Modeling Uncertainty in Prediction of Pier Scour

By Peggy A. Johnson[1] and Bilal M. Ayyub,[2] Members, ASCE

Abstract: Fuzzy regression is used to investigate the modeling uncertainty in the prediction of bridge pier scour. Fuzzy bias factors, which describe the bias between observed field data and scour estimates based on equations developed from laboratory data, were estimated. The bias exists because of the use of small-scale laboratory results to model large-scale, real-world problems. Fuzzy regression is a method of calibrating fuzzy numerical coefficients in a linear equation. Since the regression coefficients are fuzzy parameters, the output, in this case scour depth, is also a fuzzy number. The fuzzy bias factors developed from the fuzzy regression equations are compared for a variety of input data. The fuzzy bias factor provides useful information in the application of bridge pier scour equations currently available to engineers. The results of this study can be used by experimentalists in the interpretation of small-scale laboratory test results and by practicing engineers to adjust scour estimates.

INTRODUCTION

There are nearly 400,000 bridges over waterways in the United States (Harrison and Morris 1991). At many of these bridges, erosion of channel beds has developed around the pier foundations. As a result, a high percentage of bridge failures in recent years have been attributed to scour. Pier scour, the erosion of the streambed in the vicinity of pier foundations, may eventually undermine the pier foundations and cause bridges to become unstable.

Engineers are currently assessing scour conditions at existing bridges and determining the need for scour mitigation. They are also responsible for the design of new bridges that should be safe from scour. To accomplish these tasks, engineers need to estimate the expected maximum scour depth. The engineer must rely on experience and available scour equations to make decisions regarding (1) the appropriate footing depth for a new pier; and (2) the need for scour mitigation at new and existing piers.

Commonly used scour equations and models are based on laboratory data and are valid only for noncohesive channel beds of infinite depth and steady-state flow. They do not account for many variables that are typically encountered in a field setting. Although the derived equations work quite well for the laboratory setting, the use of these equations in the field is uncertain because of the greatly simplified laboratory conditions, a limited range of data, the use of ratios, and distortions in the physical model due to sediment size. Therefore, the models contain considerable uncertainty at the field scale.

Since scour prediction equations can result in estimates of scour that are different from observed field values, an assessment of this difference is needed. The difference can be considered modeling uncertainty and can be expressed in the form of bias factors between predicted and observed field values. In this study, fuzzy regression is used to quantify the bias that results when laboratory-based scour models are used for field applications. Fuzzy regression is used in this application for a variety of reasons. First, this method of calibrating a fuzzy problem is quite flexible and capable of accounting for various uncertainties. Fuzzy regression can be used to combine information from crisp data points with assumed relationships based on engineering experience and judgment, as shown in Fig. 1. The crisp data primarily include experimental results, whereas the fuzzy data can be based on judgement. Second, the technique is useful in modeling fuzzy problems. The method yields intervals for the dependent variable in which the estimated dependent variable is expected to occur. These advantages are expanded upon in the following sections.

TYPES OF UNCERTAINTY

Uncertainties in engineering systems can be mainly attributed to ambiguity and vagueness in defining the architecture, parameters, and governing prediction models for the systems. The ambiguity component is generally due to noncognitive sources. These sources include (1) physical randomness; (2) statistical uncertainty due to the use of sampled information to estimate the characteristics of system parameters; (3) lack of knowledge of the system or process; and (4) modeling uncertainty that results from simplifying assumptions in analytical and prediction models, simplified methods, and idealized representations of real processes. The vagueness related uncertainty is due to cognitive sources that include (1) the definition of system parameters, for example, structural performance (failure or survival), quality, deterioration, skill and experience of construction workers and engineers, environmental impact of projects, and conditions of existing structures; (2) other human factors; and (3) definition of the interrelationships among the parameters of the problems, especially for complex systems. Other sources of uncertainty can include conflict in information, and human and organizational errors. Statistical regression accounts for ambiguity; fuzzy regression, described in the next section, accounts for vagueness.

LINEAR FUZZY REGRESSION

Fuzzy regression is a method of calibrating fuzzy numerical coefficients (Kaufmann and Gupta 1985) in a linear equation. This method was developed by Tanaka et al. (1982) and has

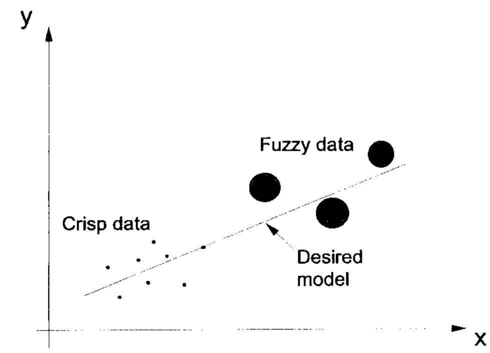

FIG. 1. Mixed Types of Information

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[1]Asst. Prof., Dept. of Civ. Engrg., Univ. of Maryland, College Park, MD 20742.

[2]Prof., Dept. of Civ. Engrg., Univ. of Maryland, College Park, MD.

