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Comparative and Uncertainty Assessment of Design Criteria for Stiffened Panels

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Stiffened and gross steel panels (plates) are very important components in ship and offshore structures, and therefore they should be designed for a set of failure modes that govern their strength. They form the backbone of most ships' structure, and they are by far the most commonly used element in a ship. They can be found in bottom structures, decks, side shell, and superstructures. To evaluate the strength of a stiffened or gross panel element, it is necessary to review various strength-predicting models and to study their biases, applicability, and limitations for different loading conditions acting on the element. In this paper, strength limit states for various failure modes of ship panels are presented. For each limit state, commonly used strength models were collected from many sources for evaluating their limitations and applicability and to study their biases and uncertainties. Wherever possible, the different types of biases resulting from these models were computed. The bias and uncertainty analyses for these strength models are needed for the development of load and resistance factor design (LRFD) rules for stiffened and gross panels of ship structures. The uncertainty and biases of these models were assessed and evaluated by comparing their predictions with ones that are more accurate or real values. The objective of this paper is to summarize strength prediction models of stiffened and gross panels that are suitable for LRFD development for ship structures. Monte Carlo simulation was used to assess the biases and uncertainties for these models. Recommendations for the use of the models and their biases in LRFD development are provided.

Introduction

THE MAIN TYPE of framing system found in ships nowadays is a longitudinal one, which has stiffeners running in two orthogonal directions (Fig. 1). Deck and bottom structures panels are reinforced mainly in the longitudinal direction with widely spaced heavier transverse stiffeners. The main purpose of the transverse stiffeners is to provide resistance to the loads induced on bottom and side shell by water pressure. The types of stiffeners used in the longitudinal direction are the T beams, angles, bulbs, and flat bars, whereas the transverse stiffeners are typically T beam sections. This type of structural configuration is commonly called gross stiffened panel or grillage (Vroman 1995). Besides their use in ship structures, these gross stiffened panels are also widely used in land-based structures, such as box and plate girders. A typical longitudinal stiffened subpanel, as shown in Fig. 1, is bounded on each end by a transverse structure, which has significantly greater stiffness in the plane of the lateral load. The sides of the panel are defined by the presence of a large structural member that has greater stiffness in bending and much greater stiffness in axial loading.

In ship structures, there are three types of loading that can affect the strength of a plate-stiffener panel: negative bending moment, positive bending moment, and in-plane compression or tension. Negative bending loads are the lateral loads due to lateral pressure. They cause the plate to be in tension and the stiffener flange in compression. Positive bending loads are those loads that put the plating in compression and the stiffener flange in tension. The third type of loading is the uniform in-plane compression. This type of loading arises from the hull girder bending and will be considered to be positive when the panel is in compression. The three types of loading can act individually or in combination with one another.

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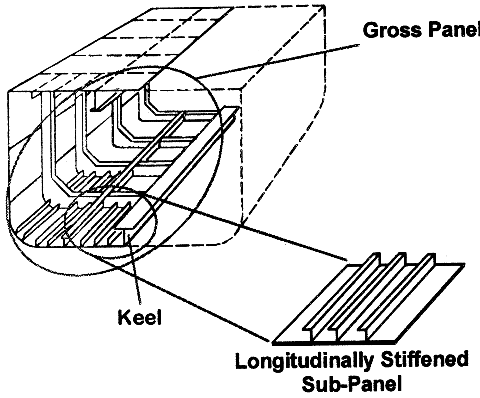

Fig. 1 Portion of the hull girder showing the gross panel and a longitudinally stiffened subpanel (Hughes 1988)

A stiffened panel can normally be idealized into an assembly of various types of simpler "mechanical model," each type of which behaves similarly under a given load application, and the assembly of which behaves in (nearly) the same way as the actual panel. Typical examples of structural idealizations to model a stiffened panel are three, namely (1) plate-stiffener (beam) combination model, (2) plate-stiffener (beam) separation model, and (3) orthotropic plate model (e.g., Paik and Thayamballi 2002). Which type of idealization is more appropriate may depend on structural dimensions and failure modes possible. It has been recognized that the plate-stiffener combination model may be relevant when stiffeners (or support members) are of medium or larger structural dimension so that they would behave as a beam-column together with the associated plating. The plate-beam combination model may also be applied to a gross panel as a system of discrete intersecting beams (or called grillage), each beam being composed of stiffener and associated effective plating. This paper is primarily concerned with the first type (i.e., plate-stiffener combination) of structural models noted above.

To evaluate the strength of a stiffened or gross panel element, it is necessary to review various strength prediction models and to study their applicability and limitations for different loading conditions acting on the element. The uncertainties that are associated with a numerical analysis are generally a result of experimental approximation or numerical inaccuracies that can be reduced by some procedures. However, the uncertainties that are associated with a strength design model are different and cannot be eliminated because they result from not accounting for some variables that can have strong influence on the strength. For this reason, the uncertainty and the bias of a design equation should be assessed and evaluated by comparing its predictions with more accurate ones. Wherever possible, the different types of biases resulting from these models were computed. In doing so, these prediction models were classified as follows (Atua and Ayyub 1996): (1) prediction models that can be used in load and resistance factor design (LRFD) methods, (2) advanced prediction models that can be used for various analytical purposes, (3) some experimental results from model testing, and (4) some real measurements based on field data during the service life of a ship. Furthermore, the relationships and uncertainty analyses for these models are required. The relationships can be defined in terms of biases (bias factors). In this paper, only selected strength models that are deemed suitable for LRFD design format are highlighted and presented.

Stiffeners

Stiffeners are used to strengthen plates and to increase their load-carrying capacity. In ship structures, most of gross stiffened panel or grillage failures are due to the collapse of one or more of the longitudinal and transverse stiffeners. Thus, the first and most basic principle with regard to stiffeners is that they should be designed at least as strong as the plating. Also, they should be sufficiently rigid and stable so that neither local stiffener buckling nor overall buckling occurs before local plate buckling. A plate stiffener can be subjected to a variety of primary and secondary loads and load combinations that cause the plate stiffener to fail in one of the following types of buckling: (1) column buckling, (2) beam-column buckling, and (3) flexural-torsional buckling. Numerous strength models for stiffeners are available according to the type of stiffener buckling involved, and can be found in API (1993), Assakkaf (1998), and Atua (1998).

Stiffened panels

In this section, different strength models for longitudinally stiffened panels under various types of loading are collected from various sources. They are presented herein and evaluated in terms of their applicability, limitations, and biases with regard to ship structures. In addition, the models used by different classification agencies, such as the AISC (1994), ASSHTO (1994), API (1993), and the Navy practices as provided in Atua (1998) were reviewed for further assessment and evaluation of biases.

Strength models for longitudinal stiffened panels

Watanabe et al (1981). A uniaxially stiffened panel subjected to uniaxial compressive stress acting in the same direction as the stiffeners is considered herein. The ultimate limit state is achieved when the applied in-plane compressive stress f equals F_u. The ultimate stress is given by Watanabe et al (1981).

$$F_u = \begin{cases} F_y & \text{if } \lambda \le 0.5 \\ F_y(1.5 - \lambda) & \text{if } 0.5 \le \lambda \le 1.0 \\ F_y\left(\dfrac{0.5}{\lambda}\right) & \text{if } \lambda > 1.0 \end{cases} \tag{1}$$

