Reliability and Uncertainty Evaluation for Longitudinal Bending of Hull Girders of Surface Ships

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Structural reliability and uncertainty assessment of hull-girder bending of surface ships requires the consideration of the following three aspects: (1) structural strength, (2) loads, and (3) methods of reliability analysis. A methodology was developed to assess the reliability of hull girders subjected to extreme bending moments. The methodology is based on ultimate strength assessment of hull-girder bending using an incremental strain compatibility method. Two reliability assessment methods—(1) advanced second moment (ASM) method, and (2) Monte Carlo simulation (MCS) method with variance reduction techniques—were employed for structural reliability assessment of hull-girder bending of surface ships. This study demonstrates the structural reliability evaluation of hull-girder bending of ships (1) by considering its strength parameters as random variables, and (2) by considering a non-closed performance function using both reliability methods. This technique can also be applied to a closed-form expression of the performance function. Utilizing such a methodology, ultimate strength and loads for hull-girder bending can be developed individually as modules, and then combined into a non-closed form in a performance function. Simulation methods are also used to assess the uncertainty in hull girder strength due to uncertainties in basic random variables. Examples using the ASM and MCS methods for reliability assessment are also presented.

1. Introduction

STRUCTURAL reliability assessment of ships requires the consideration of the following three aspects: (1) loads, (2) structural strength, and (3) methods of reliability analysis. The loads need to be defined in terms of their probabilistic characteristics based on a sea-operational profile of a ship. Extreme analysis and stochastic load combinations are also needed for ship structural reliability assessment. The probabilistic characteristics of the structural strength of the ship need to be evaluated based on its basic strength variables, prediction models, and associated uncertainties including modeling errors. Serviceability and strength failure modes need to be considered at different levels of the ship, i.e., hull girder, grillage, panel, plate, and details. Reliability assessment methods need to be selected based on the available information, desired accuracy, and available resources for performing the assessment. The reliability assessment methods result in reliability indices and failure probabilities for the failure modes of interest at the different levels. The resulting reliability can be compared with target reliability levels as a checking procedure. Reliability assessment is needed for reliability-based analysis of existing structures as well as in reliability-based design or the development of reliability-based design criteria.

A probability-based reliability and uncertainty assessment methodology for the hull-girder bending of ships was developed, and is described in this paper. The computational procedure of the methodology was developed in a modular form. The development of reliability-based analysis for ship structures needs to be performed at several levels of structural response and strength. These levels are the hull girder, grillage, panel, plates, and details. The development, in this paper, of the reliability assessment methodology for hull-girder bending of ships can be viewed as a prototype approach. Similar procedures can be developed for other failure modes and/or levels.

The development of the methodology required the definition of a performance function for hull-girder bending, development of a library of probability functions (Ayyub & Chao 1994), development of the strength module, development of structural reliability assessment module, development of user interfaces, and selection and performance of test cases.

2. Structural reliability assessment

The reliability of an engineering system can be defined as its ability to fulfill its design purpose for some time period. The theory of probability provides the fundamental basis to measure this ability. The reliability of a structure can be viewed as the probability of its satisfactory performance according to some performance functions under specific service and extreme conditions within a stated time period. In estimating this probability, system uncertainties are modeled as random variables with mean values, variances, and probability distribution functions. Many methods have been proposed for structural reliability assessment purposes, such as the advanced second moment (ASM) method, and computer Monte Carlo simulation (Ayyub & Haldar 1984). In this section, two probabilistic methods for reliability assessment are described. They

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are (1) advanced second moment (ASM) method, and (2) the Monte Carlo simulation (MCS) method with variance reduction techniques (VRT) using conditional expectation (CE) and antithetic variates (AV).

Advanced second moment (ASM) method

The reliability of a structure can be determined based on a performance function in terms of basic random variables X_i's for structural strength and relevant loads. Mathematically, the *performance function* Z can be described as

$$\begin{aligned} Z &= Z(X_1, X_2, \ldots, X_n) \\ &= \text{Structural strength} - \text{Load effect} \end{aligned} \tag{1}$$

The failure surface (or the *limit state*) can be defined as $Z = 0$. Accordingly, when $Z < 0$, the structure is in the failure state, and when $Z > 0$ it is in the safe state. The failure probability P_f of a structure can be given by the integral

$$P_f = \int \cdots \int f_{X_1, X_2, \ldots, X_n}(x_1, x_2, \ldots, x_n)\, dx_1 dx_2 \ldots dx_n \tag{2}$$

where $f_{X_1, X_2, \ldots, X_n}(x_1, x_2, \ldots, x_n)$ is the joint probability density function (PDF) for the basic random variables X_i's, and the integration is performed over the region in which $Z < 0$. In general, the joint PDF is unknown, and the integral is a formidable task. For practical purposes, alternative methods of evaluating P_f are necessary.

Reliability index—Instead of using the direct integration given by equation (2), the performance function Z in equation (1) can be expanded using a Taylor series about the mean value of X's and then truncated at the linear terms. Therefore, the first-order approximate mean and variance of Z can be shown, respectively, as

$$\bar{Z} \cong Z(\bar{X}_1, \bar{X}_2, \ldots, \bar{X}_n) \tag{3}$$

and

$$\sigma_Z^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \left(\frac{\partial Z}{\partial X_i}\right) \left(\frac{\partial Z}{\partial X_j}\right) Cov(X_i, X_j) \tag{4}$$

where $Cov(X_i, X_j)$ is the covariance of X_i and X_j; $\bar{Z}$ = mean of Z; and σ^2 = variance of Z. The partial derivatives of $\partial Z/\partial X_i$ are evaluated at the mean values of the basic random variables. For statistically independent random variables, the variance can be simplified as

$$\sigma_Z^2 = \sum_{i=1}^{n} \sigma_{X_i}^2 \left(\frac{\partial Z}{\partial X_j}\right)^2 \tag{5}$$

A measure of reliability can be estimated by introducing the *reliability index* β that is based on the mean and standard deviation of Z as

$$\beta = \frac{\bar{Z}}{\sigma_Z} \tag{6}$$

If Z is normally distributed, then it can be shown that the failure probability P_f is

$$P_f = 1 - \Phi(\beta) \tag{7}$$

where Φ is the cumulative distribution function (CDF) of standard normal variate. The aforementioned procedure of equations (3) to (7) produces accurate results when the random variables are normally distributed and the performance function Z is linear.

Nonlinear performance functions—For nonlinear performance functions, the Taylor series expansion of Z is linearized at some point on the failure surface called *the design point* or *the most likely failure point* rather than at the mean. Assuming the original basic variables X_i's are uncorrelated, the following transformation can be used:

$$Y_i = \frac{X_i - \bar{X}}{\sigma_{X_i}} \tag{8}$$

If X_i's are correlated, they need to be transformed to uncorrelated random variables (e.g., Thoft-Christensen & Baker 1982, Ang & Tang 1990). The reliability index β is defined as the shortest distance to the failure surface from the origin in the reduced Y-coordinate system. The point on the failure surface that corresponds to the shortest distance is the most likely failure point. Using the original X-coordinate system, the reliability index β and design point $(X_1^*, X_2^*, \ldots, X_n^*)$ can be determined by solving the following system of nonlinear equations iteratively for β:

Nomenclature

A = effective cross-sectional area
ASM = advanced second moment (method)
AV = antithetic variates
CDF = cumulative distribution function
CE = conditional expectation
COV = coefficient of variation
Cov = covariance
d = total derivative
E = expectation
F = cumulative distribution function or force
f = probability density function
FORM = first-order reliability method
L = load
M = moment
MCS = Monte Carlo simulation
n = total number of components
N = number of simulation cycles
NA = neutral axis
P = probability
PDF = probability density function
RV = random variable
Stddv = standard deviation
U = random variable
ULTSTR = ultimate strength
V = random variable
Var = variance
VBUSA = visual BASIC with ULTSTR, Simulation, and ASM
VRT = variance reduction techniques
X = random variable
$\bar{X}$ = mean value of X
y = distance to the centroid for a structural component
Y = random variable
Z = performance function
α = directional cosine
β = reliability index
Δ = very small quantity
δ = acceptable tolerance
ε = strain
Φ = cumulative distribution function of standard normal variate
ϕ = probability density function of standard normal variate
σ = stress
θ = curvature
∂ = partial derivative