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been used primarily for cases in which too few data points were available for a standard statistical regression. In fuzzy regression, the regression parameters are fuzzy numbers with central values and ranges described by the degree of acceptance of the values of the parameter. Since the regression parameters are fuzzy numbers, the dependent variable is also a fuzzy number (Bardossy et al. 1990). The objective of fuzzy regression is to minimize the vagueness of the dependent variable y (Bardossy 1990; Bardossy et al. 1990). A fuzzy linear regression results in the following model:

$$Y^* = A_1^*X_1 + A_2^*X_2 + \cdots + A_n^*X_n \tag{1}$$

where an asterisk denotes a fuzzy number; and A_k^* = a fuzzy parameter consisting of the ordered pair (α_k, c_k), α_k being the center of fuzzy parameter A_k^*, and c_k the width or tolerance of the coefficient, that is, fuzziness of the parameter (Heshmaty and Kandel 1985), for $k = 1, 2, \ldots, n$. Fig. 2 shows the membership function of a fuzzy number A_k^*. For any value α_k', the corresponding h' is the degree of belief that this value is taken by A_k^*. A central assumption of fuzzy linear regression is that the residuals (i.e., the difference between the predicted and observed values) are due to fuzziness of the system parameters.

The objective of a linear fuzzy regression can best be met by minimizing the sum of the widths of the fuzzy regression coefficients, that is, min Σc_k. A degree of fitting (or belief) or level of credibility h must also be established. The degree of fitting can be thought of as a threshold value such that the fuzzy number Y^* must include the observed value Y at a degree of belief of at least h. This is shown in Fig. 3. Bardossy et al. (1990) recommend $0.5 < h < 0.7$. By increasing h, the values of c_k increase. Bardossy et al. (1990) showed that fuzzy regression analysis can be reduced to the following linear programming problem, assuming triangular membership functions for A_k^*:

$$\text{minimize: } J = \sum_{k=1}^{n} c_k \tag{2}$$

subject to:

$$(1 - h) \sum_{k=1}^{n} c_k|x_{ik}| + \sum_{k=1}^{n} \alpha_k x_{ik} \geq y_i + (1 - h)e_i \tag{3a}$$

$$(1 - h) \sum_{k=1}^{n} c_k|x_{ik}| - \sum_{k=1}^{n} \alpha_k x_{ik} \geq -y_i + (1 - h)e_i \tag{3b}$$

for all samples i, where n = number of variables; x_{ik} = kth variable in the ith sample point; e_i = width of the ith sample point; and $c_k \geq 0$. As an example, a data set with a sample size of 10 will require a set of 20 constraints, according to (3). In (3), all independent variables x are nonfuzzy (known with certainty); however, the dependent variable y is fuzzy (see Heshmaty and Kandel 1985). For nonfuzzy y, $e = 0$.

Fig. 3 shows the membership function of the predicted value of y. The value of a selected grade of membership or threshold h in the figure can be derived as follows:

$$\frac{1 - h}{1} = \frac{|y_i - y_i^*|}{\Sigma c_k x_{ik}} \tag{4}$$

where y^* is estimated at the central value of the fuzzy membership function. Solving for h

$$h = 1 - \frac{|y_i - y_i^*|}{\Sigma c_k x_{ik}} \tag{5}$$

At $h = 1$, the estimated value of y exactly approximates, relative to the magnitude of the system uncertainty (which is

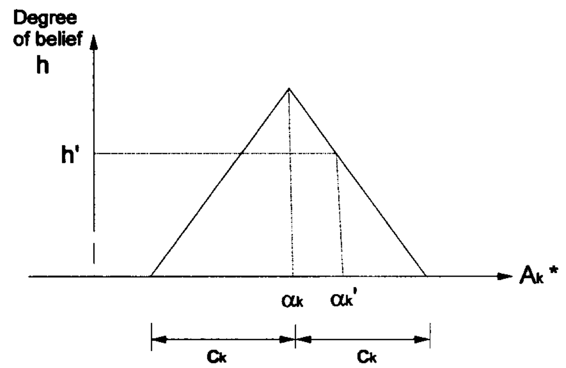

FIG. 2. Example Fuzzy Number and Threshold Level *h*

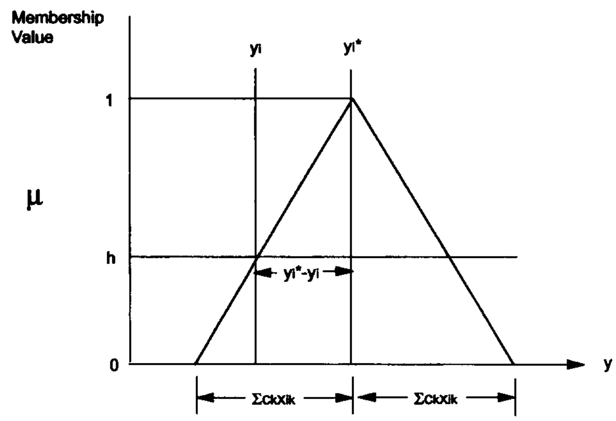

FIG. 3. Fuzzy Number *Y and Threshold Level *h***

minimized by the fuzzy regression procedure), the actual value of y. As c becomes very small, h approaches zero, and the estimated value of y does not estimate the actual value of y well. In this respect, h can be used as one indicator of the goodness of fit. The aspect is addressed in a later section.

Based on this derivation of h, the selection of h can be interpreted as follows. For a selected grade of membership h, or higher, the estimation of y is considered to be a good outcome. When $h = 0$, there is a zero degree of belief that the estimation of y is a good estimation of the actual value of y. When $h = 1$, we have a high degree of belief of our estimation of y. If h is selected as, say, 0.7, then the degree of belief that y^*, in the form of a fuzzy number with a triangular membership function, is a good estimate of y is at least 0.7.