The buckling coefficient is given by

$$k = \min(k_R, k_F) \tag{2}$$

$$k_R = 4n^2 \tag{3}$$

$$k_F = \begin{cases} \dfrac{(1+\alpha^2)^2 + n\omega}{\alpha^2(1+n\delta)} & \text{if } \alpha \le (1+n\omega) \\ \dfrac{2[1+(1+n\omega)^{1/2}]}{1+n\omega} & \text{if } \alpha > (1+n\omega) \end{cases} \tag{4}$$

$$\omega = \frac{EI_s}{bD} \tag{5}$$

$$\delta = \frac{A_s}{bt} \tag{6}$$

where I_s is the moment of inertia of one stiffener about an axis parallel to the plate surface at the base of the stiffener, and D is the plate flexural rigidity. The plate flexural rigidity is given by

$$D = \frac{Et^3}{12(1 - \nu^2)} \tag{7}$$

Paik and Lee (1996). An empirical formula for predicting the ultimate strength of longitudinal stiffened subpanels based on 130 collapse test data for stiffened plates with initial imperfections is presented. This empirical formula has been derived considering the average level of initial imperfections in the form of initial deflection and residual stresses. The formula also expresses the ratio of ultimate strength of the subpanel to its yield strength in terms of the plate slenderness ratio, β, and the stiffener slenderness ratio, λ, as follows:

$$F_u = F_{y(panel)}\,[0.995 + 0.936\lambda^2 + 0.170B^2 + 0.188\lambda^2 B^2 - 0.067\lambda^4]^{-0.5} \quad \text{for } F_u \le \frac{F_{y(panel)}}{\lambda^2} \tag{8}$$

where $F_{y(panel)}$ = yield strength of the whole panel and is given by

$$F_{y(panel)} = \frac{F_{yp} + \zeta F_{ys}}{1 + \zeta} \tag{9}$$

where

$$\zeta = \frac{d_w t_w + f_w t_f}{bt} \tag{10}$$

The plate slenderness ratio, B, is given by

$$B = \frac{b}{t}\sqrt{\frac{F_y}{E}} \tag{11}$$

The stiffener slenderness ratio, λ, is given by

$$\lambda = \frac{a}{\pi r}\sqrt{\frac{F_y}{E}} \tag{12}$$

in which a = span (length) of stiffener, r = radius of gyration of one stiffener with fully effective plating and is given by

$$r = \sqrt{\frac{I}{A}} \tag{13}$$

where A = sectional area of the plate and the stiffener and it is given by

$$A = bt + d_w t_w + f_w t_f \tag{14}$$

The moment of inertia of one stiffener with fully effective plating (I) is given by

$$I = \frac{bt^3}{12} + bt\left(z_0 - \frac{t}{2}\right)^2 + \frac{{d_w}^3 t_w}{12} + d_w t_w\left(z_0 - t - \frac{d_w}{2}\right)^2 + \frac{f_w {t_f}^3}{12} + f_w t_f\left(z_0 - t - d_w - \frac{t_f}{2}\right)^2 \tag{15}$$

where z_0 = distance of neutral axis from the base line of plate, t = thickness of plate, t_w = thickness of stiffener web, t_f = thickness of stiffener flange, d_w = stiffener web height, b = spacing between stiffener, and f_w = stiffener flange width. The formula was compared with experimental and numerical data (Paik & Lee 1996, Paik & Thayamballi 2002, Paik 1997) and proved to predict the strength value reasonably.

Herzog (1987). Based on reevaluation of 215 tests by various researchers and on empirical formulation, Herzog (1987) developed two simple models (formulas) for the ultimate strength of stiffened panels that are subjected to uniaxial compression with or without lateral loads. The first model considers the case of uniaxial compression alone, while the other considers the combination of uniaxial compression with lateral pressure. These models are presented in the next two sections.

Uniaxial compression. The ultimate strength F_u of a longitudinally stiffened plate is given by the following empirical formula (Herzog 1987):

$$F_u = \begin{cases} mF_y\left[0.5 + 0.5\left(1 - \frac{a}{r\pi}\sqrt{\frac{F_y}{E}}\right)\right] & \text{for } \frac{b}{t} \le 45 \\ mF_y\left[0.5 + 0.5\left(1 - \frac{a}{r\pi}\sqrt{\frac{F_y}{E}}\right)\right] \times \left[1 - 0.007\left(\frac{b}{t} - 45\right)\right] & \text{for } \frac{b}{t} > 45 \end{cases} \tag{16}$$

where r as given by equation (13), m = 1.2, 1.0, or 0.8, depending on whether the designer considers, respectively, (1) no or average imperfection and no residual stress, (2) average imperfection and average residual stress, and (3) average or large imperfection and high value for the residual stress. The remaining symbols are given in the list of notations.

The 215 tests evaluated by Herzog belong to three distinct groups. Group 1 (75 tests) consisted of small values for imperfection and residual stress, group 2 (64 tests) had average values for imperfection and residual stress, and group 3 (76 tests) consisted of higher values for imperfection and residual stress.

The statistical uncertainty (coefficient of variation, COV) associated with the Herzog model of equation (16) is 0.218. The mean value μ, standard deviation σ, and COV of the measurement to prediction are given in Table 1.

Biaxial compression with lateral pressure. Based on test results, Herzog (1987) proposed an empirical formula for predicting the strength of stiffened plates subjected to both biaxial compression and lateral pressure. This formula is given by the following interaction equation:

$$F_u = F_y\left(1.15 - 0.5\frac{f_p}{f_{py}}\right) \le F_y \tag{17}$$

where f_p = applied stress due to lateral pressure, f_{py} = limit stress as given by yield-line theory, and the remaining symbols are given in the list of notations. The above equation is based on evaluation of 11 British tests on unstiffened plate models and 4 tests on ship grillages.

Table 1 Statistics of 215 tests conducted on longitudinally stiffened plates in uniaxial compression (Herzog 1987)

Group	Number of Tests	Mean Value (μ)	Standard Deviation (σ)	COV
I	75	1.033	0.134	0.130
II	64	0.999	0.100	0.100
III	76	0.981	0.162	0.169
All	215	1.004	0.136	0.135

COV = coefficient of variation.

Adamchak (1979). The model developed by Adamchak was intended to estimate the ultimate strength of conventional surface ship hulls or hull components under longitudinal bending or axial compression. The model itself is very complex for hand calculation, and therefore it is not recommended for use in a design code without some computational tools or a computer program. The recent version of this model is intended for preliminary design and based on a variety of empirically based strength of material solutions for the most probable ductile failure modes for stiffened and unstiffened plate structures. The probable ductile failure modes include section yielding or rupture, interframe Euler beam-column buckling, and interframe stiffener tripping (lateral-torsional buckling). The model also accounts for the effects of materials having different yield strength in plating and stiffeners, for initial out-of-plane distortion due to fabrication, and for lateral pressure loading.

Longitudinally stiffened panel elements can fail either by material yielding, material rupture (tension only), or by some form of structural stability. The instability failure modes for this model include Euler beam-column buckling and stiffener lateral torsional buckling (tripping). These modes of failure are illustrated in Fig. 2. Euler beam-column buckling is actually treated in this model as having two distinct types of failure patterns, as shown in Fig. 3. Type I is characterized by all lateral deformation occurring in the same direction. Although this type of failure is dependent on all ge-

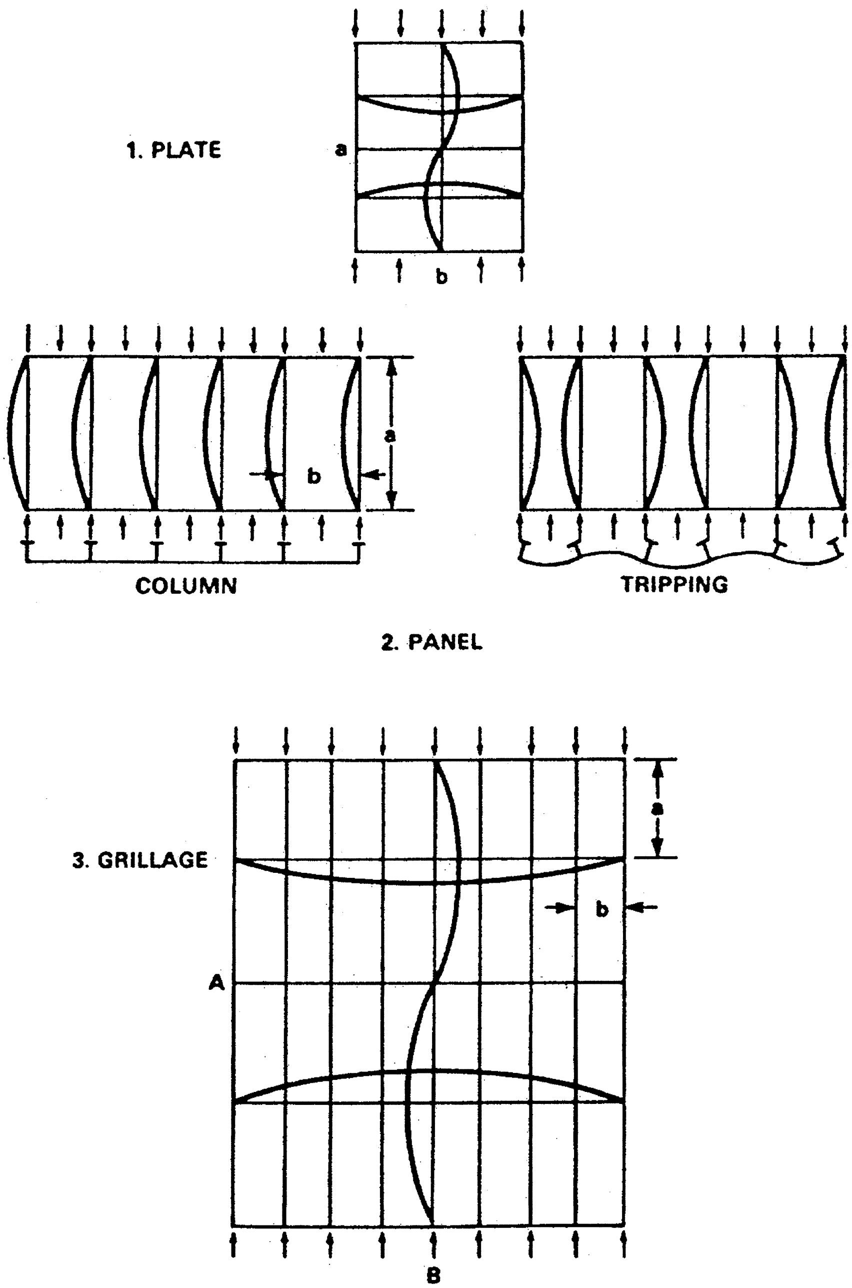

Fig. 2 Instability failure modes (Adamchak 1979)

