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Reliability-Based Design Format for Marine Structures

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Recently there has been increased effort by classification societies and design authorities to bring reliability analysis into the design process. Typically these efforts involve using some reliability analysis method to calculate "partial safety factors" for design equations. The mean-value first-order second-moment method (MVFOSM), the advanced second-moment method (ASM), and an "exact" method have been used, or are being proposed for use, in the design of marine structures. In some recent work it has been shown that design formats based on the first two methods may result in engineering designs of different reliability levels than the ones specified in developing the design formats. These three methods are evaluated and a "reliability-conditioned (RC) method" is proposed in this paper. The proposed method overcomes the shortcomings of the MVFOSM and ASM methods and extends the concepts of the "exact" method in a more useful form to handle general types of problems. The RC method is believed to result in partial safety factors which give engineering designs of reliability levels equal to the specified ones.

Introduction

IN RECENT YEARS there has been a considerable effort invested to include reliability analysis in engineering designs. Typically this effort has focused on bringing into the design codes some statistical parameters to account for the various uncertainties in the design process. These parameters usually take the form of "safety" or "partial safety" factors which account for uncertainty without requiring the individual designer to perform a probabilistic analysis. This is a very reasonable approach and has been adopted by various organizations for proposed revisions to their codes [1–6].[3]

The principal difficulty with this approach is in determining a set of "partial safety" factors which will give the desired level of reliability. The limitations here include not only the oftentimes insufficient data on which to make statistical inferences, but also the manner in which the data are handled when available. The most commonly used methods for determining the partial safety factors include the mean-value first-order second-moment method (MVFOSM) and the advanced second-moment method (ASM). Recently Mansour et al [7] extended the "exact method" [8–11] to calculate partial safety factors. The present authors [12] and others [7] observed that the first two methods are unsatisfactory in many cases, sometimes providing less reliability than specified. While the approach of reference [7] gives accurate results, it is somewhat limited in scope. These weaknesses will be discussed in detail in the following sections.

The aim of this paper is to introduce a new approach, called the "reliability-conditioned (RC) method," which will more consistently and accurately evaluate the required partial safety factors for a wide variety of limit states. This method will be used to evaluate the longitudinal strength of a variety of ships and will be compared with the MVFOSM, ASM, and "exact" methods.

Existing reliability-based design methods

The purpose of engineering design is to insure the safety or performance of a given system for a given period of time and/or under a specified loading. The absolute safety of a system cannot be guaranteed due to the number of uncertainties involved. In structural design these uncertainties can be due to the simplified assumptions used in predicting the behavior of the structure under loading, our inability to determine in-place material properties accurately, randomness of loadings, etc. However, through probabilistic analysis we can limit the risk of unacceptable consequences.

That is not to say that all engineers and designers need to be deeply versed in probabilistic analysis. Rather the design criteria which they use should be developed in a format which is familiar to the users and which should produce desired levels of uniformity in safety among groups of structures without departing drastically from existing general practice.

One of the more popular formats for this type of design criterion is the load and resistance factor design (LRFD) as proposed for use by the American Petroleum Institute [1], the National Bureau of Standards [6], and other organizations [12]. The format can be expressed as

$$\phi R \geq \sum_{i=1}^{n} \gamma_i L_i \tag{1}$$

where ϕ is the resistance R reduction factor and γ_i the partial load effect L_i amplification factor.

Each of the methods of reliability analysis mentioned earlier will be used to evaluate the partial safety factors in equation (1) for ship longitudinal strength analysis. This will allow a comparison of the effectiveness of each method for reliability-based design. Before presenting the results of these applications a brief discussion of each method and how it determines the partial factors is provided. A detailed summary of the methods is given in reference [7].

Mean-value first-order second-moment method

For a structural element there are usually several limit states which constrain a design. These limit states are associated with the different modes of structural behavior and may be further classified into ultimate limit states and serviceability limit states. It is important that during the design process all applicable limit states be addressed. In general, these limit states or performance functions can be expressed as

$$M = g(X_1, X_2, \ldots, X_n) \tag{2}$$

where X_i are load and strength parameters considered as random variables

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The X_i's are called basic random variables and the limit state function $g(.)$ relates these variables for the limit state of interest. Failure according to the limit state occurs when $M < 0$.

The approximate mean and variance of M in equation (2) can be estimated by expanding $g(.)$ in a Taylor series about the mean values of X_i's and truncating the series at the linear terms [13–16] (thus mean value and first order). A measure of the safety is usually estimated in terms of a "safety index" β^* from the knowledge of the mean and variance of M

$$\beta^* = \mu_m/\sigma_m \tag{3}$$

If the distribution of M is known, then the probability of failure P_f can be found from the density function of M, $F_M(m)$. Usually the distribution of M is not known and an estimation of P_f is made by assuming M to be normally distributed, such that

$$P_f \cong 1 - \Phi\left(\frac{\mu_m}{\sigma_m}\right) \tag{4}$$

where Φ is the cumulative distribution function of the standard normal.

For the design format of equation (1), the limit state equation can be written as

$$M = R - L_1 - L_2 - \ldots - L_N \tag{5}$$

To evaluate the partial safety factors ϕ and γ_i, the statistics for the loads, that is, mean values and variances, the coefficient of variation of the resistance, and a specified level of risk in terms of β^* are needed. Using equations (3) and (5), and the Taylor series expansions for $g(.)$, the mean value of the resistance can be shown to be [13,14].

$$\bar{R} = (\bar{L}_1 + \bar{L}_2 + \ldots + \bar{L}_N) + \beta^* [\sigma_R^2 + (\sigma_{L_1}^2 + \sigma_{L_2}^2 + \ldots + \sigma_{L_N}^2)]^{1/2} \tag{6}$$

making use of the approximation that

$$(\sigma_i^2 + \sigma_j^2)^{1/2} \cong 0.75\,(\sigma_i + \sigma_j)$$

equation (6) can be rewritten as

$$(1 - 0.75\,\beta^*\,\Omega_R)\,\bar{R} \cong (\bar{L}_1 + \bar{L}_2 + \ldots + \bar{L}_N) + 0.75\,\beta^*\,(\sigma_{L_1}^2 + \sigma_{L_2}^2 + \ldots + \sigma_{L_N}^2)^{1/2} \tag{7}$$

where Ω_R is the coefficient of variation of the resistance.