Subscripts

f = failure
i = ith iteration (or component)
s = success or still water
w = wave
y = yield
Z = performance function

Superscripts

N = equivalent normal distribution
(1), (2) = components of simulation cycle
-1 = inverse
$*$ = design point

$$\alpha_i = \frac{\left(\frac{\partial Z}{\partial X_i}\right)\sigma_{X_i}}{\left[\sum_{i=1}^{n}\left(\frac{\partial Z}{\partial X_i}\right)^2 \sigma_{X_i}^2\right]^{1/2}} \tag{9}$$

$$X_i^* = \bar{X}_i - \alpha_i \beta \sigma_{x_i} \tag{10}$$

$$Z(X_1^*, X_2^*, \ldots, X_n^*) = 0 \tag{11}$$

where α_i is the directional cosine, and the partial directives are evaluated at design point. Then, equation (7) can be used to evaluate P_f. However, the above formulation is limited to normally distributed random variables. The above ASM method is also called the first-order reliability method (FORM) as described by Ang & Tang (1990), Ayyub & Haldar (1984), and White & Ayyub (1985).

Equivalent normal distributions—If a random variable X is not normally distributed, then it needs to be transformed to an equivalent normally distributed random variable. The parameters of the equivalent normal distribution, $\bar{X}_i^N$ and σ_X^N, can be estimated by imposing two conditions (Rackwitz & Fiessler 1976,1978). The cumulative distribution functions (CDF) and probability density functions (PDF) of a non-normal random variable and its equivalent normal variable should be equal at the design point on the failure surface. The first condition can be expressed as

$$\Phi\left(\frac{X_i^* - \bar{X}_i^N}{\sigma_{X_i}^N}\right) = F_i(X_i^*) \tag{12a}$$

The second condition is

$$\phi\left(\frac{X_i^* - \bar{X}_i^N}{\sigma_{X_i}^N}\right) = f_i(X_i^*) \tag{12b}$$

where F_i = non-normal CDF, f_i = non-normal PDF, Φ = the CDF of standard normal variate, and ϕ = the PDF of standard normal variate. The standard deviation and mean of equivalent normal distributions can be shown, respectively, to be

$$\sigma_{X_i}^N = \frac{\phi(\Phi^{-1}[F_i(X_i^*)])}{f_i(X_i^*)} \tag{13}$$

and

$$\bar{X}_i^N = X_i^* - \Phi^{-1}[F_i(X_i^*)]\sigma_{X_i}^N \tag{14}$$

Having determined $\sigma_{X_i}^N$ and $\bar{X}_i^N$ for each random variable, β can be solved using the same procedure of equations (9) to (11).

The ASM method is capable of dealing with nonlinear performance functions and non-normal probability distributions. However, the accuracy of the solution and the convergence of the procedure depend on the nonlinearity of the performance function in the vicinity of design point and the origin. If there are several local minimum distances to the origin, the solution process may not converge onto the global minimum. The failure probability calculated from the reliability index β using equation (7) is based on normally distributed performance functions. Therefore, the resulting failure probability P_f based on the ASM is approximate except for linear performance functions because it does not account for any nonlinearity in the performance functions.

Numerical algorithms—The ASM method can be used to accurately assess, for all practical purposes, the reliability of a structure with a nonlinear performance function that may include non-normal random variables. Also, the performance function can be in a form of closed or non-closed expression. The ASM algorithm can be summarized by the following steps:

ALGORITHM 1

1. Assign the mean value for each random variable as a starting design point value, i.e., $(X_1^*, X_2^*, \ldots, X_n^*) = (\bar{X}_1, \bar{X}_2, \ldots, \bar{X}_n)$.
2. Compute the standard deviation and mean of the equivalent normal distribution for each non-normal random variable using equations (13) and (14).
3. Compute the partial derivative $\partial Z/\partial X_i$ of the performance function with respect to each random variable evaluated at the design point as needed by equation (9).
4. Compute the directional cosine α_i for each random variable as given in equation (9) at the design point.
5. Compute the reliability index β by substituting equation (10) into equation (11) and satisfying the limit state $Z = 0$ in equation (11) using a numerical root-finding method.
6. Compute a new estimate of the design point by substituting the resulting reliability index β obtained in step 5 into equation (10).
7. Repeat steps 2 to 6 until the reliability index β converges within an acceptable tolerance.

End of Algorithm 1.

Simulation methods

The Monte Carlo simulation (MCS) technique is basically a sampling process that can be used to estimate the failure probability of a structure. The direct simulation comprises drawing samples of the basic variables according to their probabilistic characteristics and then feeding them into the performance function Z as given by equation (1). Assuming N_f to be the number of simulation cycles for which $Z < 0$ in a total N simulation cycles, then an estimate of the mean failure probability $\bar{P}_f$ can be expressed as

$$\bar{P}_f = \frac{N_f}{N} \tag{15}$$

The estimated $\bar{P}_f$ should approach the true value for the population when N approaches infinity. The variance of the $\bar{P}_f$ can be approximately computed as

$$Var(\bar{P}_f) = \frac{(1-\bar{P}_f)\bar{P}_f}{N} \tag{16}$$

Therefore, the coefficient of variation of the estimate failure probability is

$$\mathrm{COV}(\bar{P}_f) = \frac{1}{\bar{P}_f}\sqrt{\frac{(1-\bar{P}_f)\bar{P}_f}{N}} \tag{17}$$

These equations show that the direct simulation can be economically (in terms of computation time) prohibitive in some cases, especially for small failure probabilities. In order to achieve the desired accuracy of simulation and reduce the sample error without increasing sample size, combined variance reduction techniques (VRT) are used for this purpose (Ayyub & Haldar 1984).

Conditional expectation (CE) variance reduction technique—The conditional expectation (CE) variance reduction technique (VRT) reduces the variance of an estimated value by conditioning on all random variables except one or more random variables with relatively large variability, therefore removing the effect of their variability on the sampling procedure. The theory behind this technique is based on the fact that the total variance of X can be given in terms of conditional means and variances of X conditioned on Y as follows (Ang & Tang 1975):

$$Var(X) = E_Y[Var(X \mid Y)] + Var_Y[E(X \mid Y)] \tag{18}$$

The subscript Y on E and Var shows that the expectation and variance are with respect to Y. Rearranging the above equation produces the following:

$$Var_Y[E(X \mid Y)] = Var(X) - E_Y[Var(X \mid Y)] \leq Var(X) \tag{19}$$

Equation (19) indicates that computing $E(X \mid Y)$ analytically from the random variable Y results in a smaller variance than directly computing X (Ayyub & Haldar 1984). In simulation, this concept can be utilized by not generating random numbers for those random variables with large variability. For instance, consider a performance function with three random variables as follows:

$$Z = Z(X_1, X_2, X_3) = X_1 - X_2 - X_3 \tag{20}$$

If X_3 has the largest variability, then the failure probability P_f of equation (20) can be written as

$$P_f = 1 - P_s = 1 - P(X_1 - X_2 - X_3 > 0) \tag{21}$$

For the ith simulation cycle

$$\begin{aligned} P_{f_i} &= 1 - P(X_3 < X_1 - X_2 \mid X_1 = x_{1_i}, X_2 = x_{2i}) \\ &= 1 - F_{X_3}(x_{1i} - x_{2i}) \end{aligned} \tag{22}$$

Therefore, the failure probability can be calculated by not generating random number for X_3. The average failure probability can then be computed for N simulation cycles.