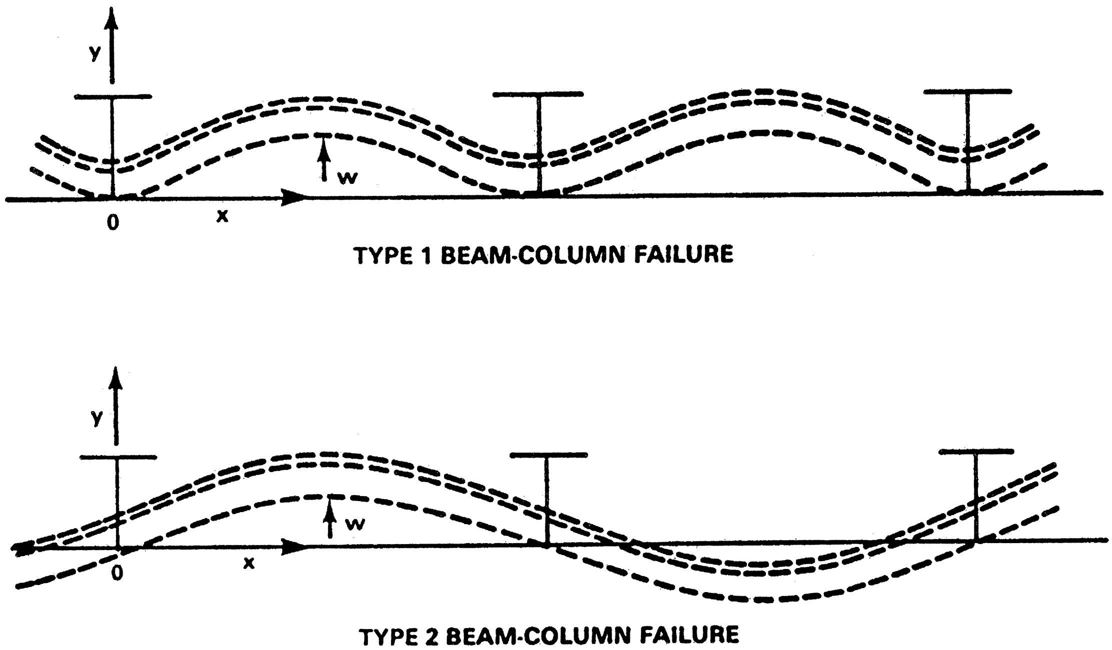

Fig. 3 Types of beam-column failure (Adamchak 1979)

ometrical and material properties that define the structural element, it is basically yield strength dependent. Type I failure is assumed to occur only when either lateral pressure or initial distortion, or both, are present. On the other hand, type II failure is modulus (E) dependent, as far as initial buckling is concerned. This type of failure can be initiated whether or not initial distortion or lateral pressure, or both, are present. Type III failure is a stiffener tripping or lateral-torsional buckling. Therefore, the ultimate axial strength (stress) for longitudinally stiffened panel under various types of loading (including material fabrication distortion) is the minimum value of the axial compressive stress computed from the expressions for the three types (modes) of failures, that is

$$F_u = \min(F_{uI}, F_{uII}, F_{uIII}) \tag{18}$$

Detailed mathematical expressions for the three modes of failures as implemented in the program ULTSTR can be found in Adamchak (1979) and Assakkaf (1998).

Hughes (1988). According to Hughes (1988), three types of load effects must be considered for determining the ultimate strength of longitudinally stiffened panels. These types of load effects are (1) lateral load causing negative bending moment of the plate-stiffener combination (the panel), (2) lateral load causing positive bending moment of the panel, and (3) in-plane compression resulting from hull girder bending. The sign convention to be used throughout this section is that of Hughes (1988). Bending moment in the panel is considered positive when it causes compression in the plating and tension in the stiffener flange, and in-plane loads are positive when in compression (Fig. 4). The deflection, w_0, due to the lateral load (i.e., lateral pressure) M_0 and initial eccentricity, Δ, are considered positive when they are toward the stiffener, as shown in Fig. 5. In beam-column theory, the expressions for the moment M_0 and the corresponding deflection w_0 are based on an ideal column, in which the beam-column is assumed simply supported.

Disregarding plate failure in tension, there can be three distinct modes of collapse (see Fig. 4) according to Hughes (1988). These modes are due to (1) compression failure of the stiffener (mode I collapse), (2) compression failure of the plating (mode II collapse), and (3) combined failure of stiffener and plating (mode III collapse). These modes of failure or collapse will be discussed in detail in the subsequent sections. In addition, the mathematical expressions of the ultimate axial stress for these modes of failure will be provided.

The ultimate axial strength (stress) F_u for a longitudinally stiffened panel under a combination of in-plane compression and lateral loads (including initial eccentricities) can be, therefore, defined as the minimum of the collapse (ultimate) values of applied axial stress computed from the expressions for the three types (modes) of failure. Mathematically, it can be given as

$$F_u = \min(F_{a,uI}, F_{a,uII}, F_{a,uIII}) \tag{19}$$

where $F_{a,uI}$, $F_{a,uII}$, and $F_{a,uIII}$ correspond to the ultimate collapse value of the applied axial stress for mode I, mode II, and mode III, respectively. Detailed descriptions of these failure modes with their mathematical expressions can be found in Hughes (1988) and Assakkaf (1998).

API (1993). The API Bulletin 2V (1993) adopts the Watanabe et al (1981) model for the ultimate stress F_u for multiple stiffened plates under pure compression. The Watanabe et al model was discussed here in detail, in the first subsection of "Strength models for longitudinal stiffened panels." The ultimate strength F_u, as described in that subsection (Watanabe et al 1981) for multiple stiffened plates under pure compression, is given by equations (1) through (7).

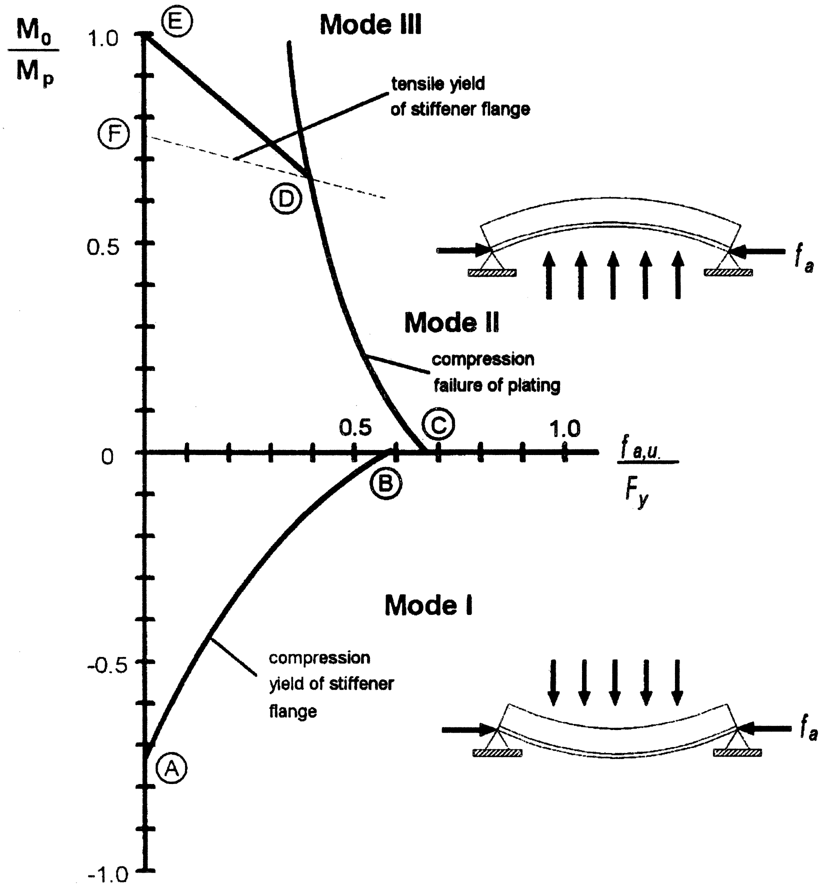

Fig. 4 Interaction diagram for collapse mechanism of a stiffened panel under lateral and in-plane loads (Hughes 1988)