Equation (7) can also be expressed as

$$\phi R = \gamma_1 L_1 + \gamma_2 L_2 + \ldots + \gamma_N L_N \tag{8}$$

where

$$\phi = 1 - 0.75\,\beta^*\,\Omega_R \tag{9}$$

$$\gamma_i = 1 + 0.75\,\beta^*\,\alpha\,\Omega_{L_i} \tag{10}$$

$$\alpha = \frac{(\sigma_{L_1}^2 + \sigma_{L_2}^2 + \ldots + \sigma_{L_N}^2)^{1/2}}{\sigma_{L_1} + \sigma_{L_2} + \ldots + \sigma_{L_N}} \tag{11}$$

There are three basic shortcomings with the MVFOSM method:

- If $g(.)$ is nonlinear and the linearization takes place at the mean values of the basic random variables, it is possible for the linearization point not to lie on the failure surface.
- The method fails to be invariant for different equivalent formulations of the same problem [7,12].
- Equation (4) is valid only when the basic random variables are normally distributed and the function $g(.)$ is linear in X_i's or when $g(.)$ is the product of log normally distributed basic random variables.

Advanced second-moment method

In order to overcome the aforementioned shortcomings of the MVFOSM method, the ASM method was proposed [17–19], in which the Taylor series expansion of $g(.)$ is linearized at some point on the failure surface rather at the mean values, say point $(X_{1^*}, X_{2^*}, \ldots, X_{n^*})$. The linearizing point is called the design point or the failure point. The selection procedure for the design point can be explained as follows. With the limit state and its variables as given by equation (2), the random variables' X's are first transformed to reduced uncorrelated variables with zero mean and unit variance. If the original basic variables' X's are uncorrelated, the transformation is given by

$$Y_i = \frac{X_i - \bar{X}_i}{\sigma_{X_i}} \tag{12}$$

If X's are correlated, they must be transformed to uncorrelated random variables. The procedures are quite involved and are beyond the scope of this paper [18].

The safety index β, according to the advanced second-moment method, is defined as the shortest distance to the failure surface from the origin in the reduced Y-coordinate system [17,18]. It has been further shown that the point on the failure surface at the minimum distance from the origin in the reduced coordinate system is the "most probable failure point." In the ASM method an iterative solution can be performed until converging on a minimum value of β by appropriately choosing the design point on the failure surface and performing a "first-order analysis."

The safety index β can be calculated in both the reduced coordinate system (Y-coordinate) or in the original coordinate system (X-coordinate). Using the original coordinate system, the failure point $(X_{1^*}, X_{2^*}, \ldots, X_{n^*})$ and the safety index β are determined by solving iteratively the following system of equations [15,17,18]:

$$\alpha_i = \frac{(\partial g/\partial X_i)\,\sigma_{X_i}}{\left[\sum_{i=1}^{n} (\partial g/\partial X_i)^2\,\sigma_{X_i}^2\right]^{1/2}} \tag{13}$$

$$X_{i^*} = \bar{X}_i - \alpha_i\,\beta\sigma_{X_i} \tag{14}$$

$$g(X_{1^*}, X_{2^*}, \ldots, X_{n^*}) = 0 \tag{15}$$

where the derivatives $\partial g/\partial X_i$ are evaluated at $(X_{1^*}, X_{2^*}, \ldots, X_{m^*})$,

Nomenclature

M = measure of structural performance (margin of safety)
μ_m, σ_m = mean value and standard deviation of M
ϕ, γ_i = resistance and load partial safety factors
X_i = basic random variable of load or strength
β^* = safety index using MVFOSM method
β = safety index using ASM method
P_f = probability of failure
F_x, f_x = cumulative distribution function (CDF) and probability density function (PDF) of variable x
Φ = cumulative distribution function of the standard normal variate
R, L_i = resistance and load variables
$\bar{R}_i, \bar{L}_i$ = mean values of resistance and load variables
Ω_R, Ω_{L_i} = coefficient of variation of a random variable R and L_i
α_x = directional cosine of random variable x
X_n^* = value of variable x_n on failure surface
Y_i = reduced uncorrelated variables with zero mean and unit standard deviation
m_0 = value of deterministic stillwater bending moment
λ = mean value of wave bending moment (long-term analysis)
L_{wl} = length of the waterline, ft

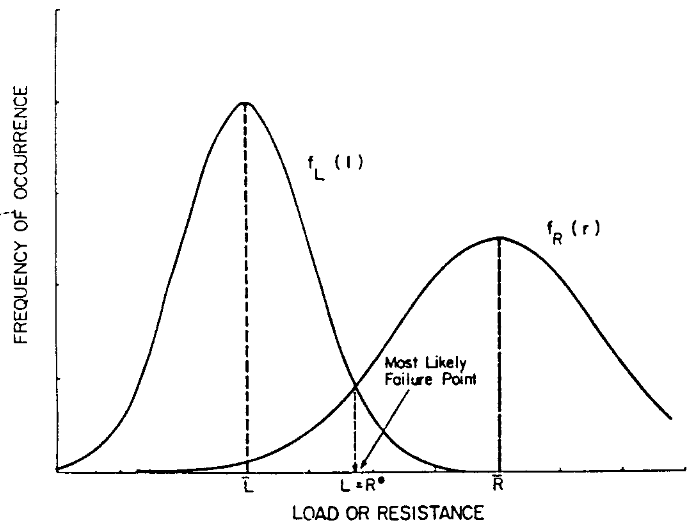

Fig. 1 Most likely failure point for two random variables

and α_i is the directional cosine of random variable X_i. The relationship between the probability of failure, P_f, and β is the same as given by equation (4). For applications of this method, see references [12,15,18–20].

