to: equal importance for criteria			equal importance for decision set	unequal importance for criteria			unequal importance for decision set	Rankings according to: equal importance for criteria	Rankings according to: unequal importance for criteria
	C_1	C_2	C_3	R	C_1^2	$C_2^{0.667}$	$C_3^{0.333}$	R		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
X_1	0.5	0.5	0.2	0.2	0.25	0.63	0.58	0.25	3	2
X_2	0.7	0.4	0.01	0.01	0.49	0.54	0.22	0.22	4	3
X_3	0.3	0.8	0.6	0.3	0.09	0.86	0.84	0.09	2	4
X_4	0.6	0.4	0.9	0.4	0.36	0.54	0.97	0.36	1	1

3. Interval analysis

One way to model and propagate uncertainty is by using the interval analysis technique. In the interval analysis, uncertainty is propagated by modeling input quantities as intervals and using the mathematics of intervals, resulting in interval output quantities. An interval value is a range of numbers with lower and upper limits that is assigned to some or all of the input quantities in a risk model to account for uncertainties in these parameters. For example, $I = [a, b]$ is an interval value ranging from a lower limit (a) to an upper limit (b). The operations of interval analysis can be used to determine uncertainty propagation in the analytical process of risk estimation [1]. The algebraic operations of interval values are extensions of operations on real numbers [12, 13]. For example, if $I_1 = [a, b]$ and $I_2 = [c, d]$ are two interval values where, $d > c$ and $b > a$, then the following operations are defined:

$$[a, b] \times 0 = 0, \tag{9}$$

$$[a, b] + [c, d] = [a + c, b + d], \tag{10}$$

$$[a, b] - [c, d] = [a - d, b - c], \tag{11}$$

$$[a, b] \times [c, d] = [ac, bd], \tag{12}$$

$$[a, b]/[c, d] = [a, b] \times [1/d, 1/c] \quad \text{if } 0 \notin [c, d]. \tag{13}$$

These operations are considered a special case of fuzzy arithmetic [14]. In order to rank two quantities where their magnitudes are given by interval values, for example $I_1 = [a, b]$ and $I_2 = [c, d]$, the following logic operations are suggested:

$$\text{IF } d > b \text{ THEN } I_2 > I_1, \tag{14}$$

$$\text{IF } d = b \text{ AND } c > a \text{ THEN } I_2 > I_1, \tag{15}$$

$$\text{IF } d = b \text{ AND } c = a \text{ THEN } I_2 = I_1, \tag{16}$$

$$\text{ELSE } I_2 < I_1, \tag{17}$$

where $I_2 <$ (or $>$) I_1 means that the quantity I_1 is ranked at a higher (or lower) severity in magnitude level than the quantity I_2, and $I_2 = I_1$ means that the quantities I_1 and I_2 are ranked at the same level [14].

4. Logic diagrams and uncertainty in consequences

Risk assessment requires the characterization of hazardous events, estimation of their probability of occurrence and the consequence associated with their occurrence. If every event that is being analyzed has the same consequence level, then the probability of an event is the only necessity to characterize the risk of the event's occurrence. If the events have different consequence levels, then the events' probabilities are not enough, and an assessment of the consequences is also necessary to describe the risk associated with the events [15].

In certain situations, a failure of a component might not result in any consequences beyond the repair or replacement cost of the component.