Antithetic variates (AV) variance reduction technique—The antithetic variates (AV) variance reduction technique reduces the variance of an estimated mean value by introducing a negative correlation between two sets of samples. Considering two unbiased estimates $X_i^{(1)}$ and $X_i^{(2)}$ of a mean $\bar{X}$ from two separate samples, these two estimates can be combined to form another estimate by taking the average as

$$X_i = \frac{X_i^{(1)} + X_i^{(2)}}{2} \tag{23}$$

The expected value of X_i is

$$\begin{aligned} E(X_i) &= E\left[\frac{1}{2}\left(X_i^{(1)} + X_i^{(2)}\right)\right] \\ &= \frac{1}{2}\left[E\left(X_i^{(1)}\right) + E\left(X_i^{(2)}\right)\right] \\ &= \frac{1}{2}(\bar{X} + \bar{X}) = \bar{X} \end{aligned} \tag{24}$$

which means that X_i is an unbiased estimate of $\bar{X}$. It also can be shown that the corresponding variance is

$$\begin{aligned} Var(X_i) = \frac{1}{4}\Big[&Var\left(X_i^{(1)}\right) + Var\left(X_i^{(2)}\right) \\ &+ 2Cov\left(X_i^{(1)}, X_i^{(2)}\right)\Big] \end{aligned} \tag{25}$$

Therefore if $X_i^{(1)}$ and $X_i^{(2)}$ are negatively correlated, i.e., $Cov(X_i^{(1)}, X_i^{(2)}) < 0$, the variance of the estimate X_i can be reduced. Thus if $X_i^{(1)}$ is a random variable uniformly distributed in (0, 1), then $X_i^{(2)} = 1 - X_i^{(1)}$ is also uniformly distributed in (0, 1) and the covariance of $X_i^{(1)}$ and $X_i^{(2)}$ is negative. Consequently, the variance of X_i can be reduced.

Combined variance reduction techniques (VRT) of conditional expectation (CE) and antithetic variates (AV)—The variance of an estimated quantity using simulation can be reduced even more by combining the conditional expectation (CE) and antithetic variates (AV). The result of this combination is as good as many other methods (Ayyub & Haldar 1984). The sample mean of the probability of failure is given by

$$\bar{P}_f = \frac{1}{N}\sum_{i=1}^{N} P_{fi} \tag{26}$$

This can be considered an unbiased estimation of the population mean. The uncertainty associated with this estimation can be expressed in terms of its variance as

$$Var(\bar{P}_f) = \frac{Var(P_f)}{N} = \frac{1}{N}\left[\frac{1}{N-1}\sum_{i=1}^{N}(P_{fi} - \bar{P}_f)^2\right] \tag{27}$$

The algorithm of the combined VRT of CE and AV can be summarized as follows:

ALGORITHM 2

1. Identify the basic variable with the most variability in the performance function in equation (1).
2. Condition the variable in step 1 with respect to all the remaining variables X_i's in the performance function.
3. Generate a uniformly distributed random deviate U_i for each remaining variable X_i as its value of the cumulative distribution function (CDF), and compute $V_i = 1 - U_i$ for each remaining X_i as its second uniformly distributed and negatively correlated CDF.
4. Compute the inverse of U_i and V_i by calling the inverse function of CDF to obtain the corresponding random variables $X_i^{(1)}$ and $X_i^{(2)}$.
5. Compute the performance function $Z_i^{(1)}$ and $Z_i^{(2)}$ in step 1.
6. Compute the failure probability $P_{fi}^{(1)}$ and $P_{fi}^{(2)}$ using the probabilistic characteristics of the variable identified in step 1.
7. Compute the average failure probability $P_{fi} = (P_{fi}^{(1)} + P_{fi}^{(2)})/2$.
8. Repeat steps 3 to 7 N times.
9. Compute the statistics of the failure probability for N simulation cycles.

End of Algorithm 2.

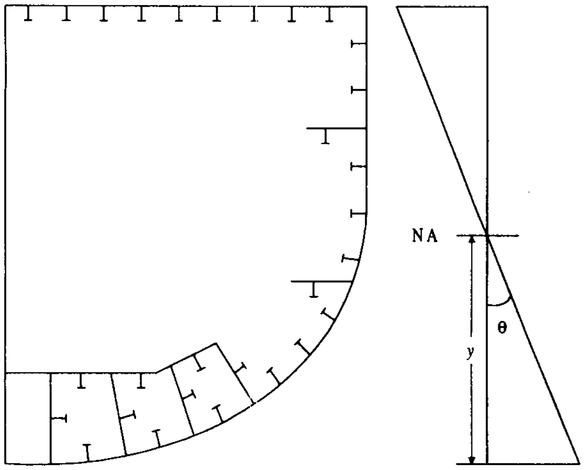
3. Ultimate strength and loss for hull-girder bending

Ultimate strength of hull-girder bending

The strength of a hull girder subjected to a bending moment can be described using a moment-curvature relationship which provides the strength-moment development of the hull girder at different levels of curvature. A typical moment-curvature relationship includes two segments, a hogging moment segment and a sagging moment segment. Figure 1 shows a moment-curvature relationship at a section under a hogging loading condition. The maximum moment strength in the moment-curvature relationship can be defined as the ultimate strength of the hull-girder bending.

The behavior of a hull-girder subjected to a longitudinal bending moment is very much dependent on that of its individual elements (Paik 1993). A ductile failure behavior of a hull girder can be assumed as a result of a sequence of failure of local components rather than of an overall concurrent instability of the entire cross section (Adamchak 1982). These local components composed of the cross section of a hull girder can be represented as a single plate-beam combination, an individual gross panel composed of several plate-beams, or a complete cross-stiffened grillage.

The computation of the ultimate strength of a hull-girder bending of surface ships primarily contains the following steps: (1) dividing the cross section of a hull-girder into a set of *gross panels* and *hard corners*, (2) imposing an assumed curvature on the hull-girder in a small finite increment, and (3) solving for the strength moment on the midship crosssection. Steps 2 and 3 should be repeated until a predefined failure condition is reached, then the moment-curvature relationship of hull-girder bending can be obtained. This condition can be, for example, the peak of the moment-curvature curve.



(a) Example Half Section of Hull Girder (b) Strain (ε) Diagram

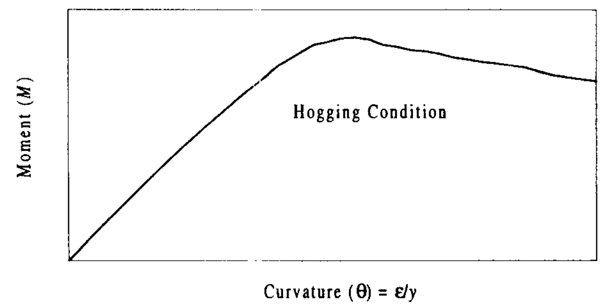


Fig. 1 Moment-curvature relationship at a cross section

The approach for calculating the resisting moment of a hull-girder's cross section at each assumed increment of curvature is based on the linear-strain assumption along the depth of the cross section. However, the stress distribution along the depth of the section is not necessarily linear. The location of zero strain is referred to as the *instantaneous* or *incremental* neutral axis (NA) for the curvature condition imposed on the cross section. At each curvature level, the location of the instantaneous neutral axis can be determined by satisfying the condition of static force equilibrium using numerical methods. The static force equilibrium state can be described as the state where the summation of all force components in the cross section corresponding to various strain and stress levels along the cross section is equal to zero. This condition of statics can be expressed as follows:

$$\sum_{i}^{n} F_i = 0 \tag{28}$$

where F = force, subscript i = ith structural component, and n = total number of components. In the computer program, this condition is used to solve for the location of the instantaneous neutral axis using a numerical (iterative) procedure such that the net axial force ΔF on the cross section is zero, i.e.