In many structural engineering problems, the design variables are non-normal. According to Rackwitz and Fiessler [21] non-normal probability distributions may be incorporated in the aforementioned reliability analysis by transforming the non-normal variables into equivalent normal variables on the failure surface. An equivalent normal distribution is one where the cumulative distribution function (CDF) and the probability density function (PDF) are the same for both the actual and equivalent normal distributions at the failure point. This requires that a new equivalent normal distribution be calculated at each iteration. This approximation of non-normal distributions becomes more and more inaccurate if the original distributions are increasingly skewed.

For design problems then, given the probability distributions and statistics of the load effects, the distribution and coefficient of variation of the resistance, a linear limit state equation [equation (5)], and a specified safety index, the mean value of the resistance and the partial safety factors ϕ and γ_i's can be shown to be [18]

$$\phi = 1 - \alpha_R^* \beta \Omega_R \tag{16}$$

$$\gamma_i = 1 - \alpha_{L_i}^* \beta \Omega_{L_i} \tag{17}$$

$$\bar{R} = R^*/\phi \tag{18}$$

where Ω is the coefficient of variation of the random variable and α_i^* is the direction cosine [equation (13)] evaluated at the most likely failure point in the reduced coordinates.

At the failure point $(R^*, L_1^*, \ldots, L_N^*)$ then, the limit state equation is given by

$$M = R^* - L^*_1 - L^*_2 - \ldots - L^*_N = 0 \tag{19}$$

It should be noted, however, that the method can be used for both linear and non-linear limit state equations [18,19].

The main shortcoming of the ASM method is that the iterative numerical solution of equations (13) to (15) may not converge to a correct β, especially for cases where the limit state equation has many variables, and/or some non-normal random variables are highly skewed [15].

Additionally, recent work by the authors [12] has shown that it is possible for both the MVFOSM and ASM methods to find partial safety factors which when used in a design equation produce designs of a reliability level other than that originally specified. This was increasingly noticed in cases involving highly non-normal load distributions and seems to be a result of the manner in which the partial safety factors depend on α_i^* in equations (16) and (17). It can be shown [12] that the directional cosines α_i^* to which the ASM method convenges in equations (13) to (15) are not unique. That is, there exists for any point on the failure surface a set of directional cosines α_i which could also satisfy those equations. Thus, a solution for partial safety factors which involves the directional cosines [equations (16) and (17)] may be inconsistent and somewhat arbitrary.

The "exact" Level III method

This method has been used to determine the probability of failure for oceangoing vessels [7,9,11]. The method simplifies the exact multiple integration required to evaluate the probability of failure in order to get an *approximate* closed-form solution [8]. A linear limit state equation is given by

$$M = g(X_1, X_2) = g(R, L) = R - L \tag{20}$$

where R is the strength or resistance and L is the load, made up of a stillwater component and a wave component.

With the strength normally distributed, the wave component exponentially distributed, and a deterministic stillwater component, the probability of failure can be closely approximated by [8]:

$$P_f \cong \left[1 - \Phi\left(\frac{\mu_r - m_0}{\sigma_r}\right)\right] + \Phi\left(\frac{\mu_r - m_0}{\sigma_r} - \frac{\sigma_r}{\lambda}\right) \times \exp\left[-(\mu_r - m_0)/\lambda + \sigma_r^2/2\lambda^2\right] \tag{21}$$

where

Φ = CDF of standard normal variate
μ_r = mean value of strength
σ_r = standard deviation of strength
m_0 = deterministic stillwater component
λ = mean value of wave component

The derivation of this equation and a similar one for the case of normally distributed stillwater loads is given in detail in reference [9].

This method was extended to determine partial safety factors in reference [7]. The key for extending the method is that for the limit state as given in equation (20) with the load and strength considered statistically independent, the joint probability density function is given by

$$f_{R,L}(r, \ell) = f_R(r) \cdot f_L(\ell) \tag{22}$$

Then the most probable failure point is that point on the failure surface where $r = \ell$ and

$$f_R(r) = f_L(\ell) \tag{23}$$

This is shown in Fig. 1. One can then simply solve for the point $x^* = R^* = L^*$ by equating the PDF of the load and strength. Once the coordinates of the failure point (R^*, L^*) are known, then equation (18) can be used to find ϕ and λ_L. Note that the values for R^* will be less than $\bar{R}$, so ϕ will be less than one; and values of L^* will be greater than $\bar{L}$, so γ_L will be greater than one.

The weakness with this approach is that it is limited to cases where the limit state equation can be expressed in terms of two variables, that is, equation (20). It does, however, point the way to a more general solution and has been shown to provide partial safety factors which give levels of safety equal to those specified.

Reliability-conditioned partial safety factors

In order to develop a more widely applicable and consistent method to accurately determine the required partial safety factors, a slightly different approach to the problem is taken. First of all, the method would be most likely used by design authorities or classification societies to generate partial safety factors for design codes; thus a simple design code format like that given by equation (1) would be the limit state.

For cases where a new limit state is being evaluated or an existing code doesn't give adequate safety, the design authorities would be looking for a level of resistance or strength which would provide the necessary level of safety. They would also want to generate the partial safety factors which, when given the loads and their statistics, would be able to arrive at that value of resistance. In this case there are two steps to the problem: first, finding a mean value of R, given the statistics of the loads, the coefficient of variation and distribution of R, and the required level of safety in terms of β or P_f, and second, after finding $\bar{R}$, determining the most likely failure point and then the partial safety factors.

For cases where the existing code provides a level of strength deemed adequate and all that is desired is to change to an LRFD format, the first of the two aforementioned steps would be skipped. This is called code calibration.

To solve the problem of finding a mean value of R for the desired level of safety, the authors recommend using simulation with variance reduction techniques (VRT's). This method is discussed by the authors in references [15] and [20]. The mean value of $\bar{R}$ can be found using an appropriate iterative scheme and the simulation with VRT's.