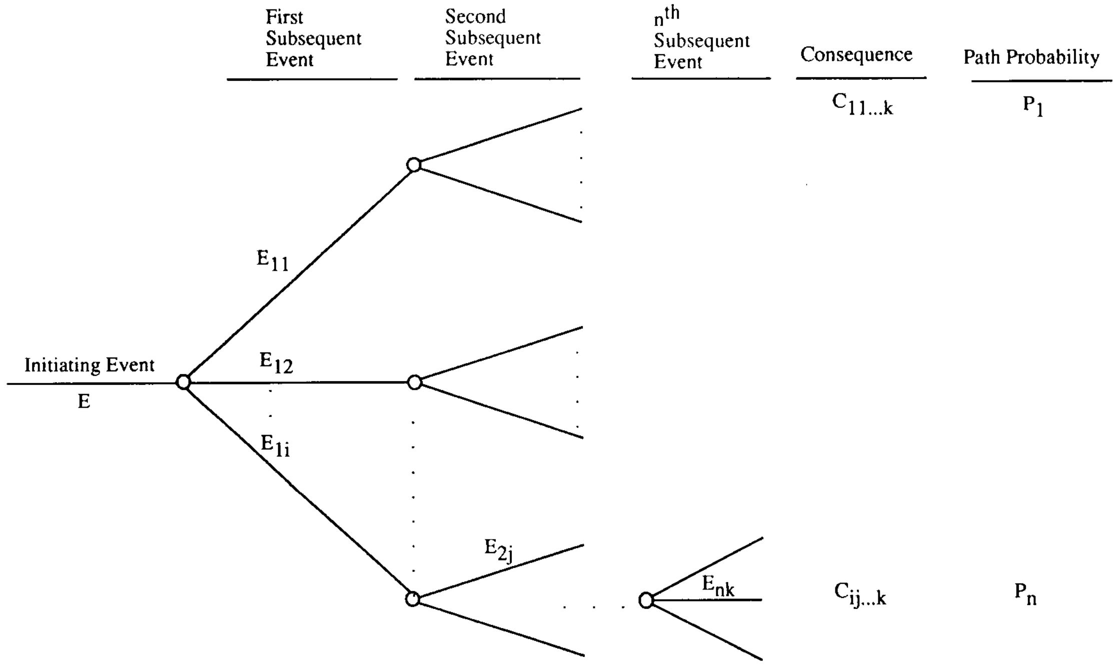

Fig. 2. Event tree model.

This can be due to the presence of redundant components. In such cases, the failure of a component shifts the load (or demand) to other redundant components, possibly resulting in the survival of the system. Therefore, consequences of significant magnitudes due to a sequence of component failures, i.e., path of failure can be prevented. Generally, the failure of a component can lead to sequences of component failures resulting in several potential failure paths. As a result, each failure path has some consequence level. The identification of these paths can be accomplished systematically and effectively through the use of logic diagrams, e.g., event trees. The event tree model is an inductive logic technique that can be used to identify potential chains of events necessary for the occurrence of accidents or failures [16]. It is a powerful technique to determine the resultant consequences and their probabilities of occurrence [17].

A general event tree model is shown in Figure 2 with an initiating event, E, and a number of possible consequences, $C_{ij\cdots k}$. According to this model, a particular consequence depends on the subsequent events along its path following the initiating event. Given an initiating event, e.g., a component failure, there may be several 'first subsequent events' that would follow. These subsequent events are mutually exclusive, i.e., the occurrence of one event precludes the occurrences of other events. The mutual exclusion principle is applicable to the case of failure or survival of a component. Each path in the event tree represents a specific sequence of subsequent events, resulting in a particular consequence. The probability associated with the occurrence of a specific path due to failure of a component is the product of the conditional probabilities of all the events on that path. As a result, each path represents a possible consequence with probability of occurrence and estimated magnitudes of human fatality and property damage. Due to the number of available paths which can occur, i.e., alternatives, ambiguity type of uncertainty exists. In this case, the uncertainty associated with the estimated consequences as a result from the component failure can be measured using U-uncertainty [18–20]. The U-uncertainty is given by

$$U = \sum_{i=1}^{k} (\rho_i - \rho_{i+1}) \log_2 \rho_i \tag{18}$$

where ρ_i is the possibility of occurrence of path i ($i = 1$ to the total number of paths k) and $\rho_i \geqslant \rho_{i+1}$. Thus as the possibility distribution ρ of the consequences approaches a state of equally possible or the number of paths increases the measured uncertainty increases. In this case, the possibility of occurrence of a path is considered to be equal to its probability. This assumption is consistent with the principle that any probable event is possible and not vice versa. Therefore, the U-uncertainty encountered in the possible paths as a result of the failure of any component can be computed using equation (18). The magnitude of the consequences associated with the failure of the component can be estimated as the interval covering all the consequences of all paths [20]. This can be expressed as:

$$\text{consequence interval} = [\min M_i, \max M_i] \qquad (19)$$

where M_i is the magnitude of consequence of path i, and $i = 1$ to k.