Fuzzy regression has been used in a variety of applications, predominantly where the sample size of the data set was too small for a statistical regression. Shiraishi et al. (1988) developed a model for estimating the fatigue life of bridge structures using a limited set of data based on the results of a fuzzy regression analysis. Bardossy et al. (1990) used fuzzy regression to develop a relationship between soil resistivity (ohm-meters) and the permeability of the soil (cm/s 10^{-9}). Bardossy et al. also described other potential uses of fuzzy regression in hydrology. Kaneyoshi et al. (1990) developed a system identification method applied to the construction of a cable-stayed bridge. They used fuzzy regression as a way to include measurement error for field data.

LABORATORY-BASED EQUATION FOR PIER SCOUR

Laboratory data from Chiew (1984), "Mechanics" (1966), Jain and Fischer (1979), Shen et al. (1969), and Chabert and Engeldinger (1956) were used to develop a traditional regression equation for pier scour for the purpose of comparing the results to a fuzzy regression equation based on limited field data to quantify the modeling uncertainty. The data consist of

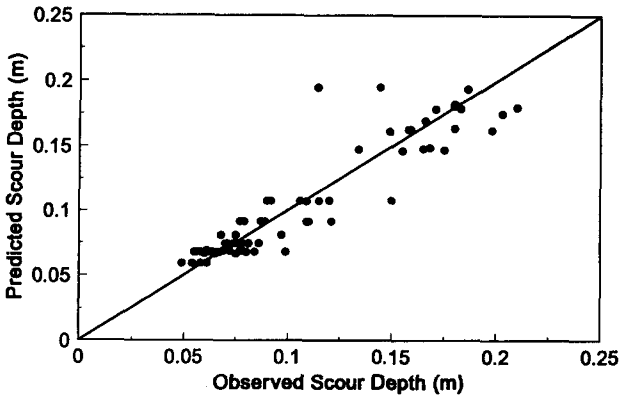

FIG. 4. Observed and Predicted Scour Depths Based on Eq. (6)

TABLE 1. Field Scour Data (Froehlich 1988)

Pier width (m) (1)	Flow depth (m) (2)	Sediment gradation (3)	Scour depth (m) (4)
4.5	18.8	2.2	4.3
4.5	17.4	2.2	3
8.2	4.9	11.5	3.7
8.2	4.3	11.5	4.3
1.5	1.3	5.3	0.4
1.5	1	5.3	0.4
1.5	0.9	5.3	0.5
1.5	0.9	5.3	0.4
1.5	0.7	5.3	0.4
3.9	3.5	2.3	2.8
16.3	4.1	8.3	7.3
22.4	3.4	8.3	6.8
25.2	5.4	8.3	8.5
14.2	16.3	18.7	7.9
12.7	11.6	18.7	4
14.2	13.4	18.7	7.6
9.4	19.5	6.1	6.1
28.7	11.3	6.3	10.4

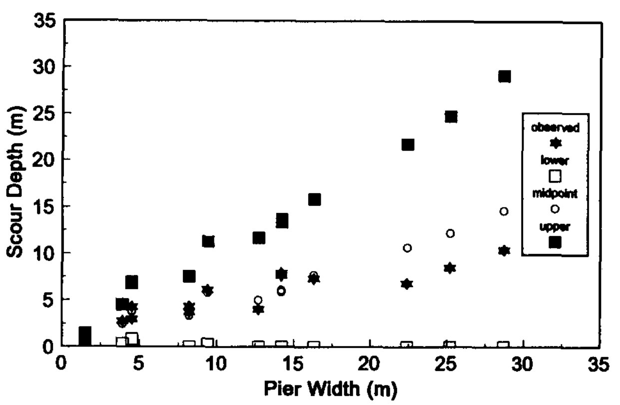

FIG. 5. Observed and Predicted Scour Depths Based on Eq. (7)

flow depth y, pier width b, and scour depth D with average values of 0.16 m, 0.11 m, and 0.13 m, respectively, and standard deviations of 0.09 m, 0.09 m, and 0.13 m, respectively. All piers were cylindrical, the bed material was uniform sediment, the flow was subcritical, and the flow velocity was greater than the critical velocity for sediment movement (i.e., live-bed conditions). The ratio of b/d_{50} (where d_{50} = median sediment size) for all data was greater than 60 to avoid problems associated with a large sediment size relative to the pier size (Chiew 1984). The data were arranged in dimensionless ratios, according to convention, to avoid problems with systems of units and to incorporate scaling into the equation. A power regression of the 54 data points resulted in the following equation:

$$\frac{D}{y} = 1.06\left(\frac{b}{y}\right)^{0.667} \quad (6)$$

Velocity was not included in (6), since the data represent only live-bed conditions and scour is not dependent on velocity under these conditions (Melville and Sutherland 1988). Fig. 4 shows the predicted and observed scour depths. The statistics for (6) are the correlation coefficient $R = 0.94$ and the ratio of the standard error to standard deviation $Se/S_D = 0.19$.

Field data from Davoren (1985) are used to demonstrate the results of (6). In this example, $b = 1.5$ m, $y = 1.3$ m, and the measured scour depth = 0.9 m. Eq. (6) resulted in an estimated scour depth of 1.52 m. One reason for this overprediction may be that the channel bed sediment at this bridge is not uniform, as in the laboratory experiments. The geometric standard deviation for this case is 5.3, which may result in a reduced scour depth.