To find the most likely failure point when the limit state equation contains only two random variables, equation (23) must be solved. To do this, first choose a value of R^* which is less than $\bar{R}$. Then solve the limit state equation for L^*, insuring that R^* and L^* are on the failure surface

$$R^* - L^* = 0 \tag{24}$$

Evaluate if equation (23) is true, that is

$$f_R(R^*) \overset{?}{=} f_L(L^*) \tag{25}$$

If not, pick a new value of R^* and repeat. Once equation (25) is within acceptable limits of being true, then load and resistance factors are found from

$$\phi = \frac{R^*}{\bar{R}}; \qquad \lambda = \frac{L^*}{\bar{L}} \tag{26}$$

This is essentially a generalization of the "exact" method for distributions other than those specified in equation (21).

When the limit state equation has more than two random variables, that is, $(R, L_1, L_2, \ldots, L_N)$, then some additional steps are required. The most likely failure point is still defined as in equations (24) and (25), but now looks like

$$f_R(R^*) - f_{L_1, L_2, \ldots, L_N}(L_{1*}, L_{2*}, \ldots L_{N*} = 0) \tag{27}$$

where the second term is the joint probability density function of the loads evaluated at the failure point. This is shown graphically in Fig. 2.

For statistically independent loads equation (27) becomes

$$f_R(R^*) - f_{L_1}(L_{1*}) \cdot f_{L_2}(L_{2*}) \cdot \ldots \cdot f_{L_N}(L_{N*}) = 0 \tag{28}$$

The values of the basic variables are still required to be on the failure surface, that is

$$R^* - L_{1*} - L_{2*} - \ldots - L_{N*} = 0 \tag{29}$$

and to further bound the possible combinations of values which satisfy equations (28) and (29), a percentile requirement on the

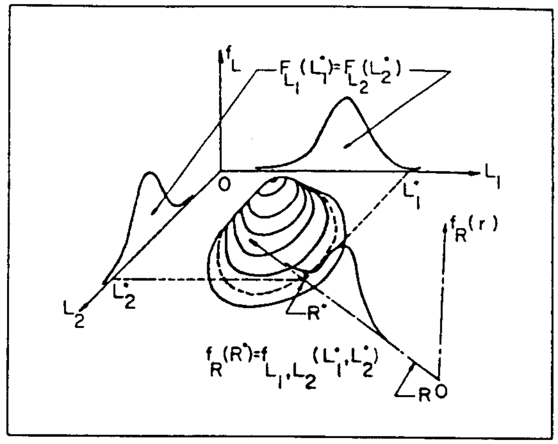

Fig. 2 RC most likely failure point for three random variables

loads is specified. That is, the likelihood of exceeding the failure value of any load effect L_i is the same as the likelihood of exceeding the failure value of any other load effect. This is given by

$$F_{L_i}(L_{i*}) - F_{L_{i+1}}(L_{i+1*}) = 0, \quad i = 1, N-1 \tag{30}$$

This set of equations (28,29,30) must be solved simultaneously for the most likely failure point. A proposed solution scheme takes advantage of the form of the equations and is given as follows:

1. Choose a value of $R^* < \bar{R}$. A value of $\bar{R} - (2 \cdot \sigma_R)$ has been found to be a good starting point.
2. Solve equations (29) and (30) using an iterative Newton's method for nonlinear simultaneous equations. For the case of linear limit state equations and multiple loads, this method is shown, in matrix form, to be

$$\begin{bmatrix} \frac{\partial M}{\partial L_1} & \cdots & \frac{\partial M}{\partial L_{N-1}} & \frac{\partial M}{\partial L_N} \\ f_{L_1}(L^*_{1,k-1}) & \cdots & 0 & -f_{L_N}(L^*_{N,k-1}) \\ \cdot & & \cdot & \cdot \\ \cdot & & \cdot & \cdot \\ \cdot & & \cdot & \cdot \\ 0 & \cdots & f_{L_{N-1}}(L^*_{N-1,k-1}) & -f_{L_N}(L^*_{N,k-1}) \end{bmatrix}$$

$$\times \begin{bmatrix} \Delta L_{1,k-1} \\ \Delta L_{2,k-1} \\ \cdot \\ \cdot \\ \cdot \\ \Delta L_{N,k-1} \end{bmatrix} = \begin{bmatrix} -R^* + (L^*_{1,k-1} + L^*_{2,k-1} + \ldots + L^*_{N,k-1}) \\ -F_{L_1}(L^*_{1,k-1}) \quad + F_{L_N}(L^*_{N,k-1}) \\ \cdot \\ \cdot \\ \cdot \\ -F_{L_{N-1}}(L^*_{N-1,k-1}) + F_{L_N}(L^*_{N,k-1}) \end{bmatrix} \tag{31}$$

where $L^*_{i,k}$ is the failure value for the ith load in the kth iteration, and $\Delta L_{i,k}$ is the incremental change in the failure value. Then the most likely failure point for the Kth iteration is given by

$$L^*_{i,k} = L^*_{i,k-1} + \Delta L_{i,k-1}, \quad i = 1, \ldots, N \tag{32}$$

The initial estimate of the values for the failure point for the $k = 1$ iteration should be positive and larger than the mean values of the loads.