$$\Delta F = \sum_{i}^{n} \sigma_i A_i = 0 \tag{29}$$

where σ_1 = stress of ith structural component, and A_i = effective cross-sectional area of the ith structural component. In the iteration process, the position of the instantaneous neutral axis can be determined such that the value of the net force is less than some predefined acceptable tolerance δ. Therefore, the equation (29) can be written as follows:

$$\Delta F = \sum_{i}^{n} \sigma_i A_i \leq \delta \tag{30}$$

Once the neutral axis is determined, the strain for each component can be determined by using similar triangles based on the linear strain diagram. Also, the corresponding stresses and forces for the structural components can be obtained through their corresponding stress-strain relationships and effective areas. The resisting bending moment (M) is computed by summing the force contributions of all the elements of the cross section with respect to an arbitrary, but computationally convenient, reference position. The resulting moment does not depend on the choice of this reference position since the section is under pure bending. The moment condition can be as

$$M = \sum_{i}^{n} F_i y_i = \sum_{i}^{n} \sigma_i A_i Y_i \tag{31}$$

where y_i is the distance from the reference position (for example the bottom or top of the cross section) to the centroid of the ith structural component.

The moment-curvature predication for a section is, therefore, based on the assumption that once instability is detected in a given mode for one element, the behavior follows through to ultimate failure of the cross section in that same mode for the element (Adamchak 1982). In hull-girder analysis, gross panel elements of a cross section can fail either through material yielding, material rupture, or through some form of structural instability.

A computer program called ULTSTR (ULTimate STRength) was used to determine the moment-curvature relationship at any section of the hull girder. This program was developed by Adamchak (1982) for the U.S. Navy. Analytical models used in this computer program with detailed formulation are provided by, for example, Evans (1975); Faulkner, Adamchak, Snyder, & Vetter (1973); Adamchak (1975,1979,1982); Dow, Hugill, Clark & Smith (1981); Ostapenko (1981); and Clarkson (1965). Two main instability failure modes were incorporated in ULTSTR. These two failure modes are Euler beam-column buckling and stiffener lateral-torsional buckling (tripping). Actually, in ULTSTR, Euler beam-column buckling was subdivided into two distinct types of failure patterns. The first type is characterized by all lateral deformations occurring in the same direction. This failure type is primarily a yield-strength dependent behavior, and is assumed to be a possible failure mode when either lateral loading or initial fabrication distortions, or both, are present. The second type is characterized by an alternating buckling pattern in direction. This failure type is primarily Young's modulus dependent concerning initial buckling, and it can occur whether or not lateral load or initial distortions or both are present (Adamchak 1982).

Plate buckling and grillage general instability are not included separately as failure modes in the current version of ULTSTR. The influence of plate buckling failure on longitudinal hull-girder collapse is considered in an indirect form through using effective width or breadth of plates of gross panels that form the basis of the ultimate strength analysis. Grillage general instability failure is not currently included because it is seldom found as the primary failure mode in geometric structural proportions that are typical to surface ships currently in design or service (Adamchak 1982).

The relationship between an applied axial force (load) and strain for an element can be described by its load-shortening curve consisting of three zones. The first zone is called the stable zone in which the relationship between the load and strain is very nearly linear. (The stress-strain behavior is assumed linear. However, because the plating effectiveness is stress dependent, the relationship between load and strain deviates slightly from linearity.) When the applied load approaches its critical (i.e., buckling load) or yield value of a structural element, this quasi-linear relationship is not valid anymore. The second zone is called the plateau that occurs after the loading reaches the critical or yield value of a structural element. In this zone, the structural element continues to deform without any increase in the applied load. The third zone is called the unloading zone and is appropriate for compression only. Reducing the applied load in order to maintain a weakened element (after its failure) in static equilibrium is called unloading. This unloading zone can significantly affect the behavior of the overall hull-girder cross section. Two recent enhancements of ULTSTR were made. In the second zone, a rupture strain can now be defined so that the capacity of a gross panel or a hard corner under tension can be reduced to zero upon rupture. In the third zone, users now can define the unloading type for a hard corner under compression (Adamchak 1982).

Loads for hull-girder bending

Load effects on a ship's hull-girder subjected to a sea and operational environment can be classified into effects due to (1) stillwater, (2) passing wave, (3) wave whipping, and (4) wave slamming. In structural reliability assessment, extreme analysis and stochastic combination of these load effects are needed. Information on loads, extreme analysis and load combination is provided in e.g., Ayyub, Beach, & Packard (1995).

In this paper, only two types of loads, (1) stillwater bending and (2) wave bending, were selected to demonstrate the development of reliability assessment of hull-girder bending of ships. Therefore, the load component in equation (1) can be expressed as

$$L = L_s + L_w \tag{32}$$

where L = total load; L_s = stillwater bending; and L_w = wave bending. The stillwater bending is usually considered as having a normal distribution. The wave bending follows, for example, a Rayleigh distribution for a short time period, and it follows an exponential distribution for a long time period (e.g., Mansour 1972, Boe et al 1974).

In order to illustrate the analytic method for structural reliability assessment of hull-girder bending, however, these two loads are treated as two random variables with normal probability distributions. The coefficient of variation (COV) of wave bending was assumed to be larger than that of the stillwater bending for illustration; although recent studies have shown that stillwater can have a larger COV than that of the lifetime extreme wave bending. The stillwater and wave bending in future development can be revised and replaced by any module or any load function that can involve more elaborate load calculations. Also, other load combinations can be used based on similar concepts.

4. Reliability assessment for hull-girder bending

Advanced second moment method

The ASM method described earlier requires a closed-performance function in equation (1), but the failure modes for hull-girder bending of ships cannot generally be expressed using a simple closed form for the strength side of the performance function. However, the ultimate moment in the moment-curvature relationship can be obtained from an ultimate strength assessment program for hull-girder bending such as ULTSTR. Therefore using this ultimate moment, the performance function Z of equation (1) can be expressed as

$$Z = M(X_1, X_2, \ldots, X_{n-2}) - L_s - L_w \tag{33}$$

where M is the ultimate resisting moment for a section from ULTSTR, L_s is the stillwater bending moment that corresponds to the section with strength M, L_w is the wave bending moment that corresponds to the section with strength M, and X_i is a basic strength random variable such as geometry or material property. The loads L_s and L_w are treated in this section as single random variables, whereas in reality these loads are also functions of several basic random variables. Future work in this area can be directed towards generalizing equation (33) by treating these loads as functions of other random variables. However, the reliability assessment methods as presented in this section are still applicable. The objectives of this section are to introduce the needed changes to the ASM in order to handle non-closed forms of the performance function.

The first change to the method is needed in computing the directional cosines as given in equation (9) for each random variable. These directional cosines require the computation of the partial derivatives of Z with respect to the random variables. The partial derivatives with respect to X_i, L_s, and L_w can be determined, respectively, as

$$\frac{\partial Z}{\partial X_i} = \frac{\partial Z}{\partial M}\frac{\partial M}{\partial X_i} = 1 \cdot \frac{\partial M}{\partial X_i} = \frac{\partial M}{\partial X_i} = \frac{\partial M(X_1, X_2, \ldots, X_{n-2})}{\partial X_i} \tag{34}$$

$$\frac{\partial Z}{\partial L_s} = -1 \tag{35}$$

and

$$\frac{\partial Z}{\partial L_w} = -1 \tag{36}$$

The partial derivative with respect to each X_i in equation (34) cannot be determined analytically. Therefore, a numerical method for computing these partial derivatives is needed as described in Algorithm 3 following equation (40). After finishing from Algorithm 3, the directional cosine as given by equation (9) for each random variable can be determined.