FUZZY REGRESSION OF PIER SCOUR

Fuzzy regression was used to develop an equation with field data compiled by Froehlich (1988). The data set is provided in Table 1. The data represent live-bed scour and only those data that were representative of conditions similar to those of the laboratory experiments (e.g., round-nosed piers, subcritical flow) were used to develop the equation. The sample size of 18 included the flow depth, pier width, sediment geometric standard deviation, pier length, angle of streamflow attack, and scour depth. Pier width, pier length, and angle of attack were combined to form a single parameter known as the effective pier width, according to Froehlich (1988). The width of the fuzzy observed scour depth was taken as 50% of the actual measured scour depth; this value was chosen because there is considerable fuzziness due to the inability to accurately measure scour during a flood. Using $h = 0.5$, fuzzy regression of the field data resulted in the following equation:

$$D^* = 0.64 + [0, 0.98]b' + [0.03, 0.12]y - 0.15G \quad (7)$$

where G = sediment gradation; b' = effective pier width (m) = $b\cos(\alpha) + L\sin(\alpha)$; α = angle of streamflow attack (deg), and L = pier length (m). Ratios were not used here since scaling is unnecessary for prototype equations. Eq. (7) provides an interval for the fuzzy predicted scour depth. Fig. 5 shows the results of the fuzzy regression in terms of observed and predicted scour depths. The observed value lies between the upper and lower values of the fuzzy predicted scour depth, as required by (3). The linear form of (7) permits the possibility

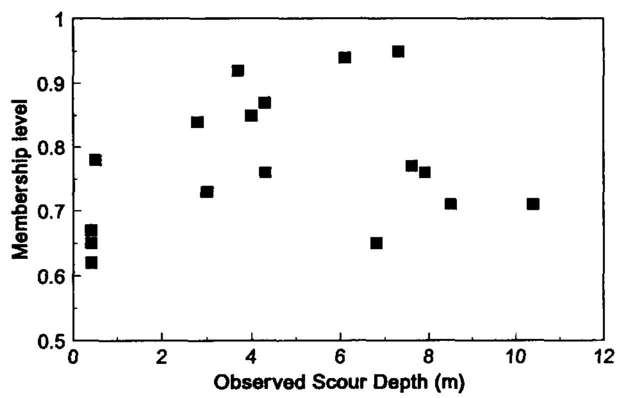

FIG. 6. Membership Value versus Scour Depth for Calibration Data

of negative scour depths. This is not physically possible; therefore, in Fig. 5, all negative scour depths were truncated at zero. A scour depth of zero implies that it is possible that no scour will occur at $h = 0.5$. The implications of having zero in the range of possible scour depths are discussed in a later section.

Field data from Davoren (1985) were again used to test (7). The scour depth for this example was 0.9 m, with $b = 1.5$ m, $y = 1.3$ m, and $G = 5.3$. Eq. (7) provides a range of scour depths from 0 to 1.49 m. The observed depth falls within this range. The central value of D, which is associated with a membership value of 1 (i.e., the highest degree of belief) is 0.75 m, which is somewhat less than the observed scour depth of 0.9 m. The scour depth of 1.52 m estimated from (6) falls outside of the fuzzy range.

The goodness of fit of (7) can be examined by computing the membership level μ or degree of belief for the 18 data used in calibration and replacing h with μ. Fig. 6 shows the values of μ plotted against the observed scour depths. All μ are greater than the specified threshold level h of 0.5, as required by (3), with an average of 0.77. More than 72% of the values of μ are greater than 0.70, which shows that the data fit the central point of the fuzzy intervals reasonably well.

Additional data not used in the calibration of (7) were used to calculate μ as a way of testing the predictive ability of (7). Fig. 7 shows that μ ranges from 0.45 to 0.98. All values fall close to or above the threshold level of 0.5 with an average of 0.75, again demonstrating a reasonably good predictive ability.

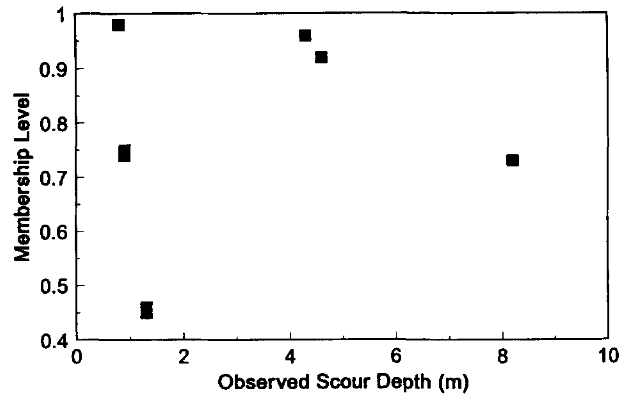

FIG. 7. Membership Value versus Scour Depth for Test Data

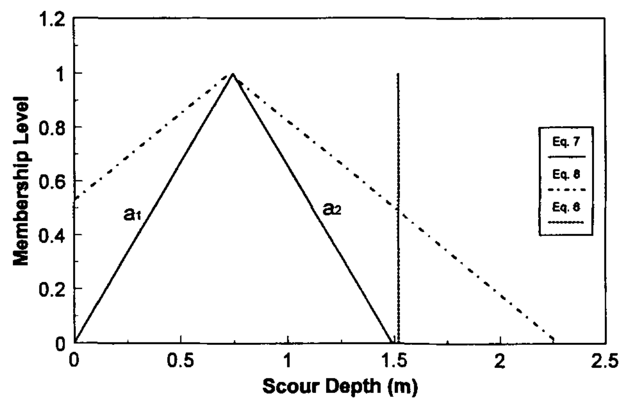

FIG. 8. Comparison of Scour Depths Predicted from Eqs. (6)–(8)