5. Applications

In this section, the methodology of multi-criteria decision making was applied to rank components according to their priority for inspection purposes. The priority rankings for inspection purposes of the components were established based on their probability of failure, magnitude of fatality, magnitude of damage, economic risk, human risk and uncertainty associated with the estimation of the consequences of failure. The notations C_1, C_2, C_3, C_4, C_5 and C_6 are used in this section for these criteria, respectively. The rankings were not based on a single criteria at a time [2], but were based on a combination of the above criteria.

A system of 9 components is shown in Table 3. For each component, the probability of failure was expressed in the form of an interval as shown in columns 2 and 3 for the lower and upper values, respectively. The magnitudes of fatality and damage were assessed as intervals using the event tree diagram and equation (19). These consequence intervals are shown in columns 4, 5, 6 and 7 for the lower and upper boundaries for fatality and damage, respectively. The risk of failure was modeled as the product of the probability of failure and the corresponding component consequences. The resulting estimates of human and economic component risks were expressed in the form of intervals. The mathematical operation that was used for this purpose is given by equation (12). The risk intervals are shown in columns 8, 9, 10 and 11 for the human and economic risks, respectively. The assessed uncertainties in the process of estimating the consequences were calculated based on the event tree analysis and equation (18) for each component. The resultant U-uncertainties are expressed as intervals since the probability of failure was expressed in intervals. The lower and upper boundaries for the U-uncertainty for each component are shown in columns 12 and 13, respectively.

The objective herein is to rank the candidates, i.e., components $X_1, X_2, \ldots, X_9$, such that a higher inspection rank for a component reflects a higher combinational level for the component based on the 6 criteria $C_1, C_2, \ldots, C_6$.

5.1. Equally-important criteria

The fuzzy set $C_i(X)$, $i = 1, 2, \ldots, 6$, is established by representing the degree to which components X_j, $j = 1, 2, \ldots, 9$, satisfy the criterion C_i. The values assigned to the components can be based on the probabilities of failure, fatality and damage magnitudes, human and economic risks, and uncertainty estimates. For example, consider the nine components of the system in Table 3. The degree of belief that component X_1 satisfies the probability of failure criterion $\hat{C}_1$ can be considered of the same magnitude as the probability of failure. In this case, the degrees of belief were represented by interval estimates. For example, the degree of belief that component X_1 satisfies the probability of failure criterion $\hat{C}_1$ is [1.00 E −13, 11.00E − 07]. In an equation format, the degree of belief for $\hat{C}_1(X)$ can be expressed as

$$\hat{C}_1(X) = \left\{ \frac{(1.00\,\text{E}-13, 1.00\,\text{E}-07)}{X_1}, \right.$$

$$\frac{(1.00\,\text{E}-10, 1.00\,\text{E}-07)}{X_2},$$

$$\frac{(1.00\,\text{E}-10, 1.00\,\text{E}-04)}{X_3},$$

Table 3. Interval analysis

System components	Probability of failure		Magnitude of casualities		Magnitude of damage		Human risk		Economic risk		*U*-uncertainty	
(1)	Lower (2)	Upper (3)	Lower (4)	Upper (5)	Lower (6)	Upper (7)	Lower (8)	Upper (9)	Lower (10)	Upper (11)	Lower (12)	Upper (13)
1	1.00 E −13	1.00 E −07	0.00 E +00	0.00 E +00	1.00 E +00	1.20 E +03	0.00 E +00	0.00 E +00	1.00 E −13	1.20 E −04	1.98	2.01
2	1.00 E −10	1.00 E −07	2.00 E +00	3.00 E +03	5.00 E +01	1.00 E +03	2.00 E −10	3.00 E −04	5.00 E −09	1.00 E −04	0.00	0.00
3	1.00 E −10	1.00 E −04	2.00 E +02	1.10 E +06	1.00 E +03	6.00 E +03	2.00 E −08	1.10 E +02	1.00 E −07	6.00 E −01	1.13	1.32
4	1.00 E −10	1.00 E −07	0.00 E +00	1.00 E +02	2.00 E +01	5.00 E +03	0.00 E −00	1.10 E −05	2.00 E −09	5.50 E −04	1.54	2.30
5	1.00 E −08	1.00 E −04	9.00 E +02	1.20 E +03	0.00 E +00	1.00 E +02	9.00 E −06	1.20 E −01	0.00 E +00	1.00 E −02	2.97	3.05
6	1.00 E −04	1.00 E −02	1.00 E +05	1.10 E +06	0.00 E +00	0.00 E +00	1.00 E +01	1.10 E +04	0.00 E +00	0.00 E +00	1.97	2.45
7	1.00 E −07	1.00 E −04	0.00 E +00	0.00 E +00	1.00 E +05	2.00 E +06	0.00 E +00	0.00 E +00	1.00 E −02	2.00 E +02	1.65	2.98
8	1.00 E −13	1.00 E −05	5.00 E +02	7.00 E +03	1.00 E +03	2.00 E +06	5.00 E −11	7.00 E −02	1.00 E −10	2.00 E +01	2.45	3.87
9	1.00 E −14	1.00 E −05	1.00 E +05	1.10 E +06	9.99 E +02	1.00 E +05	1.00 E −09	1.10 E +01	9.99 E −12	1.00 E +00	0.00	0.00