AASHTO (1994). *Compressive strength.* The ultimate strength of a stiffened panel subjected to uniaxial compressive strength is given by AASHTO (1994):

$$F_u = \begin{cases} F_y(0.66)^\lambda & \text{for } \lambda \le 2.25 \\ \dfrac{0.88F_y}{\lambda} & \text{for } \lambda > 2.25 \end{cases} \tag{20}$$

where

$$\lambda = \left(\frac{ak}{r\pi}\right)^2 \frac{F_y}{E} \tag{21}$$

The limiting width/thickness ratios for axial compression is to satisfy

$$\frac{b}{t} \le k\sqrt{\frac{E}{F_y}} \tag{22}$$

where b = spacing between stiffener, a = length of panel, E = Young's modulus, F_y = weighted yield strength, and k = plate buckling coefficient, as specified in Table 2.

Box sections in flexure. Box sections are common structures to support the longitudinal spans of bridges. There could be one, two, or four box girders underneath a bridge span. In the United States, box girders are usually made of a bottom longitudinally

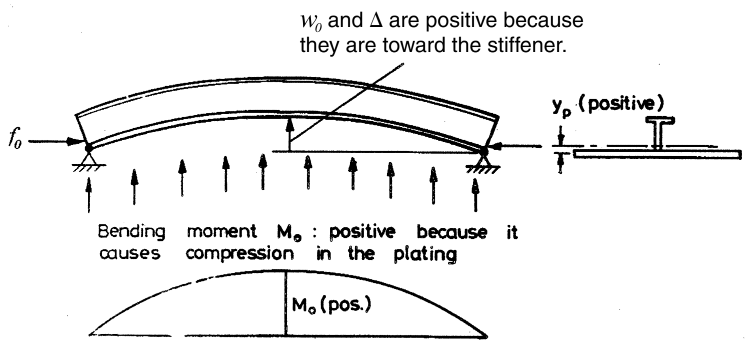

Fig. 5 In-plane compression and positive bending moment (Hughes 1988)

Table 2a Limiting width/thickness ratios for plates supported along one edge (unstiffened plates) as given by AASHTO LRFD specifications (1994)

Case	B	b
Flanges and projecting legs or plates	0.56	• Half-flange width of I sections • Full-flange width of channels • Distance between free edge and first line of bolts or welds in plates • Full width of an outstanding leg for pairs of angles in continuous contact
Stems of rolled tees	0.75	• Full depth of tee
Other projecting elements	0.45	• Full width of outstanding leg for single-angle strut or double-angle strut with separator • Full projecting width for others

Table 2b Limiting width/thickness ratios for plates supported along two edges (stiffened plates) as given by AASHTO LRFD specifications (1994)

Case	B	b
Box flanges and cover plates	1.40	• Clear distance between webs minus inside corner radius on each side for box flanges • Distance between lines of welds or bolts for flange cover plates
Webs and other plate elements	1.49	• Clear distance between flanges minus fillet radii for webs of rolled beams • Clear distance between edge supports for all others
Perforated cover plates	1.86	• Clear distance between edge supports

stiffened plate (bottom flange), two inclined transversely stiffened plates (web with inclination not to exceed 1 to 4), and two top narrow flanges adjacent to the bottom of the bridge main slab. In flexure, the bottom flanges are subjected to both bending stress and uniaxial compression stress due to the bending moment stress distribution about the neutral axis of the box girder. The flexural resistance for the longitudinally stiffened plate (bottom flange) under compression in a single box section negative flexure is determined by

$$F_u = \begin{cases} R_b R_h F_y & \text{for } \dfrac{w}{t} \le 0.57\sqrt{\dfrac{kE}{F_y}} \\ 0.592 R_b R_h F_y \left(1 + 0.687 \sin\dfrac{c\pi}{2}\right) & \text{for } 0.57\sqrt{\dfrac{kE}{F_y}} < \dfrac{w}{t} \le 1.23\sqrt{\dfrac{kE}{F_y}} \\ 26200 R_b R_h k \left(\dfrac{t}{w}\right)^2 & \text{for } \dfrac{w}{t} > 1.23\sqrt{\dfrac{kE}{F_y}} \end{cases} \tag{23}$$

where F_y = specified minimum yield strength of the compressive flange (ksi), t = plate thickness (in), and R_b, R_h = flange stress reduction factors to be taken as 1.0 in this case (no webs connected to the subpanels), and c = coefficient for determining the bending resistance and is given by

$$c = \frac{1.23 - \dfrac{w}{t}\sqrt{\dfrac{F_y}{kE}}}{0.66} \tag{24}$$

where k = plate buckling coefficient and is given by

$$k = \begin{cases} \left(\dfrac{8I_s}{wt^3}\right)^{1/3} \le 4.0 & \text{for } n = 1 \\ \left(\dfrac{14.3I_s}{wt^3n^4}\right)^{1/3} \le 4.0 & \text{for } n = 2, 3, 4, \text{ and } 5 \end{cases} \tag{25}$$

where w = the larger of the width of compression flange between longitudinal stiffeners or the distance from a web to the nearest longitudinal stiffener, n = number of equally spaced longitudinal compression flange stiffeners, and I_s = moment of inertia of a longitudinal stiffener about an axis parallel to the bottom flange and taken at the base of the stiffener.

The projection spacing of longitudinal stiffener, b, should satisfy

$$b \le 0.48 t_s \sqrt{\frac{E}{F_y}} \tag{26}$$

where t_s = thickness of stiffener. The moment of inertia of each stiffener about an axis parallel to the flange and taken at the base of the stiffener must satisfy

$$I_I \ge \psi w t_f^{\,3} \tag{27}$$

where w = the greater of the width of compression flange between longitudinal stiffeners or the distance from a web to the nearest longitudinal stiffener, t_f = compression flange thickness, and

$$\psi = \begin{cases} 0.125k^3 & \text{for } n = 1 \\ 0.07k^3n^4 & \text{for } n = 2, 3, 4, \text{ and } 5 \end{cases} \tag{28}$$

where n = number of equally spaced longitudinal stiffeners, t_f = thickness of stiffened panel plating, and b_f = width of stiffened panel plating. If the width of the subpanel exceeds one fifth of its span (length), only a width equal to one fifth of the span is considered effective in resisting bending moment.

Navy practice. In the Navy practice, a column configuration frequently encountered in structural design of ships is the plate-stiffener combination consisting of structural shape (tee, angle) welded to a plate (Ayyub et al 1998). The strength model is based on the work introduced by Bleich and Ramsey (1965) and Klitchieff (1951). The width of the plating that may be considered effective can be calculated according to the following expression:

$$b_e = 2t\sqrt{\frac{E}{F_y}} \tag{29}$$

This means if the above equation is used, then the value for the effective width b_e will be $60t$ for MS, $50t$ for HTS, $38.5t$ for HY80, $34.5t$ for HY100, $42.5t$ for AL-5086, and $39t$ for AL-5456. It is also necessary that the effective breadth (width) shall not exceed one-half the sum of spacing on each side of the member or 33% of the unsupported span length a, whichever is less. For girders and webs along a hatch opening, an effective breadth of plating is not to exceed one-half the spacing or 16.5% of the unsupported span length a; whichever is less is to be used.

According to Ayyub et al (1998), there exists a procedure for determining column strength F_u. This includes plate-stiffener combination or longitudinally stiffened panel with an effective plating b_e as provided by equation (29). The procedure for determining the strength F_u of plate-stiffener combination with an effective width b_e is as follows:

1. Calculate the column factor C from

$$C = k_c \left(\frac{a}{r}\right)\sqrt{\frac{F_y}{E}} \quad (30)$$

2. Determine column-to-yield strength ratio (F_c/F_y) from Fig. 6.
3. With (F_c/F_y) determined from step 2, the column strength is given as

$$F_u = \left(\frac{F_c}{F_y}\right)F_y \quad (31)$$

The coefficient k_c for end condition may vary between 0.5 and 2.0 as provided in Table 3.