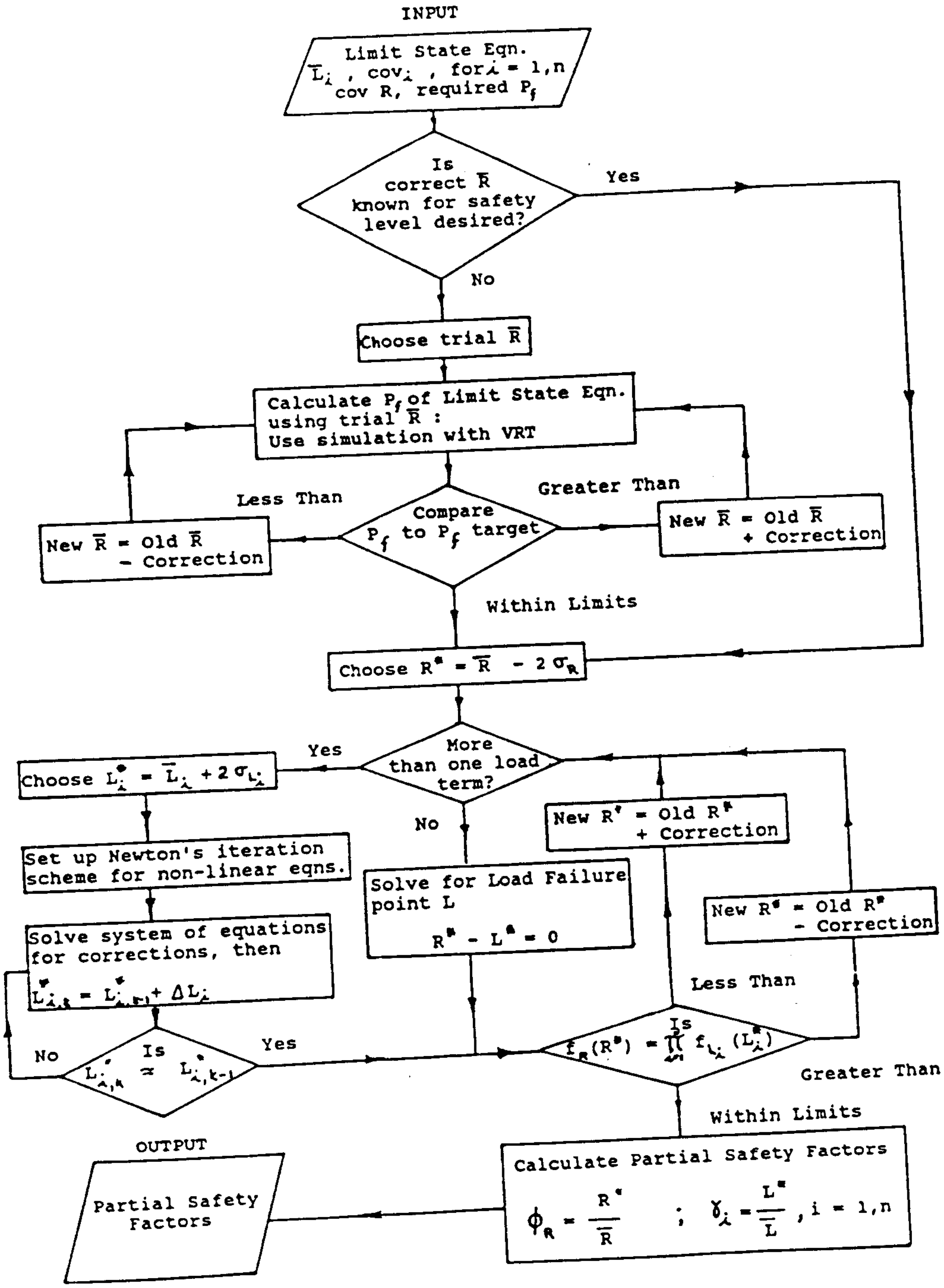

Fig. 3 Flow chart for RC method

3. Check the values from equation (32) to see if they satisfy equation (28). If not, modify R^* accordingly and go back to Step 2.

4. Find the partial safety factors using equation (26).

The RC method is shown here for a linear limit state equation of the form of equation (5). This form of limit state was chosen for an example because the RC method is intended for use by design authorities for the generation of partial safety factors for LRFD format design equations. However, as long as the limit state equation can be expressed as "resistance" minus "load," where the loads can be assumed to be statistically independent, nonlinear load relationships can be handled by the RC algorithm. A flow chart illustrating the ease of computer implementation of this approach is shown in Fig. 3. Further examples and discussion of this method is provided by the authors in reference [12].

Comparison of methods by examples

Two-variable limit state

In order to best explain the merits and shortcomings of each method, two sets of example problems were investigated. The first set, presented by Mansour [7], calculates the partial safety factors for the longitudinal strength of 18 oceangoing vessels. They are the same vessels as used in several earlier works [9–11]. The limit state equation is of the form

Table 1 Principal particulars of 18 sample ships [7]

Ship No.	LBP, ft	B, ft	d, ft	C_b	dwt (approx.)
1	1076.00	174.87	81.40	0.86	326 600
2	1069.25	163.25	58.05	0.83	206 100
3	1000.00	154.76	60.45	0.83	190 800
4	763.00	115.99	42.01	0.77	67 900
5	754.70	104.46	44.40	0.82	66 500
6	754.70	105.65	44.74	0.809	63 300
7	754.69	105.65	44.74	0.804	62 000
8	693.75	97.00	39.17	0.775	46 650
9	719.10	82.50	39.15	0.786	40 970
10	620.81	85.96	35.72	0.784	31 500
11	594.00	74.00	33.48	0.800	26 580
12	775.00	105.50	47.00	0.831	75 500
13	700.65	98.43	40.70	0.774	45 100
14	528.50	75.99	29.88	0.615	13 400
15	520.00	75.00	31.42	0.573	12 750
16	528.00	76.00	29.80	0.609	13 400
17	800.00	106.00	44.55	0.840	74 200
18	656.20	93.80	42.63	0.793	48 550

Table 2 Partial safety factors using each method—18 ships

	MVFOSM [7]		Advanced Second Moment [7]		"Exact" [7]		Reliability Conditioned	
Ship No.	ϕ_S	γ_Z	ϕ_S	γ_Z	ϕ_S	γ_Z	ϕ_S	γ_Z
1	0.348	1.086	0.370	1.135	0.533	1.663	0.533	1.663
2	0.230	1.234	0.287	1.391	0.478	2.565	0.478	2.566
3	0.208	1.352	0.289	1.596	0.474	3.087	0.474	3.087
4	0.407	1.099	0.441	1.156	0.581	1.568	0.581	1.568
5	0.349	1.075	0.366	1.116	0.529	1.630	0.529	1.630
6	0.326	1.067	0.339	1.103	0.507	1.659	0.507	1.659
7	0.330	1.067	0.343	1.103	0.510	1.648	0.510	1.648
8	0.385	1.105	0.420	1.166	0.567	1.626	0.567	1.625
9	0.342	1.117	0.378	1.189	0.539	1.762	0.539	1.762
10	0.386	1.119	0.427	1.190	0.570	1.654	0.570	1.654
11	0.349	1.219	0.438	1.354	0.563	1.941	0.563	1.941
12	0.326	1.065	0.347	1.099	0.512	1.627	0.512	1.624
13	0.304	1.154	0.350	1.252	0.519	1.973	0.519	1.973
14	0.444	1.153	0.510	1.243	0.615	1.598	0.615	1.598
15	0.432	1.071	0.452	1.109	0.590	1.461	0.590	1.462
16	0.596	1.182	0.682	1.263	0.705	1.399	0.705	1.398
17	0.200	1.394	0.287	1.671	0.472	3.290	0.472	3.292
18	0.294	1.184	0.350	1.302	0.518	2.084	0.518	2.084

$$M = R - L$$

where

R = ship longitudinal strength, ft-tons
L = load, ft-tons, made up of a deterministic stillwater component and an exponential wave component
$= L_{sw} + L_w$