Note: The numerical entries in this table are expressed in a scientific format, where, for example, 1.00 E −13 = 1.00×10^{-13}.

$$\frac{(1.00\,\mathrm{E}-10,\ 1.00\,\mathrm{E}-07)}{X_4},\ \frac{(1.00\,\mathrm{E}-08,\ 1.00\,\mathrm{E}-04)}{X_5},\ \frac{(1.00\,\mathrm{E}-04,\ 1.00\,\mathrm{E}-02)}{X_6},\ \frac{(1.00\,\mathrm{E}-07,\ 1.00\,\mathrm{E}-04)}{X_7},\ \frac{(1.00\,\mathrm{E}-13,\ 1.00\,\mathrm{E}-05)}{X_8},\ \left.\frac{(1.00\,\mathrm{E}-14,\ 1.00\,\mathrm{E}-05)}{X_9}\right\}.$$

In order to satisfy the axiom of fuzzy set theory, the membership value for each component in each criterion should be between 0 and 1. Therefore, each degree of belief, which is represented by an interval value, was normalized. The normalization was performed by dividing each interval limit value by the largest upper limit of the degrees of belief of all the components for each criterion. The membership values $C_1(X)$ for $\hat{C}_1(X)$ can therefore be expressed as

$$C_1(X) = \left\{\frac{(1.00\,\mathrm{E}-11,\ 1.00\,\mathrm{E}-05)}{X_1},\ \frac{(1.00\,\mathrm{E}-08,\ 1.00\,\mathrm{E}-05)}{X_2},\ \frac{(1.00\,\mathrm{E}-08,\ 1.00\,\mathrm{E}-02)}{X_3},\ \frac{(1.00\,\mathrm{E}-08,\ 1.00\,\mathrm{E}-05)}{X_4},\ \frac{(1.00\,\mathrm{E}-06,\ 1.00\,\mathrm{E}-02)}{X_5},\ \frac{(1.00\,\mathrm{E}-02,\ 1.00\,\mathrm{E}+00)}{X_6},\ \frac{(1.00\,\mathrm{E}-05,\ 1.00\,\mathrm{E}-02)}{X_7},\ \frac{(1.00\,\mathrm{E}-11,\ 1.00\,\mathrm{E}-03)}{X_8},\ \frac{(1.00\,\mathrm{E}-12,\ 1.00\,\mathrm{E}-03)}{X_9}\right\}.$$

Similarly, $C_2(X)$, $C_3(X)$, $C_4(X)$, $C_5(X)$ and $C_6(X)$ were constructed. The operations of interval values were performed according to equations (9) to (13). The resulting lower and upper limits of the fuzzy-based criteria C_1 to C_6 are shown in columns 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 and 13 of Table 4, respectively. The lower and upper limits of the fuzzy decision set R was calculated according to equation (1) and is shown in columns 14 and 15 of Table 4, respectively. Equations (2) and (14) to (17) were used as a basis for ranking the components. The ranking results of the components are shown in column 16 of Table 4.