Table 3 Values for the end coefficient k_c (Ayyub et al 1998, Assakkaf 1998)

End condition	k_c
Cantilever	2
Pinned ends	1
One end pinned, the other fixed	0.75
Clamped ends	0.50

Comparison and evaluation of strength models for stiffened panels

In this section, a comparison between real and predicted values of ultimate strength was performed based on real test specimens from various sources. Some of the strength models used in this comparison are adopted in the current design codes, such as the AISC LRFD (1994), AASHTO LRFD (1994), and API (1993). Other models that are also used in this comparison are those developed by different researchers, such as Hughes (1988), Adamchak (1979), Herzog (1987), Paik and Lee (1996), and Mikami and Niwa (1996).

The purpose of this comparison is to select the most appropriate model (models) for use in LRFD design format. The level of complexity associated with the above-mentioned strength models ranges from highly complex models to simple ones. The more complex theoretical models, such as that of Adamchak (equation [18]), and Hughes (equation [19]) do not necessarily lead to less uncertainty. Although they can be accurate and rigorous models, they can be more uncertain because they involve a larger number of variables, some of which may be very uncertain. On the other hand, simple empirical formulations based on real test data, such as that of Herzog (equations [16] and [17]) and Paik and Lee (equation [8]), can lead to fairly good results. Although theoretically less rigorous, they can be of practical use because they were derived from real-world stiffened plates tests. In formulating a design model, a balance must be achieved between the model accuracy, bias, applicability, and simplicity, all of which are desired features.

The uncertainty and the biases of a strength model can be assessed by comparing model prediction with the ones that are more accurate. In the following two sections, bias assessments for uniaxial strength of longitudinally stiffened panels under axial and lateral (pressure) loads, respectively, are presented.

Bias assessment for uniaxial strength models without lateral pressure. This section provides a comparison that was performed by Assakkaf and Atua (1997) on nearly 80 test specimens under uniaxial load alone. The failure axial stress and the mode of

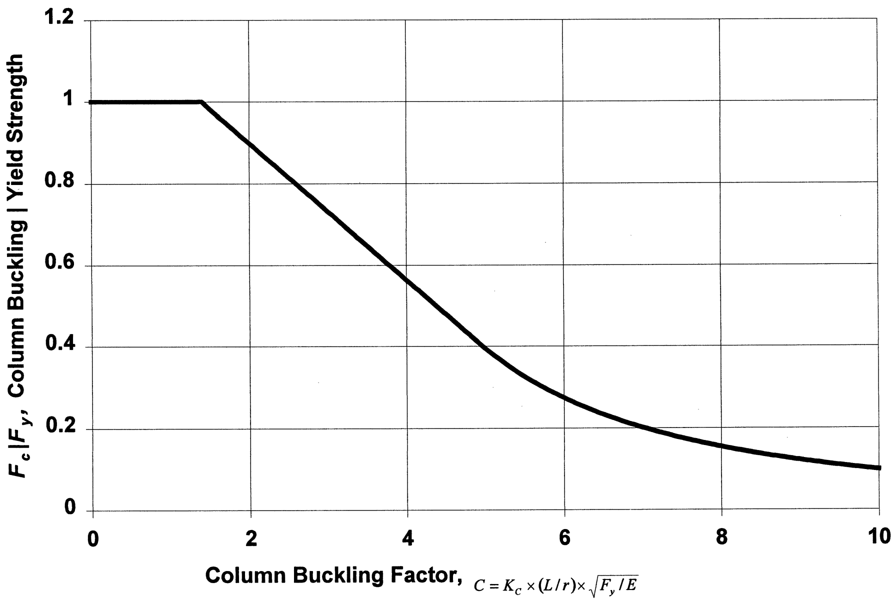

Fig. 6 Column strength (Ayyub et al 1998, Assakkaf 1998)

Table 4 Statistics of the bias (real/predicted) for strength models under uniaxial stress, without lateral pressure (Assakkaf 1998, Atua 1998)

Bias	Hughes	Paik and Lee	Herzog ($m = 1.2$)	Herzog ($m = 1.0$)	Herzog ($m = 0.8$)	Adamchak
Mean	1.085	1.030	0.837	1.004	1.255	0.844
Standard deviation	0.187	0.188	0.154	0.185	0.231	0.245
COV	0.173	0.182	0.184	0.184	0.184	0.291

COV = coefficient of variation; m = correction factor accounts for initial deformation and residual stresses.

failure for each test were reported. Table 4 provides the mean, standard deviation, and the COV of the bias (real/predicted) for Hughes (1988), Herzog (1987), Adamchak (1979), and Paik and Lee (1996) models. It is apparent from the results in this table that these models have the least bias values for the predicted strength (stress). Table 5 gives the mean, standard deviation, and the COV of the bias (real/predicted) for the strength models used in the current design codes for stiffened panels. Variations in the bias as a function of column slenderness ratio for Hughes (1988), Herzog (1987), Adamchak (1979), and Paik and Lee (1996) models are shown in Fig. 7. Figure 8 gives the variation in the bias for the current design codes.

Table 5 Statistics of the bias (real/predicted) of the current design codes for stiffened plates under uniaxial stress, without lateral pressure (Assakkaf 1998, Atua 1998)

Bias	API (1993)	AISC (1994)	AASHTO (1994)	Navy Practice
Mean	0.794	0.819	0.818	0.784
Standard deviation	0.203	0.168	0.167	0.160
COV	0.255	0.205	0.205	0.204

COV = coefficient of variation.

Bias assessment for uniaxial strength models with lateral pressure. In this section, the comparison was performed by Assakkaf (1998) and Atua (1998) on 14 test specimens subjected to a combination of uniaxial stress and uniform lateral pressure. The failure axial stress and the mode of failure for each test were reported. Table 6 gives the mean, standard deviation, and the COV of the bias (real/predicted) for Hughes (1988), Herzog (1987), Adamchak (1979), and Paik and Lee (1996) models. The results in this table suggest that these models have the least bias values for the predicted ultimate strength (stress) as compared to the values predicted by the codes. Table 7 gives the mean, standard deviation, and the COV of the bias (real/predicted) for the strength models used in the current design codes for stiffened panels. Variations in the bias as a function of the ratio of applied moment to plastic moment for stiffened panels with simply supported ends are shown in Fig. 9. Figure 10 gives the variations in the bias for the clamped case.

Summary of stiffened panels strength models for developing LRFD guidelines

In the preceding sections, comparison and evaluation of the strength models for stiffened plates, including those used in the current design codes, were performed. It was concluded that the

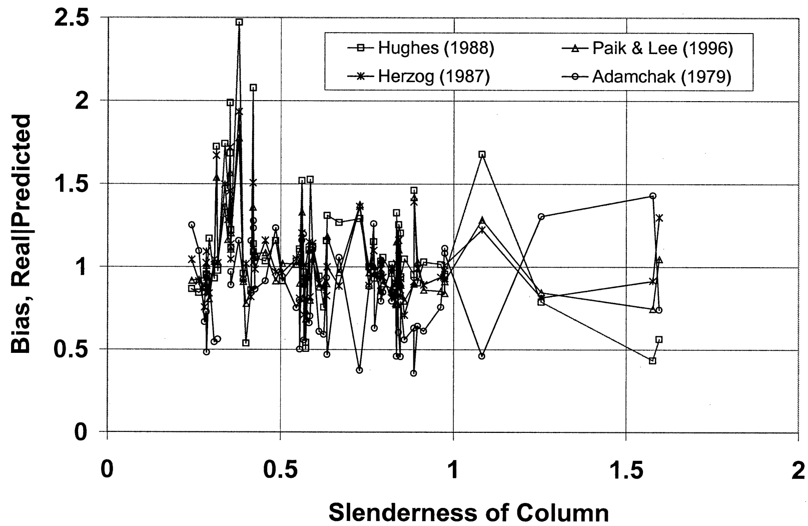

Fig. 7 Variation of bias of strength models as a function of slenderness ratio of column under uniaxial load only (Assakkaf 1998, Atua 1998)