The second change to the method is needed in solving for the reliability index β by substituting equation (10) into equation (33). This solution requires finding the root β for the following expression:

$$Z = M(X_1, X_2, \ldots, X_{n-2}) - L_s - L_w = Z(\beta) = 0 \tag{37}$$

Therefore, a numerical method for finding the roots of nonlinear equations is needed as described in Algorithms 4 and 5. After finding β, the design point is updated according to equation (10), therefore completing steps 5 and 6 in Algorithm 1.

Numerical differentiation—By definition, if $f(x)$ is continuous in the domain of x, then the derivative of $f(x)$ can be expressed as

$$\frac{df(x)}{dx} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \tag{38}$$

The accuracy of computing the derivative in equation (38) can be improved as

$$\frac{df(x)}{dx} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x - \Delta x)}{2\Delta x} \tag{39}$$

Therefore, the partial derivative for each X_i in equation (34) can be expressed as

$$\frac{\partial M}{\partial X_i} = \lim_{\Delta X \to 0} \left(\frac{M(X_1, X_2, \ldots, X_i + \Delta X, \ldots, X_{n-2})}{2\Delta X_i} - \frac{M(X_1, X_2, \ldots, X_i - \Delta X, \ldots, X_{n-2})}{2\Delta X_i} \right) \tag{40}$$

This numerical approach can be applied to both closed and non-closed expressions, and hence it meets the purposes of computer usage for reliability assessment. Based on equation (40), the numerical algorithm for computing the partial derivative for a random variable X_i in M can be summarized as follows:

ALGORITHM 3

1. Compute $X_i + \Delta X_i$, in which ΔX_i is a specified very small quantity (or increment).
2. Compute $M_1 = M(X_1, X_2, \ldots, X_i + \Delta X_i, \ldots, X_{n-2})$ using ULTSTR.
3. Compute $X_i - \Delta X_i$, in which ΔX_i is a specified very small quantity (or increment).
4. Compute $M_2 = M(X_1, X_2, \ldots, X_i + \Delta X_i, \ldots, X_{n-2})$ using ULTSTR.
5. Estimate $\partial M/\partial X_i \approx (M_1 - M_2)/2\Delta X_i$.
6. Repeat steps 1 to 5 for each X_i.

End of Algorithm 3.

Numerical solution of roots of equations—Newton's method is commonly used for root-finding, but this method does not always converge to the true values for some cases. Besides, Newton's method requires evaluation of the derivative of a function, which is the main disadvantage because those performance functions are expressed in a non-closed form and need a numerical approach to compute the derivatives. The bisection method always converges, but the speed of finding roots within an acceptable tolerance for a nonlinear equation can be slow. Therefore, a linear interpolation method or *regula falsi method* (Gerald & Wheatley 1984) is adopted herein. This method always converges, and its convergence rate is somewhat slower than Newton's method but faster than the bisection method. Assume that $f(x)$ is continuous in $[a, b]$, and $f(a)$ and $f(b)$ are of opposite signs. The algorithm of the *regula falsi method* can be stated as follows (Gerald & Wheatley 1984):

ALGORITHM 4

```
Do While (|b − a| ≥ tolerance 1 or |f(c)| ≥ tolerance 2)
    c = b − f(b) * (b − a)/(f(b) − f(a))
    If f(c) * f(a) < 0, then
        b = c
    Else
        a = c
    End If
End Do
End of Algorithm 4.
```

If $f(x)$ has a significant curvature in $[a, b]$, the convergence speed becomes small (Gerald & Wheatley 1984). The convergence speed can be improved, in this case, by using a modified linear interpolation method. According to the modified method, the value of $f(x)$ at the unchanged end position is replaced with $f(x)/2$ as described in the following revised algorithm (Gerald & Wheatley 1984):

ALGORITHM 5

```
Fa = f(a); Fb = f(b); FF = f(a)
Do While (|b − a| ≥ tolerance 1 or |Fc| ≥ tolerance 2)
    c = b − Fb * (b − a)/(Fb − Fa)
    Fc = f(c)
    If Fc * Fa < 0, then
        b = c
        Fb = Fc
        If Fc * FF > 0, then Fa = Fa/2
    Else
        a = c
        Fa = Fc
        If Fc * FF > 0, then Fb = Fb/2
    End If
    FF=Fc
End Do
End of Algorithm 5.
```

The implementation of computer programs in ASM for finding β was based on the modified linear interpolation method.

Monte Carlo simulation methods

Monte Carlo simulation (MCS) methods also can be used for reliability assessment of hull girders bending using the same model (or performance function) as was used for the ASM. In order to improve the efficiency of MCS methods, variance reduction techniques (VRT) can be used. These methods were described earlier. The combined VRT of conditional expectation and antithetic variates is employed in this section for reliability assessment. Due to the non-closed nature of M in equation (33), the basic variable X_i in M cannot be chosen as the conditioned random variable even if X_i is the one with the highest variability level. The reason is because M has to be computed based on ULTSTR in a non-closed form. The only remaining choices are L_s and L_w. If L_w has a higher variability level than L_s, then from equation (33) the survival probability P_s is given by

$$\begin{aligned} P_s &= P(Z > 0) = P(M - L_s - L_w > 0) \\ &= P(L_w < M - L_s) = E[F_{L_w}(M - L_s)] \end{aligned} \tag{41}$$

where $M = M(X_1, X_2, \ldots, X_{n-2})$, F_{L_w} is the cumulative distribution function (CDF) of L_w, and $E(.)$ is the expected value. Therefore, the failure probability P_f can be determined as

$$P_f = 1 - P_s = 1 - E[F_{L_w}(M - L_s)] \tag{42}$$

Therefore, each simulation cycle (ith cycle) is expected to produce a failure probability P_{fi} based on evaluating the CDF of L_w at generated values of $M = M(X_1, X_2, \ldots, X_{n-2})$ and L_s (i.e., M_i and L_{si}). The sample mean of the probability of failure ($\bar{P}_f$) for N simulation cycles is computed as

$$(\bar{P}_f) = \frac{1}{N}\left(\sum_{i=1}^{N} P_{fi}\right) \tag{43}$$

This estimate of P_f can be considered an unbiased estimator of the population value. The variance associated with this estimated value is

$$Var(\bar{P}_f) = \frac{Var(P_f)}{N} = \frac{1}{N}\left(\frac{1}{N-1}\sum_{i=1}^{N}(P_{fi} - \bar{P}_f)\right) \tag{44}$$

where $Var(\bar{P}_f)$ indicates the accuracy in estimating $\bar{P}_f$. A smaller value of $Var(\bar{P}_f)$ is always preferred. The coefficient of variation (COV) for the estimated failure probability is given by

$$\text{COV}(\bar{P}_f) = \frac{1}{\bar{P}_f}\sqrt{\frac{1}{N}\left(\frac{1}{N-1}\sum_{i=1}^{N}(P_{fi} - \bar{P}_f)\right)} \tag{45}$$

The use of conditional expectation and antithetic variates in Monte Carlo simulation for the purpose of reliability assessment is defined in Algorithm 2. The use of equation (33) as a performance function requires the selection of a control variable which is not randomly generated according to this algorithm. In the case of hull-girder bending, L_w was identified as the control random variable needed in step 1 of Algorithm 2.