5.2. *Unequally-important criteria*

In the previous section, components were ranked according to several combined criteria. The criteria were assumed to be equally important. In general, the criteria might not be equally important. The ranking of components according to unequally-important criteria was performed in this section. In order to construct the scalar ratings of these criteria, the A matrix according to equation (5) was constructed with the help of Table 1. The magnitude of fatality, i.e., criteria C_2, was assumed to be absolutely important, i.e., $\beta = 9$, in comparison to the failure probability of a component, i.e., C_1. Therefore, $a_{21} = 9$. Also the magnitude of damage was rated as very strongly important compared to the probability of failure, i.e., $a_{31} = 7$. The human risk criterion was rated as essential or strongly important compared to the probability of failure criterion, resulting in $a_{41} = 5$. The probability of failure criterion was rated as essential or strongly important compared to the consequence uncertainty criterion, resulting in $a_{16} = 5$. Finally the human risk was rated as more than very strongly important than the economic risk criterion, resulting in $a_{45} = 8$. In order to force a consistency in the A matrix, only the above 5 pairwise comparisons were made, i.e., $m - 1$, where $m = 6$. The rest of the A matrix entries were calculated according to $a_{ij} = 1/a_{ji}$, $a_{ii} = 1$ and $a_{ij} = a_{ik}a_{kj}$. Therefore, the A matrix is given by

$$A = \begin{bmatrix} 1 & \frac{1}{9} & \frac{1}{7} & \frac{1}{5} & \frac{8}{5} & 5 \\ 9 & 1 & \frac{9}{7} & \frac{9}{5} & \frac{72}{5} & 45 \\ 7 & \frac{7}{9} & 1 & \frac{7}{5} & \frac{56}{5} & 35 \\ 5 & \frac{5}{9} & \frac{5}{7} & 1 & 8 & 25 \\ \frac{5}{8} & \frac{5}{72} & \frac{5}{56} & \frac{1}{8} & 1 & \frac{25}{8} \\ \frac{1}{5} & \frac{1}{45} & \frac{1}{35} & \frac{1}{25} & \frac{8}{25} & 1 \end{bmatrix}.$$

Table 4. Equally-important multi-criteria ranking

System Components	Probability of failure, C_1		Magnitude of casualty, C_2		Magnitude of damage, C_3	
	Lower	Upper	Lower	Upper	Lower	Upper
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	1.00 E −11	1.00 E −05	0.00 E +00	0.00 E +00	5.00 E −07	6.00 E −04
2	1.00 E −08	1.00 E −05	1.82 E −06	2.73 E −03	2.50 E −05	5.00 E −04
3	1.00 E −08	1.00 E −02	1.82 E −04	1.00 E +00	5.00 E −04	3.00 E −03
4	1.00 E −08	1.00 E −05	0.00 E +00	9.09 E −05	1.00 E −05	2.50 E −03
5	1.00 E −06	1.00 E −02	8.18 E −04	1.09 E −03	0.00 E +00	5.00 E −05
6	1.00 E −02	1.00 E +00	9.09 E −02	1.00 E +00	0.00 E +00	0.00 E +00
7	1.00 E −05	1.00 E −02	0.00 E +00	0.00 E +00	5.00 E −02	1.00 E +00
8	1.00 E −11	1.00 E −03	4.55 E −04	6.36 E −03	5.00 E −04	1.00 E +00
9	1.00 E −12	1.00 E −03	9.09 E −02	1.00 E +00	5.00 E −04	5.00 E −02

Human risk, C_4		Economic risk, C_5		U-uncertainty, C_6		Decision set, R		
Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Ranking
(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
0.00 E +00	0.00 E +00	5.00 E −16	6.00 E −07	5.12 E −01	5.19 E −01	0.00 E +00	0.00 E +00	5
1.82 E −14	2.73 E −08	2.50 E −11	5.00 E −07	0.00 E +00	0.00 E +00	0.00 E +00	0.00 E +00	5
1.82 E −12	1.00 E −02	5.00 E −10	3.00 E −03	2.92 E −01	3.41 E −01	5.00 E −10	3.00 E −03	1
0.00 E +00	1.00 E −09	1.00 E −11	2.75 E −06	3.98 E −01	5.94 E −01	0.00 E +00	1.00 E +09	4
8.18 E −10	1.09 E −05	0.00 E +00	5.00 E −05	7.67 E −01	7.88 E −01	0.00 E +00	5.00 E −05	2
9.09 E −04	1.00 E +00	0.00 E +00	0.00 E +00	5.09 E −01	6.33 E −01	0.00 E +00	0.00 E +00	5
0.00 E +00	0.00 E +00	5.00 E −05	1.00 E +00	4.26 E −01	7.70 E −01	0.00 E +00	0.00 E +00	5
4.55 E −15	6.36 E −06	5.00 E −13	1.00 E −01	6.33 E −01	1.00 E +00	4.55 E −15	6.36 E −06	3
9.09 E −14	1.00 E −03	5.00 E −14	5.00 E −03	0.00 E +00	0.00 E +00	0.00 E +00	0.00 E +00	5

Note: The numerical entries in this table are expressed in scientific format.