Sensitivity of D^* to Choice of h

Eq. (7) yields a fuzzy number D^*. The coefficients in (7), and thus the spread of the interval containing D^* depend on the choice of h. If h is, say, 0.6 or 0.7, the coefficients will change. As h is decreased, the interval of each coefficient decreases, and, therefore, the interval of D^* decreases. To determine the sensitivity of D^* to the choice of h, (7) was recalibrated using $h = 0.7$. The values of h were chosen according to findings by Bardossy et al. (1990). Fuzzy regression of the same data set using the $h = 0.7$ resulted in the following equation:

$$D^* = 0.70 + [-0.45, 1.46]b' + [-0.04, 0.18]y - 0.16G \quad (8)$$

Eq. (8) yields a larger interval for D^* than (7). This is the expected result since as h increases, the spread of the interval must increase, according to (5). Using the example field data from Davoren (1985) applied in the previous sections yields $D^* = [-0.82, 2.28]$ m, a wider interval than obtained from (7). The negative values of D^* are not physically possible and result from the linear form of the equation. Truncating the negative values yields $D^* = [0, 2.28]$ m. This interval is approximately a 150% increase over using $h = 0.5$. Fig. 8 shows a comparison of scour depths estimated from (6)–(8). The membership functions of scour depths resulting from (7) and (8) are assumed to be triangular, with the highest degree of belief, or membership μ, occurring at the central point of the interval.

Assessment of Fuzzy Regression Models

A direct comparison of the fuzzy regression models with statistical regression models has been avoided for several reasons. First, pier scour literature abounds with examples of statistical regression of laboratory data. It need not be repeated here. Second, comparisons and differences may be found elsewhere [see, for example, Heshmaty and Kandel (1985)].

Fuzzy regression of crisp data results in a regression-type model containing fuzzy regression coefficients and producing fuzzy output. This is a very desirable attribute for most small-scale laboratory experiments where extrapolation to the field is uncertain. For example, in developing an equation for pier scour, the laboratory data from which the equation is developed are crisp, but the output from field-based input data contains a great deal of uncertainty. A fuzzy regression model is also desirable when the sample size is insufficient to reliably perform traditional regression, as in the case of the field data. However, there are a number of drawbacks to this method.

First, a level of acceptance h must be specified. The choice of h is somewhat arbitrary. A sensitivity analysis could be performed to determine the effect of various choices of h on the resulting prediction equations, but typically there is little basis for selecting a value. This problem is similar to the problem of choosing a level of significance in traditional statistical tests. Brubaker and McCuen (1990) showed that an arbitrarily assumed level of significance can lead to an incorrect model under certain conditions.

Second, the objective function is the minimization of the sum of the widths of the fuzzy regression parameters (min Σc_k). This objective function can result in unrealistic engineering predictions. An objective function should meet the following requirements in order to produce realistic engineering predictions: (1) the sum of the squares of the difference between the observed crisp points and the midpoints of the predicted fuzzy points should be minimized; (2) the sum of the difference between the observed crisp points and the midpoints of the predicted fuzzy points should be required to be zero (an unbiased model); and (3) the average spread of the predicted fuzzy points should be minimum, and can be assumed to be either constant or variable. The writers are currently pursuing these variations to the fuzzy regression model.

Third, the development of the set of constraints [(3)] is a laborious task, particularly when the data set is large. For each trial, where there is a modification of the set of independent variables or a different selection of h, a new set of constraints must be developed. Also, slack variables must be added if one or more coefficients could be negative. A revised model should depart from the approach of specifying numerous constraints that are sample dependent.

Fourth, it is difficult to assess the fit of the fuzzy regression equation. Goodness-of-fit parameters used in least squares regression do not apply for the case of fuzzy regression. By revising the model according to the second item, the sum of the squares of errors can be used as a measure of the goodness of fit. Heshmaty and Kandel (1985) suggested several criteria for evaluating the goodness of fit. One was the percentage of the relative deviations between the actual y and the computed central value of y^*, in this case D^*, that is, $|\Sigma\alpha_j x_{ij} - y_i|/y_i$. A second criterion was the relative width of the fuzzy sets compared to the actual y, that is, $\Sigma c_j|x_{ij}|/y_i$. The results for these

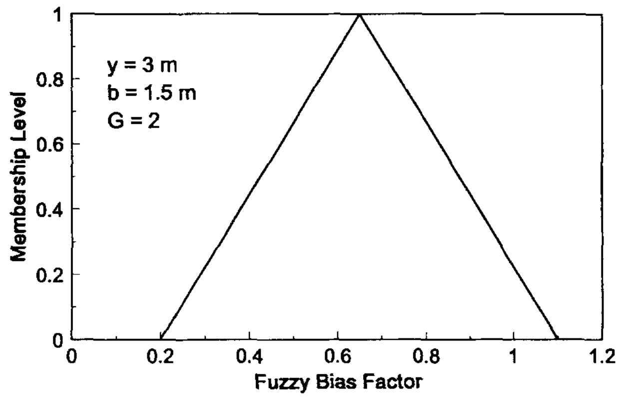

FIG. 9. Fuzzy Bias Factor for $b = 1.5$ m, $y = 3$ m, and $G = 2$ as Function of Membership Value