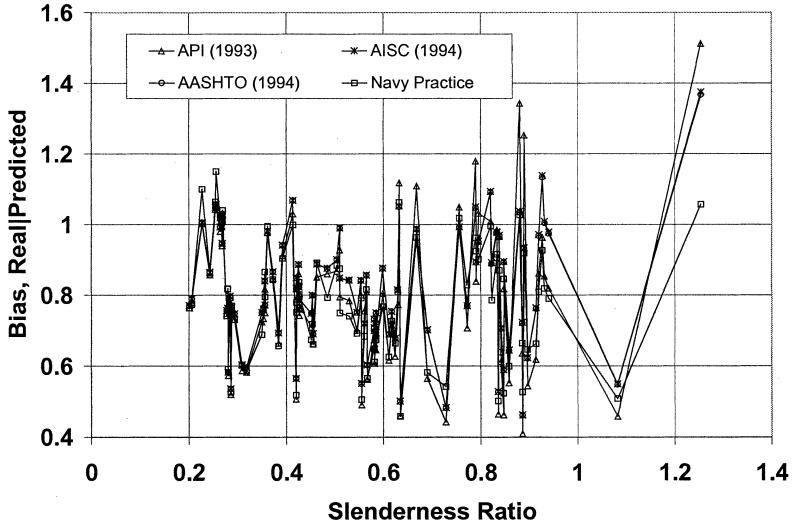

Fig. 8 Variation of bias of current design codes as a function of slenderness ratio of column under uniaxial load only (Assakkaf 1998, Atua 1998)

Table 6 Statistics of the bias (real/predicted) for strength models under uniaxial stress with lateral pressure (Assakkaf 1998, Atua 1998)

Bias	Hughes	Paik and Lee	Herzog (m =1.2)	Herzog (m =1.0)	Herzog (m = 0.8)	Adamchak
Mean	1.316	1.061	0.828	0.994	1.242	1.08
Standard deviation	0.303	0.160	0.152	0.182	0.228	.25
COV	0.230	0.151	0.183	0.183	0.183	.232

COV = coefficient of variation; m = correction factor accounts for initial deformation and residual stresses.

empirical formula proposed by Herzog (1987) for predicting the ultimate strength of stiffened panels has the least bias with a relatively low coefficient of variation among the other models. Also, the models used in the current design codes are extremely nonconservative with high COV values for the bias (in cases of combined uniaxial stress and lateral pressure). This could be attributed to the fact that these models are not developed for stiffened panels with slenderness ratios and geometry that resemble shipbuilding stiffeners.

The strength model proposed by Hughes (1988) is overconservative, especially in the case of combined uniaxial and lateral stresses. Also, this model is very complex and it has many variables that make it unsuitable for use in a design code. Based on the statistical analyses performed on the various models presented in the paper, the only model that considers the combination of both uniaxial stress and lateral pressure in the prediction of the ultimate strength and has a lower bias (real/predicted) value is that of Adamchak (1979). Although this model is very complex in nature, it can be used in a design code with the help of a computer program such as ULSTRA, or any other computational tools, to facilitate the calculation task. It is to be noted that this model gives underconservative results in the case of uniaxial stress without lateral pressure. Finally, it was concluded that the lateral pressure has a minor effect on the real value of the ultimate strength of stiffened panels.

Table 7 Statistics of the bias (real/predicted) of the current design codes for stiffened plates under uniaxial stress with lateral pressure (Assakkaf 1998, Atua 1998)

Bias	API (1993)	AISC (1994)	AASHTO (1994)
Mean	0.758	0.777	0.709
Standard deviation	0.468	0.432	0.484
COV	0.617	0.556	0.683

COV = coefficient of variation.

Table 8 provides recommended stiffened panels strength models for the LRFD development. The table also shows their probabilistic characteristics and biases.

Gross panels

The definition of a gross stiffened panel is a panel of plating that has stiffeners running in two orthogonal directions. This panel is bounded by another structure, which has significantly greater stiffness in the planes of the loads when compared to the panel longitudinal bulkheads, side shell, or large longitudinal girders. Choosing the size of the transverse stiffeners so that they provide sufficient flexural rigidity to enforce nodes at the location of the transverse stiffener can prevent the collapse of a stiffened panel. If the transverse stiffeners act as nodes, then the collapse of the stiffened panel is controlled by the strength of the longitudinally stiffened subpanel.

A typical longitudinally stiffened subpanel, as shown in Fig. 1, is bounded on each end by a transverse structure that has

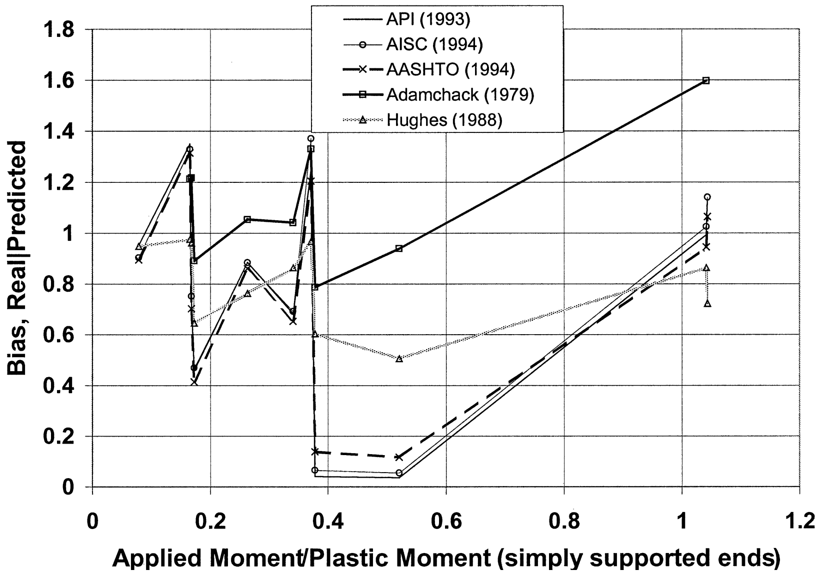

Fig. 9 Variation of bias of the current design codes as a function of applied moment to plastic moment, simply supported ends (Assakkaf 1998, Atua 1998)

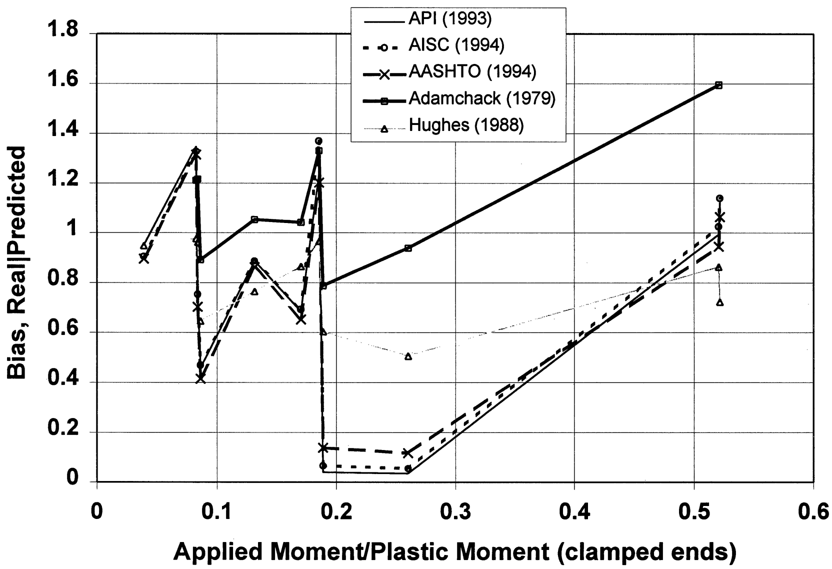

Fig. 10. Variation of bias of the current design codes as a function of applied /plastic moments ratio, clamped ends (Assakkaf 1998, Atua 1998)

Table 8 Recommended stiffened panels strength models for load and resistance factor design methods

Loading Case	Description	Reference	Distribution Type	Total Bias B_T	COV (%)
Uniaxial compression	$F_u = \begin{cases} mF_y\left[0.5 + 0.5\left(1 - \frac{a}{r\pi}\sqrt{\frac{F_y}{E}}\right)\right] & \text{for } \frac{b}{t} \le 45 \\ mF_y\left[0.5 + 0.5\left(1 - \frac{a}{r\pi}\sqrt{\frac{F_y}{E}}\right)\right]\left[1 - 0.007\left(\frac{b}{t} - 45\right)\right] & \text{for } \frac{b}{t} > 45 \end{cases}$	Herzog (1987), equation (16)	Lognormal	1.09	18
Uniaxial compression with lateral pressure	$F_u = \min(F_{uI}, F_{uII}, F_{uIII})$	Adamchak (1979), equation (18)	Lognormal	1.25	25
	$F_u = \min(F_{uI}, F_{uII}, F_{uIII})$	Hughes (1988), equation (19)	Lognormal	1.30	23

COV = coefficient of variation.

significantly greater stiffness in the plane of the lateral load. The sides of the panel are defined by the presence of a large structural member that has greater stiffness in bending and much greater stiffness in axial loading. Such structural members as keels, bottom girders, longitudinal bulkheads, and deck girders can act as the side boundaries of the panel.