Solving the A matrix for the eigenvector corresponding to the maximum eigenvalue $\lambda_{max} = 6$ and applying equations (7) and (8), the scalar factors for the criteria are

$$\alpha = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \end{bmatrix} = \begin{bmatrix} 0.26 \\ 2.37 \\ 1.84 \\ 1.31 \\ 0.17 \\ 0.05 \end{bmatrix}.$$

Substituting the scale rating factor α in equation (3) to the equally-important fuzzy-based criteria C_1 to C_6 that were used in Table 4 results in weighted fuzzy criteria. The decision set R is determined according to equation (3). The fuzzy sets C_1 to C_6, and R are summarized in Table 5 in a similar format to Table 4. The maximum operation on the set R was performed to rank the components according to equation (4). The priority ranking for the components is shown in column 16 of Table 5.

A component ranking comparison based on equally and unequally-important criteria is shown in Figure 3. It is evident from the figure that selecting the appropriate model is essential for obtaining a credible ranking of the components of a system.

6. Summary and conclusions

In this study, a risk-based methodology with uncertainty evaluation and propagation was developed for the purpose of creating an inspection strategy. The ranking priority list for inspection purposes was based on the assessments of the probabilities of failure, resulting consequences, expected human and economic risks and the uncertainties associated with their assessments. The consequences included property damages, injuries and fatalities.

In order to utilize the impact of the

Table 5. Unequally-important multi-criteria ranking

System components	Probability of failure, $C_1^{0.26}$		Magnitude of casualty, $C_2^{2.37}$		Magnitude of damage, $C_3^{1.84}$	
	Lower	Upper	Lower	Upper	Lower	Upper
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	1.28 E −03	4.84 E −02	0.00 E +00	0.00 E +00	2.55 E −12	1.18 E −06
2	7.87 E −03	4.84 E −02	2.63 E −14	8.59 E −07	3.41 E −09	8.43 E −07
3	7.87 E −03	2.98 E −01	1.42 E −09	1.00 E +00	8.43 E −07	2.28 E −05
4	7.87 E −03	4.96 E −02	0.00 E +00	2.75 E −10	6.31 E −10	1.63 E −05
5	2.64 E −02	2.98 E −01	4.98 E −08	9.83 E −08	0.00 E +00	1.22 E −08
6	2.98 E −01	1.00 E +00	3.44 E −03	1.00 E +00	0.00 E +00	0.00 E +00
7	4.84 E −02	2.98 E −01	0.00 E +00	0.00 E +00	4.04 E +03	1.00 E +00
8	1.28 E −03	1.63 E −01	1.24 E −08	6.37 E −06	8.43 E −07	1.00 E +00
9	6.98 E −04	1.63 E −01	3.44 E −03	1.00 E +00	8.42 E −07	4.04 E −03

Human risk, $C_4^{1.31}$		Economic risk, $C_5^{0.17}$		U-uncertainty, $C_6^{0.05}$		Decision set, R		
Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Ranking
(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
0.00 E +00	0.00 E +00	3.05 E −03	9.49 E −02	2.69 E −02	2.73 E −02	0.00 E +00	0.00 E +00	5
8.72 E −19	1.14 E −10	1.81 E −02	9.21 E −02	0.00 E +00	0.00 E +00	0.00 E +00	0.00 E +00	5
3.71 E −16	2.35 E −03	2.96 E −02	3.85 E −01	1.54 E −02	1.79 E −02	8.43 E −07	2.28 E −05	1
0.00 E +00	1.48 E −12	1.56 E −02	1.22 E −01	2.09 E −02	3.13 E −02	0.00 E +00	1.48 E −12	4
1.14 E −12	3.01 E −07	0.00 E +00	1.96 E −01	4.04 E −02	4.15 E −02	4.98 E −08	9.83 E −08	3
1.01 E −04	1.00 E +00	0.00 E +00	0.00 E +00	2.68 E −02	3.33 E −02	0.00 E +00	0.00 E +00	5
0.00 E +00	0.00 E +00	1.96 E −01	1.00 E +00	2.24 E −02	4.05 E −02	0.00 E +00	0.00 E +00	5
1.41 E −19	1.48 E −07	9.51 E −03	6.85 E −01	3.33 E −02	5.26 E −02	1.41 E −19	1.48 E −07	2
7.23 E −18	1.14 E −04	6.51 E −03	4.19 E −01	0.00 E +00	0.00 E +00	0.00 E +00	0.00 E +00	5