Development of software VBUSA for reliability assessment

The software of VBUSA (Visual BASIC with ULTSTR, Simulations, and Advanced Second Moment reliability methods) was developed and implemented to provide ultimate strength analysis and reliability assessment of hull-girder bending of surface ships with a friendly user interface (Chao 1995; Ayyub, Chao, Bruchman, & Adamchak 1995). The VBUSA contains three main analysis functions: (1) ultimate strength analysis for hull-girder bending using ULTSTR, (2) reliability assessment using the ASM method, and (3) reliability assessment using the MCS with combined variance reduction techniques

Table 1 Example hull girder with a total of 5 random variables

Total		Random Variables (RV) =		5
Hull-girder strength			RV =	3
General section			RV =	0
	Name	Mean (in)	COV(Stddv)	Type
1	Compartment length	480.0	deterministic	Normal
2	Transverse frame spacing	96.0	deterministic	Normal
3	Effective length (Euler beam-column buckling)	96.0	deterministic	Normal
4	Effective length (Stiffener tripping)	68.0	deterministic	Normal
Node - 54				1
	Name	Mean (in)	COV(Stddv)	Type
1	Overall height	369.0	1.00E-3	Normal
2	Overall width	310.0	deterministic	Normal
Panel - 26 (6 types of thickness)			RV =	1
	Name	Mean (in)	COV(Stddv)	Type
1	Thickness of plate	0.500	0.10	Normal
2	Tension yield zone (3.0-4.5)	3.000	deterministic	Normal
Stiffener - 8			RV =	0
	Name	Mean (in)	COV(Stddv)	Type
1	Overall depth	12.16-3.50	deterministic	Normal
2	Web thickness	0.26-0.13	deterministic	Normal
3	Flange width	4.03-2.68	deterministic	Normal
4	Flange thickness	0.42-0.18	deterministic	Normal
Material - 3			RV =	1
	Name	Mean (psi)	COV(Stddv)	Type
1	Young's modulus	30.0E+6	0.10	Normal
2	Yield stress	80000-33000	deterministic	Normal
Loads			RV =	2
	Name	Mean (lb-in)	COV(Stddv)	Type
1	Stillwater bending	2.75E+09	0.15	Normal
2	Wave bending	1.00E+09	0.20	Normal

Table 2 Reliability results for example hull girder with 5 random variables

Reliability Method	Failure Probability	Reliability Index
ASM	0.657048E-03	3.22061
MCS (Cycles = 50,000)		
1 (CE+AV) (pc seed = -15120)	0.768180E-03	3.16800
2 (CE+AV) (unix seed = -17)	0.641104E-03	3.22019
3 (CE+AV) (unix seed = -371)	0.552852E-03	3.26239

pc = personal computer, unix = unix-based computer

Table 3 Example hull girder with a total of 58 random variables

Total		Random Variables (RV) =		58
Hull-girder strength			RV =	56
General section			RV =	4
	Name	Mean (in)	COV(Stddv)	Type
1	Compartment length	480.0	(0.50)	Normal
2	Transverse frame spacing	96.0	(0.25)	Normal
3	Effective length (Euler beam-column buckling)	96.0	(0.25)	Normal
4	Effective length (Stiffener tripping)	68.0	(0.25)	Normal
Node - 54			RV =	2
	Name	Mean (in)	COV(Stddv)	Type
1	Overall height	369.0	(0.50)	Normal
2	Overall width	310.0	(0.50)	Normal
Panel - 26 (6 types of thickness)			RV = 6 × 2 =	12
	Name	Mean (in)	COV(Stddv)	Type
1	Thickness of plate	0.750-0.250	0.04	Normal
2	Tension yield zone (3.0-4.5)	3.75	0.10	Normal
Stiffener - 8			RV = 8 × 4 =	32
	Name	Mean (in)	COV(Stddv)	Type
1	Overall depth	12.16-3.50	0.05	Normal
2	Web thickness	0.26-0.13	0.04	Normal
3	Flange width	4.03-2.68	0.05	Normal
4	Flange thickness	0.42-0.18	0.04	Normal
Material - 3			RV = 3 × 2 =	6
	Name	Mean (psi)	COV(Stddv)	Type
1	Young's modulus	30.0E+6	0.04	Normal
2	Yield stress	84000-34650	0.07	Normal
Loads			RV =	2
	Name	Mean (lb-in)	COV(Stddv)	Type
1	Stillwater bending	2.45E+09	0.15	Normal
2	Wave bending	1.00E+09	0.20	Normal

Table 4 Reliability results for example hull girder with 58 random variables

Reliability Method	Failure Probability	Reliability Index
ASM	0.289984E-04	4.02090
MCS (Cycles = 50,000)		
1 (CE+AV) (pc seed = 7951)	0.291184E-04	4.01986
2 (CE+AV) (unix seed = 74971)	0.399448E-04	3.94477
3 (CE+AV) (unix seed = 97361)	0.310100E-04	4.00502

pc = personal computer, unix = unix-based computer

(VRT) of conditional expectation (CE) and antithetic variates (AV). In addition to these three main analysis functions, the user interface of the software allows the preparation of input data for running these three main analysis functions, viewing the results of the analysis, and plotting the simulation results and a ship's hull-girder cross section. These three main analysis functions were all written in FORTRAN, and the interface portion is written in Microsoft Visual BASIC©.

Examples

Three examples are described to illustrate the reliability assessment using the ASM method and the MCS method with combined variance reduction techniques (VRT) of conditional expectation (CE) and antithetic variates (AV). The first two examples are for the purpose of comparing the results from the two reliability assessment methods, and the third example shows simulation results and statistical analysis of the moment-curvature performance for a hull girder based on ULTSTR.

Example 1: hull-girder bending with 5 random variables—The five random variables are shown in Table 1. Three of the five random variables are the hull-girder design parameters that include overall height, plate thickness and Young's modulus. The remaining two random variables are stillwater bending and wave bending. The results of using the MCS methods with VRT are shown in Fig. 2. The results for both ASM and MCS with VRT are summarized in Table 2. Table 2 and Fig. 2 show the effect of selecting a seed (that is needed in random number generators) on the estimated failure probability. The simulation results are at about the same level for all practical purposes for all cases, and are about the same as the results of ASM.

Example 2: hull-girder bending with 58 random variables—In this example, 58 random variables as shown in Table 3 are used. The hull girder has 56 random variables for geometry and material properties. The remaining two variables are the external loads of stillwater and wave bending. The results of MCS with VRT are shown in Fig. 3 with using the same scale as in Fig. 2 in order to compare results. The comparison indicates that the simulation results with five random variables show larger variability that the corresponding results with 58 random variables. The variability level needs to be related to the magnitude of the failure probability. In this example, the reliability index β is larger than that of Example 1, which can be mainly attributed to the lower stillwater bending as can be compared in Tables 1 and 3, respectively. Also, the nominal yield stresses in this example were multiplied by a bias factor