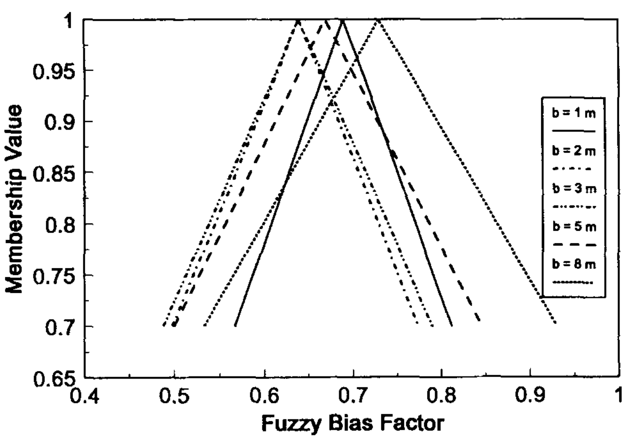

FIG. 10. Fuzzy Bias Factor as Function of Pier Diameter

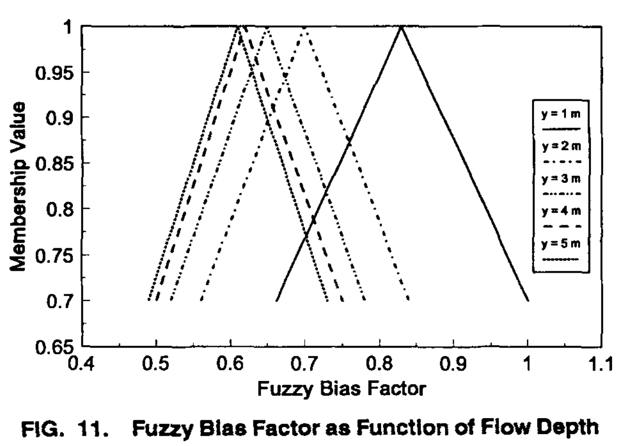

FIG. 11. Fuzzy Bias Factor as Function of Flow Depth

criteria depend on the user's choice of h. Also, the user has to specify unacceptable limits of these criteria. This is not always an easy task when dealing with fuzzy problems. In addition, these criteria do not consider the case of fuzzy y_i.

One way to deal with some of these problems of objective function and goodness of fit is to use two objective functions, one that would minimize the sum of squares of errors and one that would minimize the widths of the fuzzy coefficients. Savic and Pedrycz (1991) suggested a two-step method. In the first step a statistical linear regression is performed to obtain the central values of the regression coefficients α_i. In the second step the linear programming procedure is followed as before except that the values of α_i are known. Thus, in (3), the constraints are a function of c_i only. The first step satisfies the least-squares objective function. The second step satisfies the minimum sum-of-widths objective function. This process also simplifies the development of the constraint equations. However, the process still does not enable an unbiased goodness-of-fit measure for the interval width.

FUZZY BIAS FACTOR

Laboratory-based scour equations are typically developed as envelope curves [see Jones (1984) for a comparison of these equations] representing maximum scour depths. Eq. (6) was developed as a best-fit curve for the purpose of comparison with (7), which is based on limited field data. A comparison of (6) and (7) provides insight into the bias between the laboratory and field-based scour estimates. The bias may be computed as the ratio of the field estimate to the laboratory estimate. However, a simple division is not appropriate since the scour depth estimates are fuzzy numbers. At a selected degree of belief, a fuzzy number a ranging in value from a_1 to a_2 can be divided by a fuzzy number b, at the same degree of belief, ranging from b_1 to b_2 as follows (Kaufmann and Gupta 1985):

$$\frac{[a_1, a_2]}{[b_1, b_2]} = \left[\frac{a_1}{b_2}, \frac{a_2}{b_1}\right] \qquad (9)$$

In the case here, b = a crisp number, that is, the value does not change for any degree of belief. Let $b_1 = b_2$ = scour depth based on (6), and a_1 and a_2 represent the left and right sides of the triangular membership functions for the field-based scour estimates from (7), respectively, as shown in Fig. 8, where a and b represent scour depth as a function of membership value. The bias can then be computed from (9). As an example, let $b = 1.5$ m, $y = 3$ m, and $G = 2$. Eq. (6) yields an estimate of 2.0 m. Eq. (7) yields an estimate of [0.43, 2.17] m. From (10), the fuzzy bias ranges from 0.2 to 1.1, meaning that the actual field scour may be 0.2–1.1 times the estimate obtained from (6). The fuzzy bias factor for this example is shown in Fig. 9.