Because of the deterministic stillwater component, the distribution of L can be considered to be a shifted exponential distribution for the purpose of calculating the PDF and CDF as follows:

$$f_L(1) = \frac{1}{\mu_w} e^{-(1-m_0)/\mu_w} \quad 1 \geq m_0$$
$$= 0 \quad 1 < m_0 \tag{33}$$

and

$$F_L(1) = 1 - e^{-(1-m_0)/\mu_w} \quad 1 \geq m_0$$
$$= 0 \quad 1 < m_0 \tag{34}$$

where m_0 is the value of the stillwater bending moment in foot-tons and μ_w is the mean value of the wave bending moment.

In reference [7] the MVFOSM, ASM, and the "exact" methods were used to find the partial safety factors for the 18 ships whose characteristics are given in Table 1. The results are given in Table 2 along with the results of using the RC method. It should be noted that the γ_L values are multiplied by the sum of the mean value of the wave μ_L and the stillwater component m_0. It is obvious from Table 2 that the MVFOSM and ASM methods give partial safety factors that are unreasonably low and not consistent. The "exact" and RC methods give the same values. This is due to the fact that the MVFOSM and ASM methods do not converge on the correct "most likely" failure point. In the case of the MVFOSM method this is due to the assumption that all variables are normally distributed. In the ASM method, the approximation of the equivalent normal distribution does not adequately account for the shape of the shifted exponential distribution.

The partial safety factors for the "exact" and RC methods are the same because both approaches require the most likely failure point to satisfy equations (23) or (25). For this type of problem these two methods are just different schemes for solving the same equations. The advantage of the RC method is its ability to be applied in a much wider variety of cases and its ease of computer implementation.

Partial safety factors for code calibration

In order to demonstrate the use of these methods for code calibration and to point out some of their differences, a second set of examples is investigated. Again, this set is from Mansour [7]. The American Bureau of Shipping's (ABS) 1982 *Rules for Building and Classing Steel Vessels* were used to find the minimum hull strengths and loadings required for the longitudinal strength of ten Series 60 ships, with $C_B = 0.70$ and $L/B = 7.0$. Several assumptions were made regarding the distributions and coefficients of variation of the strength and loads as well as the exceedance level of the wave loads implied in the rules. The mean values and coefficients of variation of the parameters used for the following calculations are given in Table 3. A detailed explanation of how these values were found is given in reference [7].

The goal in this example is to determine partial safety factors for an LRFD format implementation of the ABS rules for longitudinal strength using the four methods. These factors should then provide the required mean value of strength for the safety level desired when the loads are given.

Table 3 Load and strength parameters for Series 60 ships, using ABS rules

Ship Length	Mean Still Water (ft-tons)	Mean Wave BM (ft-tons)	Mean Strength (ft-tons)
300	2.3164E+04	7.7494E+03	8.6403E+04
400	5.8329E+04	2.0212E+04	2.1943E+05
500	1.2053E+05	4.3088E+04	4.5642E+05
600	2.2137E+05	8.1002E+04	8.4017E+05
700	3.7226E+05	1.3902E+05	1.4140E+06
800	5.9405E+05	2.1885E+05	2.2145E+06
900	8.7463E+05	3.2171E+05	3.2424E+06
1000	1.2240E+06	4.4794E+05	4.5031E+06
1100	1.6519E+06	5.9620E+05	6.0001E+06
1200	2.1630E+06	7.7404E+05	7.7815E+06

NOTE: All values calculated following procedure in reference [7].

Table 5 Partial safety factors using RC method

		Reliability-Conditioned		
Ship	Length ft	ϕ_S	γ_{SW}	γ_W
1	300	0.5297	1.127	2.537
2	400	0.5158	1.119	2.371
3	500	0.5052	1.112	2.241
4	600	0.4963	1.107	2.122
5	700	0.4889	1.102	2.023
6	800	0.4833	1.095	1.916
7	900	0.4779	1.091	1.849
8	1000	0.4734	1.088	1.787
9	1100	0.4697	1.084	1.724
10	1200	0.4663	1.080	1.668

Table 4 Partial safety factors using MVFOSM and ASM methods

		Mean-Value First-Order Second Moment				Advanced Second Moment [7]			
Ship	Length ft	ϕ_S	γ_{SW}	γ_W	β*	ϕ_S	γ_{SW}	γ_W	β
1	300	0.6554	1.076	4.091	4.705	0.8605	1.031	6.515	3.420
2	400	0.5775	1.165	1.691	4.650	0.8662	1.029	6.434	3.364
3	500	0.6709	1.072	4.107	4.595	0.8710	1.028	6.348	3.311
4	600	0.6777	1.070	4.106	4.542	0.8750	1.027	6.261	3.265
5	700	0.6844	1.069	4.102	4.489	0.8780	1.027	6.185	3.227
6	800	0.6889	1.069	4.073	4.439	0.8783	1.027	6.117	3.193
7	900	0.6913	1.069	4.062	4.414	0.8807	1.027	6.085	3.179
8	1000	0.6935	1.069	4.050	4.390	0.8808	1.027	6.046	3.170
9	1100	0.6950	1.070	4.031	4.367	0.8809	1.027	6.018	3.160
10	1200	0.6967	1.070	4.016	4.346	0.8812	1.027	5.988	3.149

* All calculations assumed still water moment COV = .091, strength COV = 0.1, and Wave COV = 1.0

The LRFD format chosen is the same as in equation (1), which for this case is

$$\phi R \geq \gamma_{sw} L_{sw} + \gamma_w L_w \tag{35}$$

For this analysis the stillwater load effect is considered a normally distributed random variable with a coefficient of variation of 0.091 and therefore must be accounted for explicitly. The strength is considered to be normally distributed with a coefficient of variation of 0.10 and the wave loads are exponentially distributed (a Weibull distribution with $k = 1$ and $\lambda = \mu_w$; a long-term analysis [9]).