When the panel is located in such a way that it is in a position to experience large in-plane compression, the boundary conditions for the ends are taken as simply supported. The boundary conditions along the sides can also be considered simply supported.

As noted earlier, three types of loads affect the panel strength. Negative bending loads are the lateral loads due to uniform lateral pressure, which in effect causes the plate to be in tension and the stiffener flange to be in compression. Positive bending moment loads are the lateral loads, which put the plating in compression and the stiffener flange in tension. The third type of loading is the uniform in-plane compression. This type of loading arises from the hull girder bending moment. It is positive when the panel is in compression and negative when the panel is in tension. These loads may act individually or in combination with one another.

Strength models for gross panels

Mikami and Niwa (1996). *Elastic strength.* In this section, a method for determining the elastic strength of gross panel to avoid buckling under uniaxial compression load is presented. This method is proposed for orthogonal stiffened plates. The stiffened panel in this method is assumed simply supported along all the edges and subjected to uniaxial compression stress, as shown in Fig. 11. The orthogonal stiffened plate is considered an orthotropic plate (Giencke 1964a, 1964b). The elastic buckling strength of the panel is defined either by the overall buckling strength of the gross panel or the buckling strength of the partial panel, as shown in Fig. 11. In this case, no local buckling occurs in a single panel or in a longitudinal stiffener. If any local buckling occurs under a lower load, the orthogonal stiffened plate has postbuckling strength.

The critical elastic buckling stress of the overall panel is given by

$$F_e = \frac{\pi^2 E}{12(1-\nu^2)(1+\delta_s)\beta^2} \times \left[\gamma_s\left(\frac{m}{\alpha_p}\right)^2 + \frac{\gamma_r}{\alpha_{pr}}\left(\frac{\alpha_p}{m}\right)^2 + \left(\frac{m}{\alpha_p}+\frac{\alpha_p}{m}\right)^2\right] \tag{32}$$

where γ_r = flexural rigidity ratio of transverse stiffener and is equal to $\gamma_r = E_r I_r / bD_p$, γ_s = flexural rigidity ratio of longitudinal stiffener where $\gamma_r = E_s I_s / bD_p$, α_p = aspect ratio of gross panel, α_{po} = maximum aspect ratio of gross panel when buckled with one half-wave, α_{pr} = aspect ratio of partial panel, E_s = section modulus of longitudinal stiffeners, E_r = section modulus of transverse stiffeners, α_p = aspect ratio of gross panel, m = number of half-waves of longitudinal buckling mode of gross panel, and δ_s = cross-sectional area ratio of longitudinal stiffener of uniformly stiffened plate where $\delta_s = A_s/bt$ where A_s = cross-sectional area of longitudinal stiffener.

The overall buckling occurs when m is less than $n_r + 1$ where n_r = number of transverse stiffeners. The expression given by equation (11) is simplified by the following two expressions:

1. For $\alpha_p < \alpha_{p0}$:

$$F_e = \frac{\pi^2 E}{12(1-\nu^2)(1+\delta_s)\beta^2} \times \left[\left(\alpha_p + \frac{1}{\alpha_p}\right)^2 + (n_r+1)\,\gamma_r\alpha_{pr} + \frac{\gamma_s}{\alpha_p{}^2}\right] \tag{33}$$

2. For $\alpha_p \geq \alpha_{p0}$:

$$F_e = \frac{\pi^2 E}{12(1-\nu^2)(1+\delta_s)\beta^2}\left[1 + \sqrt{(1+\gamma_s)\left(1+\frac{\gamma_r}{\alpha_{pr}}\right)}\right] \tag{34}$$

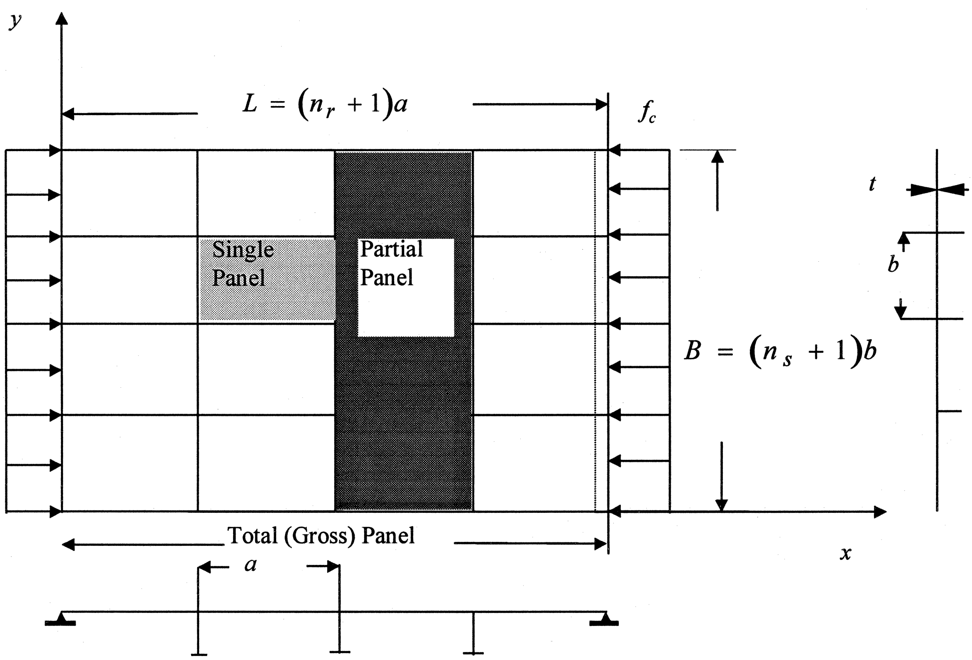

Fig. 11. Orthogonally stiffened panel (Mikami & Niwa 1996)

where f_e = elastic buckling stress, α_{po} = maximum aspect ratio of gross panel when buckled in one half-wave and is given by

$$\alpha_{po} = \sqrt[4]{\frac{1 + \gamma_s}{1 + \frac{\gamma_r}{\alpha_r}}} \tag{35}$$

The aspect ratio of the total panel given by equation (35) gives the minimum strength in the case of buckling with one half-wave in the longitudinal direction.

The buckling strength of the gross, partial, and single panel is obtained from the ultimate strength curve represented by the parameter λ, where

$$\lambda = \frac{F_y}{F_s} \tag{36}$$

This parameter is used in various design specifications, such as DASt (DASt-Richtlinie 012, 1979), FHWA (Wolchk and Mayrbauri 1980), Dubas and Gehri (1986), and BS5400 (BSI 1980). Substituting equations (33) and (34) into equation (36), yields

1. For $\alpha_p < \alpha_{p0}$:

$$\lambda_1 = \frac{R\alpha_p}{\sqrt{[\gamma_r + (n_r + 1)\gamma_r\alpha_p{}^3 + (1 + \alpha_p{}^2)^2]\,(1 + \delta_s)}} \tag{37}$$

2. For $\alpha_p \geq \alpha_{p0}$:

$$\lambda_2 = \frac{R}{\sqrt{2(1 + \delta_s)\left[1 + \sqrt{(1 + \gamma_s)\left(1 + \frac{\gamma_r}{\alpha_{pr}}\right)}\right]}} \tag{38}$$

where

$$R = B_p\sqrt{\frac{12(1 - \nu^2)}{\pi^2 E}F_y{}^*} \tag{39}$$

R = equivalent width-thickness ratio of gross panel and $F_y{}^*$ = equivalent yield stress of uniformly stiffened panel. The equivalent yield stress, $F_y{}^*$, is given by

$$F_y{}^* = \frac{1 + \frac{\delta_s F_{ys}}{F_y}}{1 + \delta_s}F_y \tag{40}$$

where F_{ys} = yield stress of longitudinal stiffener. The overall buckling stress of orthogonally stiffened plate is obtained by using the parameters λ_1 or λ_2. For convenience in practical design, the parameter λ_2 is used instead of λ_1.