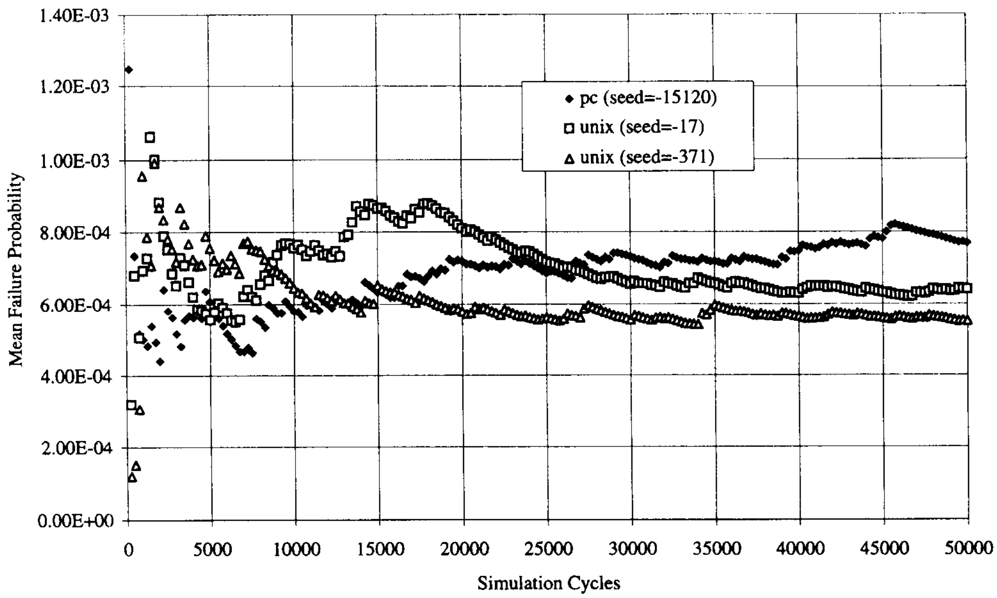


Fig. 2 Simulation results for 5 random variables

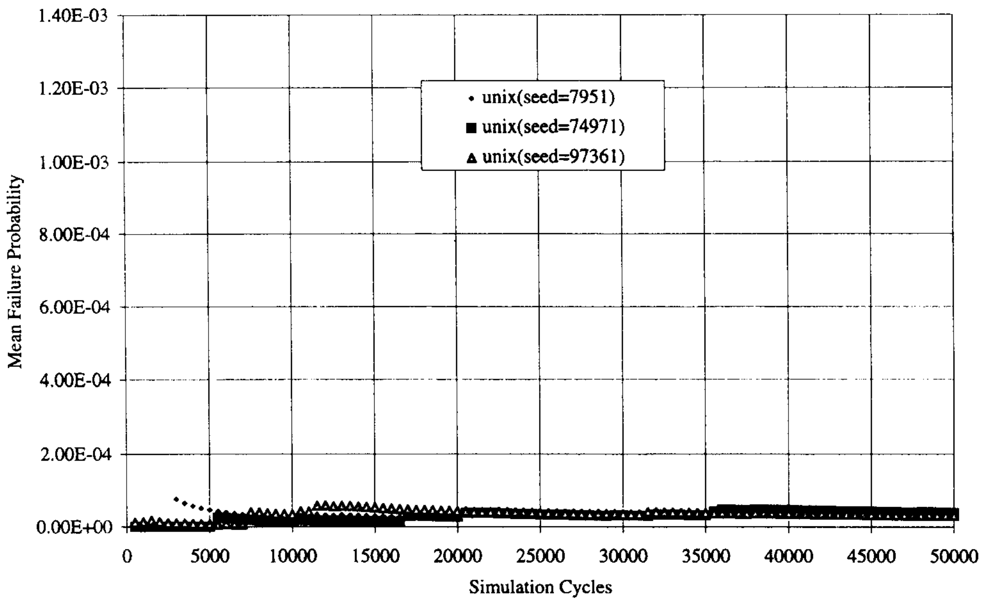


Fig. 3 Simulation results for 58 random variables

of 1.05 to obtain the tabulated mean values, whereas in Example 1 this factor was taken as 1.0. The results of ASM and MCS are summarized in Table 4. Table 4 shows the effect of selecting a seed (that is needed in random number generators) on the estimated failure probability. The simulation results are at about the same level for all practical purposes for all cases, and are about the same as the results of ASM. In conclusion, both examples show agreement in estimating the failure probability P_f for hull-girder bending.

Example 3: simulation of hull-girder strength—The same input data used in Example 2 are used in this example. However, the objective of this example is to investigate the uncertainty in the resulting moment curvature relationship based on ULTSTR using MCS with 58 random variables. Figure 4 shows the moment-curvature relationship with mean and nominal curves. The 58 input random variables were assumed to have the same mean and nominal values except for the Young's modulus and yield stress. Young's modulus was assumed to have a mean of 30.0E+06 psi, and nominal value of 29.5E+06 psi. The yield stress was assumed to have a mean value which 1.05 times its corresponding nominal value for each material. Table 3 shows the mean yield values used in this example. Under these specific assumptions, it can be observed that the nominal moment is smaller than the mean moment but larger than the mean minus one standard deviation.

The mean moment-curvature curve as shown in Fig. 4 can be normalized by dividing the mean moment value by the corresponding nominal moment at each curvature. The resulting normalized curve is shown in Fig. 5. It can be observed that within the elastic behavior, the ratio of mean to nominal moment is very close to 1.015. When the imposed curvature is increased, i.e., the corresponding strain is increased, the moment approaches its ultimate strength and the ratio has a sharp change. After reaching ultimate strength, the ratio stays in the range 1.040 to 1.055.

Figure 6 shows the relationship between the curvature and the coefficient of variation (COV) of resisting moment. Again, the COV of the moment stays in the range 3.5% to 4.0% within