The partial safety factors for the MVFOSM and the ASM method are given in Table 4. Note that the MVFOSM and ASM methods use different target safety indices for each ship. In both of these methods the ABS rule values for loads and strength are first evaluated to determine the prescribed level of safety, measured by the safety index. This safety index is then used to find partial safety factors which, when the loads are given, allow the designer to find a value of resistance which gives the prescribed level of safety. Both methods are dependent on having a β-value, which in turn requires the assumption that all random variables are normally distributed when determining the probability of failure. The safety indices may also give a false sense of the level of safety implied by use of the partial safety factors. Clearly, there is a significant difference between the safety indices using each approach. The question is, which is the correct one? Recent work [7,12,20] has shown that in many cases neither is correct, and in fact they both err on the nonconservative side.

The "exact" method cannot be used to evaluate the partial safety factors for formats such as equation (35) because the method does not account for the loads separately. This is due to the fact that there are a number of solutions to equation (27) [which is the multidimensional equivalent of equation (23)] and the method provides no means to determine one among them. Therefore, the "exact" method as it currently exists would not be suitable.

The partial safety factors as determined by the RC method are given in Table 5. It is interesting to compare the results of the RC method approach with that of the ASM. It is obvious that

Table 6 PDF's for variables at the failure points for Series 60 ships, using ABS rules for finding mean values

		Reliability-Conditioned Method					Advanced Second Moment Method				
Ship	Length ft	Still Water (1)	Wave (2)	Product (1*2)		Strength	Still Water (1)	Wave (1)	Product (1*2)		Strength
1	300	7.134E-05	1.020E-05	7.279E-10	=	7.283E-10	1.786E-04	1.911E-07	3.413E-11	<	1.745E-05
2	400	3.212E-05	4.620E-06	1.484E-10	=	1.477E-10	7.144E-05	7.900E-08	5.644E-12	<	7.429E-06
3	500	1.705E-05	2.468E-06	4.207E-11	=	4.218E-11	3.470E-05	4.060E-08	1.409E-12	<	3.804E-06
4	600	9.917E-06	1.479E-06	1.467E-11	=	1.468E-11	1.895E-05	2.360E-08	4.472E-13	<	2.174E-06
5	700	6.318E-06	9.512E-07	6.010E-12	=	6.014E-12	1.127E-05	1.480E-08	1.668E-13	<	1.340E-06
6	800	4.260E-06	6.728E-07	2.866E-12	=	2.867E-12	7.063E-06	1.010E-08	7.133E-14	<	8.592E-07
7	900	3.025E-06	4.893E-07	1.480E-12	=	1.483E-12	4.797E-06	7.110E-09	3.410E-14	<	6.040E-07
8	1000	2.251E-06	3.740E-07	8.416E-13	=	8.427E-13	3.427E-06	5.330E-09	1.827E-14	<	4.354E-07
9	1100	1.736E-06	2.993E-07	5.195E-13	=	5.196E-13	2.539E-06	4.080E-09	1.036E-14	<	3.271E-07
10	1200	1.372E-06	2.436E-07	3.343E-13	=	3.344E-13	1.940E-06	3.240E-09	6.284E-15	<	2.532E-07

* Strength COV = .10 (Normal), Still Water COV = .091 (Normal), and Wave COV = 1.0 (Exponential)

Table 7 CDF's of the load variables at the failure points for Series 60 ships, using ABS rules for finding load mean values

		Reliability-Conditioned		Advanced Second Moment	
Ship	Length ft	Still Water Bending Moment	Wave Bending Moment	Still Water Bending Moment	Wave Bending Moment
1	300	0.91879	0.92093	0.63333	0.99852
2	400	0.90386	0.90661	0.62502	0.99839
3	500	0.89089	0.89367	0.62084	0.99825
4	600	0.88025	0.88025	0.61665	0.99809
5	700	0.86776	0.86776	0.61665	0.99794
6	800	0.85276	0.85276	0.61665	0.99779
7	900	0.84258	0.84258	0.61665	0.99772
8	1000	0.83248	0.83248	0.61665	0.99763
9	1100	0.82156	0.82156	0.61665	0.99756
10	1200	0.81142	0.81142	0.61665	0.99749

* All calculations assumed still water moment COV = .091 (Normal) and wave moment COV = 1.0 (Exponential)

there is a significant difference in the values of the partial safety factors between the two approaches. Yet given the values of the loads, both sets of partial safety factors when used in equation (35) produce the same level or resistance. The advantage of the RC approach is that it is not dependent on determining a value of β. For code calibration it can quickly and easily determine the partial safety factors without first determining a probability of failure. In addition, while the ASM solution may not be unique, the RC method provides a consistent logic which defines a unique, most likely failure point in terms of the PDF's and CDF's of the random variables. In Table 6 the PDF's for the variables at the failure points are compared. For the RC method the product of the load PDF's is approximately equal to the strength PDF. This is not the case for the ASM approach, confirming that it did not select the most likely failure point. Table 7 gives the cumulative distribution functions of the load variables at the failure point. For the RC method the CDF's of the load are approximately equal, but for the ASM method they are considerably different. This would indicate that the failure point value of the stillwater load effect in the ASM case is too low, because there is a significant chance of exceeding it. If this value is too low, then to remain on the failure surface the value of the wave load failure point is too high, and there is almost no chance of exceeding the value.

The values determined for the partial safety factors are plotted in Fig. 4 along with β for each ship length. This shows that the reliability implied by the ABS rules decreases with increasing ship length [7]. But the values for ϕ and γ_{sw} remain fairly constant. Only the γ_w changes with length and that is nearly linear.