Note: The numerical entries in this table are expressed in scientific format.

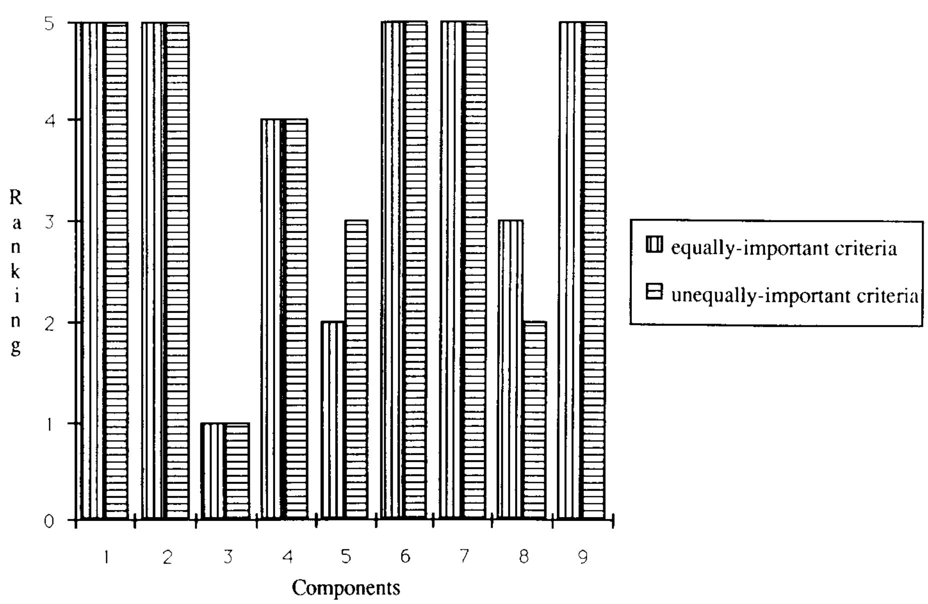

Fig. 3. Ranking summary.

uncertainty in the development of the risk-based inspection methodology for ranking purposes, interval risk estimates were considered as a suggested method for risk estimation and propagation of uncertainty. The uncertainties associated with the failure probability for components and the failure consequences were propagated to the risk of failure.

A method for estimating the resulting consequences due to failure using logic diagram was established. The uncertainty encountered in the assessment of the consequence value was estimated and treated as a criterion to be included in the decision making process.

The fuzzy-based multi-criteria decision making method was utilized for prioritizing the components of a system for inspection purposes. A method for ranking components was established such that a higher rank for a component represents a higher combination level of its probability of failure, magnitude of fatality and damage, and the human and economic risks. An account for the varying degree of importance of each criterion on the ranking decision was addressed.

The representation of the values of the probability of failures, magnitudes of consequences, risks and uncertainties by interval estimates provided a better ranking judgment for inspection purposes. The developed methodology is applicable for ranking at the levels of components, subsystems and systems. For a complex system which is composed of a large number of components, it is unnecessary to evaluate and rank all its components in a single process. In such cases, it is necessary to rank all the subsystems of the complex system. The same methodology established in this study for ranking components can be applied to rank the subsystems. The subsystems probabilities of failure, consequences, risks and uncertainties can be assessed and used as the ranking criteria. Then the components of each of the subsystems can be ranked according to the established criteria. Therefore, the resulting ranking of the components of the system is in the form of subsystems' ranking and a ranking of the components within each subsystem. Such an approach results in an efficient procedure for ranking the components and subsystems of the system.

Acknowledgements

This paper resulted from reasearch projects funded bythe American Society of Mechanical Engineers and the Nuclear Regulatory Commission.

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