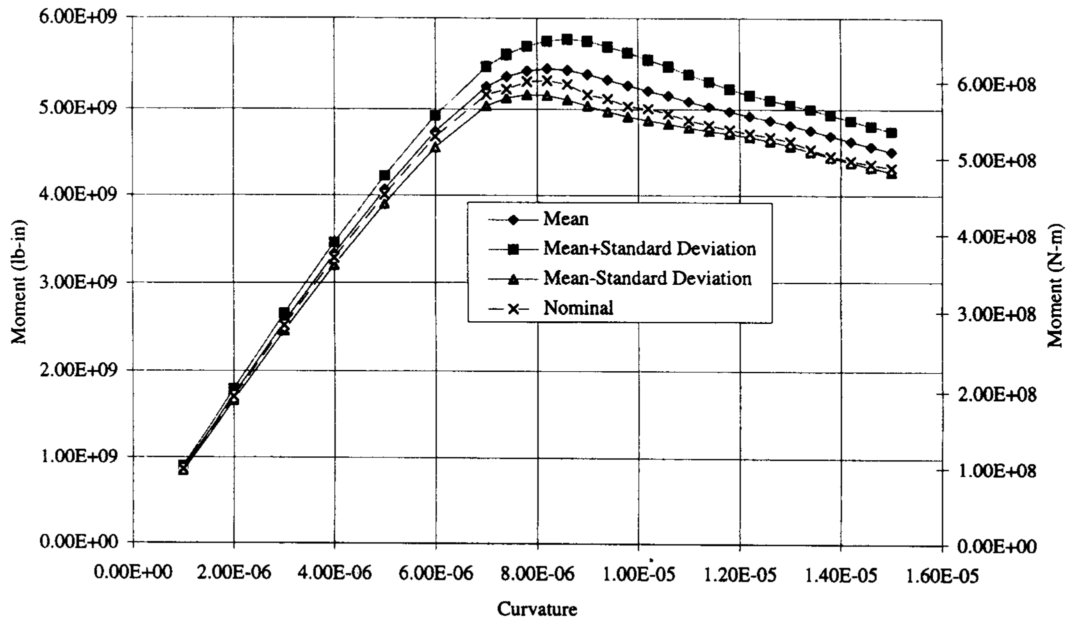


Fig. 4 Moment-curvature relationship using 10 000 cycles

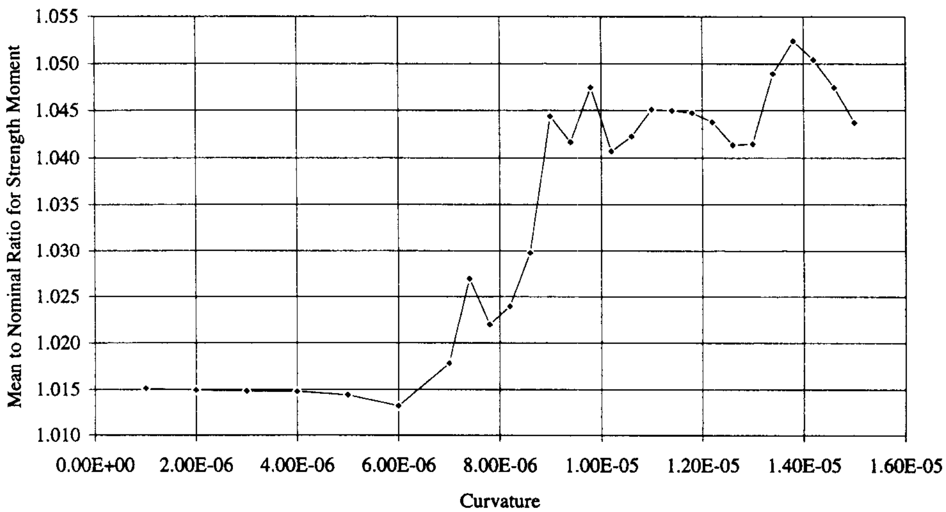


Fig. 5 Mean to nominal ratio for strength moment using 10 000 cycles

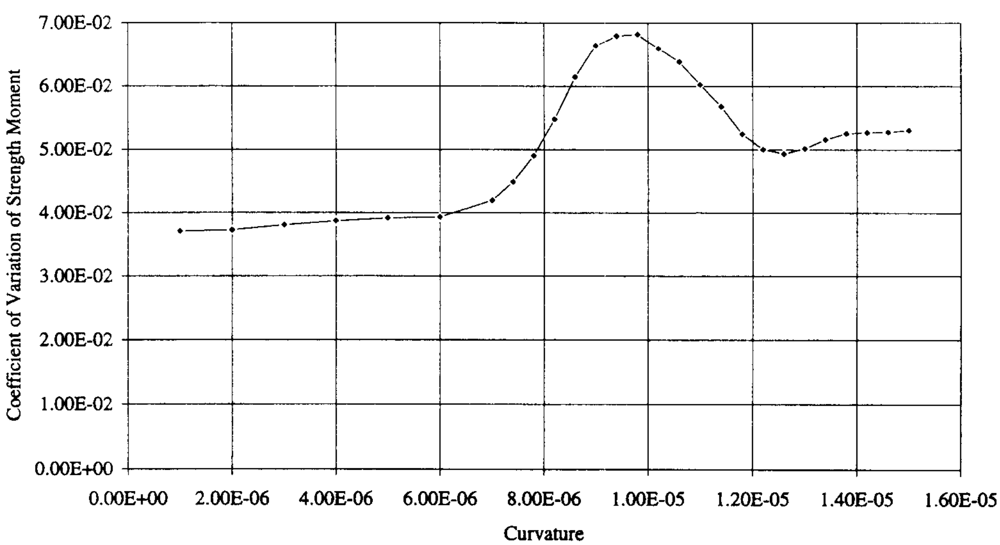


Fig. 6 Coefficient of variation of strength moment using 10 000 cycles

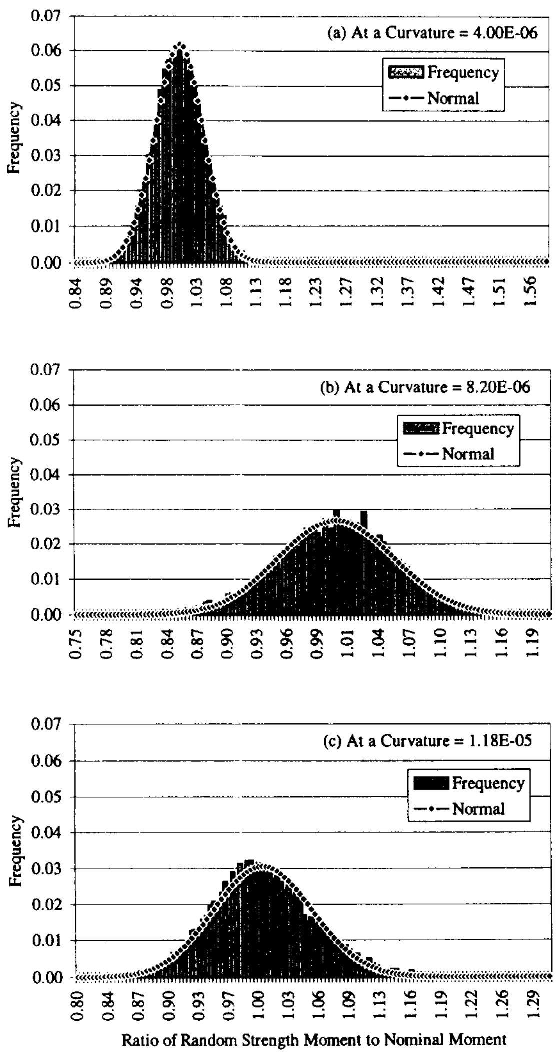


Fig. 7 Strength moment distribution using 10 000 cycles

the elastic range of the section. Upon reaching the maximum resisting moment, the COV shows a dramatic increase, but it stays less than 7%.

A frequency histogram based on 10 000 simulation cycles for the resisting moment at each curvature was developed. At each curvature level, including the elastic range, inelastic range, maximum resisting moment, and post-maximum moment, the histogram has a normal distribution with its bell shape. Figure 7 shows histograms for the resisting moment at different curvatures (4.00E-06, 8.20E-06, and 1.18E-05), which can be modeled using a normal probability distribution with a mean/nominal ratio and COV as given in Figs. 5 and 6 respectively. The strength moment distributions at curvatures 4.00E-06, 8.20E-06, and 1.18E-05 correspond to elastic behavior, ultimate moment, and inelastic behavior, respectively, in the moment-curvature relationship as shown in Fig. 8.

5. Conclusions

Structural reliability assessment of hull-girder bending of ships requires the consideration of the following three aspects: (1) structural strength, (2) loads, and (3) methods of reliability analysis. A methodology based on the ultimate strength of hull-girder bending was developed to assess the structural reliability for ships. In this paper the structural reliability of hull-girder bending of ships was assessed by considering its strength parameters as random variables. It also demonstrates that the structural reliability of hull-girder bending of ships can be assessed with a non-closed performance function using advanced second moment (ASM) method or Monte Carlo simulation (MCS) with variance reduction techniques. The methodology can be used to assess the reliability based on ultimate strength and loads for hull-girder bending that are developed individually as modules, and then combined into a non-closed form of the performance function. The COV of ultimate strength was determined to be 0.07. Based on this study, loads for hull-girder bending also can be considered by generalizing equation (33) in which loads are treated as functions of other basic random variables.

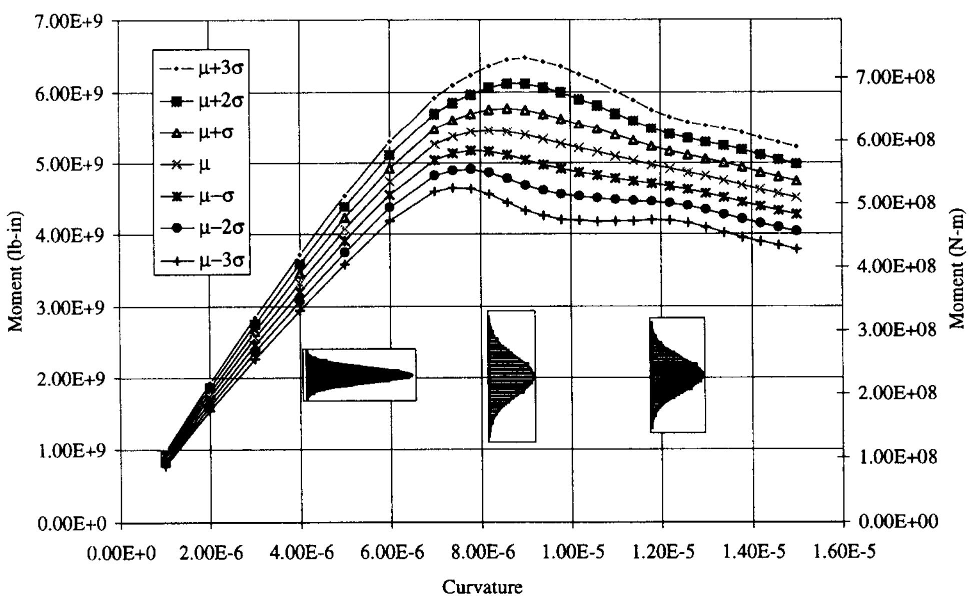


Fig. 8 Strength moment distribution at different curvatures

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