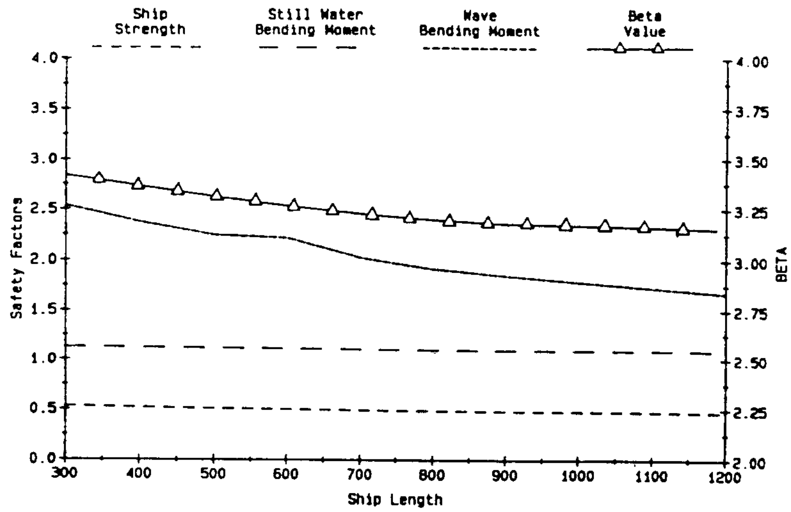

Fig. 4 Partial safety factors—calibrated using ABS rules for vessel strength: Series 60; $C_B = 0.70$; $L/B = 7.0$

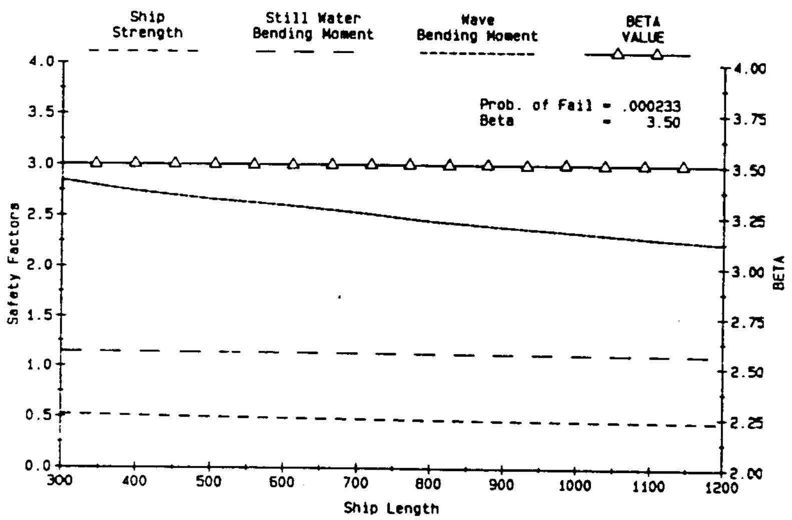

Fig. 5 Partial safety factors—constant P_f—calibrated using ABS rules for loads: Series 60; $C_B = 0.70$; $L/B = 7.0$

Using a linear regression for determining an equation in terms of ship length to fit the γ_w curve, a very reasonable equation for longitudinal strength could be written

$$0.49\,\bar{R} = 1.1\,\bar{L}_{sw} + \left(2.73 - 0.094\,\frac{L_{wl}}{100}\right)\bar{L}_w \qquad (36)$$

or

$$\phi\bar{R} = \gamma_{sw}\bar{L}_{sw} + \gamma_w\bar{L}_w \qquad (36a)$$

where

$$\phi = 0.49$$

$$\gamma_{sw} = 1.1$$

$$\gamma_w = 2.73 - 0.94\,\frac{L_{wl}}{1000} \qquad (36b)$$

L_{wl} = length of ship, ft

Equations (36*a*) and (36*b*) would represent the ABS Longitudinal Strength Rules in an LRFD format. An alternative approach would be to require a constant value of β or P_f for all lengths. Figure 5 shows the partial safety factors for the ten ships using a constant $P_f = 0.000\,233$ or $\beta = 3.5$. Linear regressions for the partial factors would result in an equation for the longitudinal strength in the same form as equation (36*a*), but with

$$\phi = \left(0.5314 - 0.0723\,\frac{L_{wL}}{1000}\right)$$

$$\gamma_{sw} = \left(1.1502 - 0.0315\,\frac{L_{wL}}{1000}\right) \qquad (37)$$

$$\gamma_w = \left(3.0 - 0.67\,\frac{L_{wl}}{1000}\right)$$

Summary and conclusion

The mean-value first-order second-moment (MVFOSM) and the advanced second-moment (ASM) methods are being used to determine partial safety factors. Neither of the methods can consistently and accurately handle random variables of other than normal distributions. The ASM method does not provide a logic which can uniquely define a failure point and thus partial safety factors. It is also possible that design formats based on these methods may result in engineering designs of different reliability levels than those specified in developing the design formats. The "exact" method has not been used to date, but shows considerable promise for the types of problems for which it is suited. This method correctly finds the most likely failure points and gives the correct partial safety factors for the reliability level specified. It is currently limited to cases of two variables. All three methods are evaluated in this paper. A reliability-conditioned (RC) method is proposed here to overcome the shortcomings of the MVFOSM and ASM methods. For two-variable problems it provides the same solutions as the "exact" method. However, the RC method has the additional capability of handling multivariable linear limit states with any probability distribution and may easily be extended to nonlinear limit states. The RC method can be utilized in the development of LRFD format design codes or in the calibration of existing codes. When developing new codes the required value of resistance for a given safety level is first found using any of several methods, including simulation with variance reduction techniques. Then the RC method is used to find the partial safety factors required to give the desired level of safety. For code calibration, only the latter step is required.

The method proposed in this paper is but another tool for use in reliability-based design. Much work remains to be done in evaluating the uncertainties associated with the loads, resistance and the mathematical procedures used. As more work is done in this area, the RC method will be available to quickly apply the new knowledge.

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