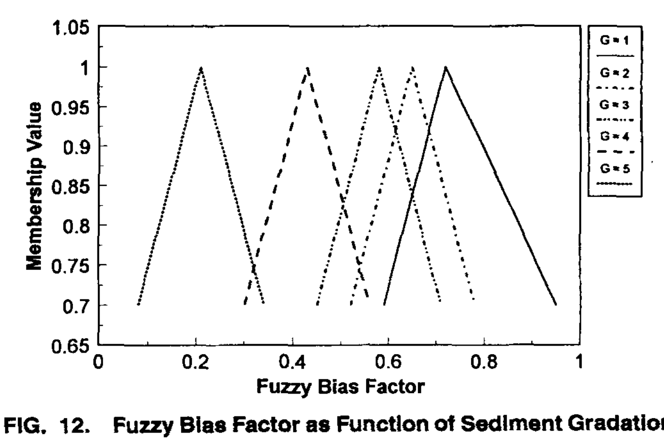

FIG. 12. Fuzzy Bias Factor as Function of Sediment Gradation

TABLE 2. Summary of Fuzzy Bias Factors for μ = 1, Pier Diameter b = 1, 3, and 8 m, and Sediment Gradation G = 2, 5, and 8

Flow depth y (m) (1)	Sediment Gradation G = 2			Sediment Gradation G = 5			Sediment Gradation G = 8		
	b = 1 m (2)	b = 3 m (3)	b = 8 m (4)	b = 1 m (5)	b = 3 m (6)	b = 8 m (7)	b = 1 m (8)	b = 3 m (9)	b = 8 m (10)
1	0.86	0.86	1.02	0.45	0.66	0.92	0.04	0.46	0.82
3	0.69	0.64	0.73	0.41	0.50	0.66	0.12	0.37	0.59
5	0.66	0.58	0.64	0.42	0.46	0.58	0.18	0.35	0.52
8	0.67	0.54	0.57	0.46	0.45	0.52	0.26	0.35	0.47

TABLE 3. Summary of Fuzzy Bias Factors for μ = 0.7, Pier Diameter b = 1, 3, and 8 m, and Sediment Gradation G = 2, 5, and 8

Flow depth y (m) (1)	Sediment Gradation G = 2			Sediment Gradation G = 5			Sediment Gradation G = 8		
	b = 1 m (2)	b = 3 m (3)	b = 8 m (4)	b = 1 m (5)	b = 3 m (6)	b = 8 m (7)	b = 1 m (8)	b = 3 m (9)	b = 8 m (10)
1	0.71	0.65	0.74	0.30	0.45	0.64	0.00	0.25	0.54
1	1.01	1.06	1.30	0.60	0.86	1.20	0.19	0.67	1.10
3	0.57	0.49	0.53	0.28	0.35	0.46	0.00	0.21	0.39
3	0.81	0.79	0.93	0.53	0.65	0.86	0.24	0.52	0.79
5	0.54	0.44	0.47	0.30	0.33	0.41	0.06	0.21	0.35
5	0.78	0.71	0.81	0.54	0.60	0.75	0.30	0.48	0.69
8	0.55	0.42	0.42	0.34	0.32	0.37	0.14	0.22	0.32
8	0.79	0.67	0.72	0.58	0.57	0.67	0.38	0.47	0.62

The fuzzy bias factor at a degree of belief of zero is not likely to be of interest to an engineer. It is more likely that the engineer will be concerned with the bias at a degree of belief of 0.7 or more. In that case, the fuzzy bias factor for the example above at a degree of belief of 0.7 is [0.52, 0.79] based on a triangular membership function for the fuzzy field scour equation [(7)] and (9). This fuzzy bias means that at a degree of belief of 0.7, (6) overpredicts, so that the actual amount of scour that can be expected in a field situation may be between 0.5 and 0.8 times that predicted by (6). In other words, for this example, the actual field scour may be at least 1.0 m and no more than 1.6 m.

Pier width has been shown to have a significant effect on pier scour in laboratory studies [see, for example, Melville and Sunderland (1988)]. The effect of pier width on the fuzzy bias can be illustrated by computing the bias based on various membership values for various pier widths. Fig. 10 shows the fuzzy bias factor computed from (9) and corresponding membership values greater than 0.7 for y = 3 m, G = 2, and b = 1, 2, 3, 5, and 8 m. As the pier width increases, the range of bias for membership values of 0.7–1.0 increases. For a larger pier width (b = 8 m), the bias is strongly dependent on the membership value.

Similar information can be derived for changes in flow depth or sediment gradation and for membership values greater than 0.7. Figs. 11 and 12 show the bias resulting from changes in flow depth and sediment gradation, respectively. Changes in flow depth produce a corresponding shift in the central point of the fuzzy bias factor; however, the range of the fuzzy bias interval does not change nearly as much as the change in range for pier width variations. This is also true of the fuzzy bias factors for various sediments gradations. The shift in the central point for changes in sediment gradation, however, is considerably greater than the shift in the central point for changes in the flow depth.

It is also useful to examine the fuzzy bias for combinations

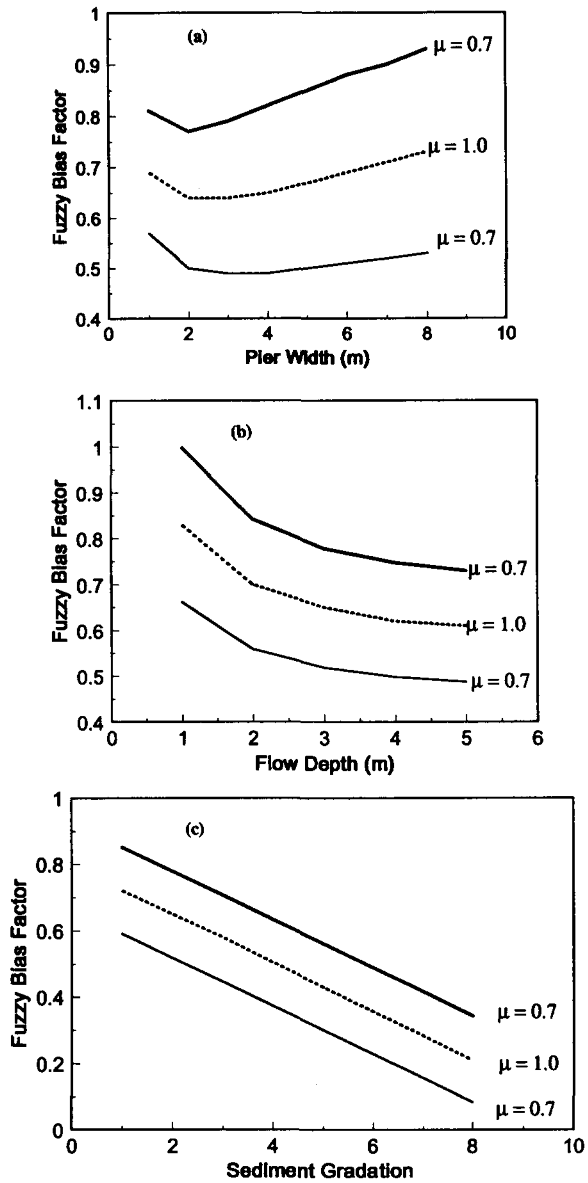

FIG. 13. Fuzzy Bias Factor as Function of Scour Parmeters and Membership Value

of values of the variables y, b, and G. Table 2 provides the bias for a membership value of 1; this is the bias with the greatest degree of belief. This shows a variability in the mean bias (for μ = 1, where μ = membership value), with a range from 0.04 for a small pier, low flow depth, and large sediment gradation to 1.02 for a large pier, low flow, and low sediment gradation. The fuzzy bias for a membership value of 0.7 is presented in Table 3. Two values of bias are given, representing the upper and lower bias at μ = 0.7. The bias at this degree of belief ranges from zero to 1.30. A fuzzy bias of zero implies that there is a possibility that no scour will occur even though the laboratory-based scour equation predicts that scour will occur. Fig. 13 shows these ranges in bias for selected cases. Variables that were held constant for each case had the following values: y = 3 m, b = 1.5 m, and G = 2. From Fig. 13(a) and Tables 2 and 3, it is clear that the bias range becomes large for large values of b; however, there is little change in the central value (at μ = 1). For y and G (Figs. 13b and c), the bias increases as the value of the variable increases.

CONCLUSIONS

Fuzzy linear regression was used to develop a bridge pier scour model based on field data for the purpose of determining modeling uncertainty in scour models based on laboratory data. The fuzzy regression equations provide a range of scour estimates for any given set of input data. Fuzzy regression is advantageous where there is considerable uncertainty in the model parameters. In the case presented in this paper, only data for round-nosed piers, subcritical flow, and live-bed conditions were used. The fuzzy regression equation developed here accounts only for effective pier width, flow depth, and sediment gradation.

The fuzzy bias associated with pier scour prediction developed here provides useful information in the application of pier scour equations currently available to engineers. The bias exists because of the use of small-scale laboratory results to predict large-scale, real-world problems. The range of values of the fuzzy bias factor represents uncertainty in the estimation of bias due to uncertainties in estimating pier scour.

The results of this study can be used to guide experimenters in their interpretation of small-scale laboratory test results. The fuzzy bias can be incorporated into laboratory-based models in the form of multiplicative correction factors to provide engineers with a better (more realistic) estimate of the predicted variable for field applications.

Engineers can use the fuzzy bias for adjusting scour estimates. Using the example for $b = 1.5$ m, $y = 3$ m, and $G = 2$, the fuzzy bias ranges from 0.52 to 0.79 for a degree of belief of 0.7, with a bias of 0.66 at $\mu = 1$. The engineer can base decisions on the design scour depth or expected scour depth using this range. On a very busy bridge, a high degree of belief as well as a high level of reliability is desired; therefore, the engineer would use the upper bias of 0.79. The scour depth estimated from the laboratory-based equation developed in this paper would then be multiplied by 0.79 to obtain the design depth. In another case, for a seldom used rural bridge, a high degree of belief would be combined with a lower reliability requirement; therefore, the mean fuzzy bias of 0.66 would be used.

The bias factors can also be used in developing reliability-based design methods of bridge piers for scour (Ang and Tang 1984). The factors are an essential step towards quantifying the reliability of real piers against scour. Once the reliability levels corresponding to current design practices are assessed, a calibration of the practices can be performed by developing safety factors to achieve desired (target) reliability levels.

Although fuzzy regression is a useful tool for incorporating uncertainty into model parameters, several points were raised to caution the user of potential problems in the application of fuzzy regression as a final prediction model. The writers are currently investigating improvements to the technique. However, fuzzy regression has been an effective tool in determining the differences between scour estimates based on laboratory analyses and field observations. Additional field data could be used to improve the model as that data become available.

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APPENDIX II. NOTATION

The following symbols are used in this paper:

A^* = fuzzy regression coefficient;
b = pier diameter;
c = width of fuzzy number;
d_{50} = mean sediment size;
D_s = scour depth;
G = sediment gradation;
h = threshold level;
R = correlation coefficient;
S_D = standard deviation of observed scour depth;
Se = standard error of estimate;
y = flow depth;
α = central value of fuzzy regression coefficient; and
μ = membership value.