The elastic buckling stress of a longitudinally stiffened subpanel (Fig. 1) is given by

$$F_e = \frac{\pi^2 E}{12(1 - \nu^2)(1 + \delta_s)B_p{}^2}\left[\gamma_s\left(\frac{m_r}{\alpha_{pr}}\right)^2 + \left(\frac{m_r}{\alpha_{rp}} + \frac{\alpha_{pr}}{m_r}\right)^2\right] \tag{41}$$

Because the overall buckling occurs when m is less than n_r, equation (41) is simplified as follows:

1. For $\alpha_{pr} < \alpha_{pro}$:

$$F_e = \frac{\pi^2 E}{12(1 - \nu^2)(1 + \delta_s)B_p{}^2}\left[\left|\alpha_{pr} + \frac{1}{\alpha_{pr}}\right|^2 + \frac{\gamma_s}{\alpha_{pr}{}^2}\right] \tag{42}$$

2. For $\alpha_{pr} \geq \alpha_{pro}$:

$$F_e = \frac{\pi^2 E}{12(1 - \nu^2)(1 + \delta_s)B_p{}^2}\left[1 + \sqrt{1 + \gamma_s}\right] \tag{43}$$

where α_{pro} = maximum aspect ratio of partial panel when buckled with one half-wave and it is given by

$$\alpha_{pro} = \sqrt[4]{1 + \gamma_s} \tag{44}$$

The aspect ratio of the partial panel given by equation (44) gives the minimum strength in the case of buckling with one half-wave in the longitudinal direction. Substituting equations (42) and (43) into equation (36) yields

1. For $\alpha_{pr} < \alpha_{pro}$:

$$\lambda_3 = \frac{R\alpha_{pr}}{\sqrt{[\gamma_r + (1 + \alpha_{pr}{}^2)^2]\,(1 + \delta_s)}} \tag{45}$$

2. For $\alpha_{pr} \geq \alpha_{pro}$:

$$\lambda_4 = \frac{R}{\sqrt{2(1 + \delta_s)(1 + \sqrt{1 + \gamma_s})}} \tag{46}$$

The partial buckling stress of the longitudinally stiffened plate can be determined using the above parameters λ_3 or λ_4. If $\alpha_{pr} < \alpha_{pro}$, the greater value of λ_2 and λ_3 corresponds to the elastic buckling stress of the orthogonally stiffened plate. If $\alpha_{pr} \geq \alpha_{pro}$, λ_4 also corresponds to the elastic buckling stress.

Ultimate strength. The ultimate strength of the orthogonally stiffened plate (gross panel) without local buckling is obtained by the following ultimate strength expression (Mikami & Niwa 1990) for the determined type of λ:

$$\frac{F_u}{F_y{}^*} = \begin{cases} 1.0 & \lambda \leq 0.3 \\ 1.0 - 0.63\,(\lambda - 0.3) & 0.3 < \lambda \leq 1.0 \\ 1.0/(0.8 + \lambda^2) & 1.0 < \lambda \end{cases} \tag{47}$$

where F_u = ultimate stress of stiffened plate without local buckling.

The ultimate strength of orthogonally stiffened plate (gross panel) with local buckling is obtained by reducing the elastic strength of the nonlocal buckling. It is based on the effective width of the locally buckled panel instead of its true width. According to Mikami and Niwa (1996), the ultimate strength for gross panel with local buckling is given by

$$\frac{F_u{}^*}{F_y{}^*} = \frac{\rho_p F_y + \delta_s\rho_s F_{ys}}{(1 + \delta_s)F_y{}^*}\,\frac{F_u}{F_y{}^*} \tag{48}$$

where $F_u{}^*$ = ultimate stress of stiffened plate with local buckling, ρ_p = effective width of single panel, and ρ_s = effective width of longitudinal stiffener. ρ_p and ρ_s can be obtained, respectively, from the following two expressions:

$$\rho_p = \begin{cases} 1.0 & F_{up} \geq F_u \\ F_{up}/F_y & F_{up} < F_u \end{cases} \tag{49}$$

$$\rho_s = \begin{cases} 1.0 & F_{us} \geq F_u \\ F_{us}/F_{ys} & F_{us} < F_u \end{cases} \tag{50}$$

where F_{up} = the local buckling strength of a single panel, and it is given by

$$F_{up} = \begin{cases} F_y & \lambda_p \leq 0.526 \\ F_y(0.526/\lambda_p) & 0.526 < \lambda_p \end{cases} \tag{51}$$

in which λ_p = buckling parameter of single panel, and it is given by

$$\lambda_p = \frac{b}{t}\sqrt{\frac{12(1-\nu^2)}{4\pi^2 E}F_y} \qquad (52)$$

The local buckling strength of a stiffener F_{us} = the local buckling strength of a stiffener is given by the following two cases:

1. For plate element (flat stiffeners) (Mikami et al 1983):

$$F_{us} = \begin{cases} F_{ys} & \lambda_s \leq 0.526 \\ F_{ys}\,(0.526/\lambda_s)^{0.7} & 0.526 < \lambda_s \end{cases} \qquad (53)$$

where λ_s = buckling parameter of each element of longitudinal stiffener, λ_{st} = buckling parameter of torsional buckling of longitudinal stiffener.
2. For stiffener as member (T-, L-, U-, and V-shaped stiffeners):

$$\frac{F_{us}}{F_{ys}} = \begin{cases} 1.0 & \lambda_{st} \leq 0.3 \\ 1.0 - 0.53(\lambda_{st} - 0.45) & 0.3 < \lambda < 1.41 \\ 1.0/\lambda_{st}^2 & 1.41 < \lambda \end{cases} \qquad (54)$$

where λ_s = buckling parameter of each element of longitudinal stiffener and it is given by

$$\lambda_s = \frac{b_{si}}{t_{si}}\sqrt{\frac{12(1-\nu_s^2)}{k\pi^2 E_s}F_{ys}} \qquad (55)$$

where k = buckling coefficient where k is 4.0 or 0.425 for simply supported plate element or outstanding element, respectively, ν_s = Poisson's ratio for transverse stiffener, and λ_{st}= buckling parameter of torsional buckling of longitudinal stiffener. The buckling parameter of torsional buckling of longitudinal stiffener is given by

$$\lambda_{st} = \sqrt{\frac{F_{ys}}{F_{crt}}} \qquad (56)$$

where F_{crt} = elastic torsional buckling stress of the longitudinal stiffener and it is given by

$$F_{crt} = \frac{G_s J_s}{I_{pc}} \qquad (57)$$

where G_s = shear modulus of longitudinal stiffener, J_s = torsion constant of longitudinal stiffener, and I_{pc} = polar moment of inertia of one longitudinal stiffener with respect to the point connected with the main plate.

Hughes 1988 (elastic and plastic). The minimum serviceability strength required to avoid elastic and plastic buckling of gross stiffened panel can be given by (Mansour 1976, 1986)

1. For elastic range, $F_s \leq F_{pr}$:

$$F_s = \begin{cases} 4\gamma_g \dfrac{\pi^2\sqrt{\dfrac{D_x}{D_y}}}{t_x L_b^2} & \text{for } \dfrac{L_a}{L_b} > 1.0 \\ \gamma_g\left(\dfrac{1}{\rho} + 2\eta + \rho^3\right)\dfrac{\pi^2\sqrt{D_x D_y}}{t_x L_b^2} & \text{for } \dfrac{L_a}{L_b} < 1.0 \end{cases} \qquad (58)$$

2. For plastic range, $F_s > F_{pr}$:

$$F_s = \begin{cases} F_y \dfrac{C_1}{C_1 + 1} & \text{For } \dfrac{L_a}{L_b} \geq 1.0 \\ F_y\gamma_g - \dfrac{F_{pr}\,(F_y - F_{pr}) t_x L_b^2}{\left(\dfrac{1}{\rho} + 2\eta + \rho^3\right)\pi^2\sqrt{D_x D_y}} & \text{For } \dfrac{L_a}{L_b} < 1.0 \end{cases} \qquad (59)$$

where

$$C_1 = \frac{\left[4\pi^2\sqrt{\dfrac{D_x D_y}{t_x L_b^2}}\right]^2}{F_{pr}(F_y - F_{pr})} \qquad (60)$$

γ_g = factor of safety associated with the buckling of the entire gross panel and includes the uncertainty in both initial imperfection and residual stresses, f_s = serviceability limit strength for plate under uniaxial compressive stress, F_{pr} = proportional (linear elastic) limit stress in compression (taken as 60% of F_y), F_y = average yield strength (stress) in compression, A = length or span of the panel (unloaded edge), B = width of the panel (loaded edge), t_x = equivalent thickness of plate and stiffener (diffused) extending in the x-direction, D_x flexural rigidity of panel in the x-direction and it is given by

$$D_x = \frac{EI_x}{S_y\,(1-\nu^2)} \qquad (61)$$

D_y = flexural rigidity of panel in the y-direction and it is given by

$$D_y = \frac{EI_y}{S_y\,(1-\nu^2)} \qquad (62)$$

ρ = virtual aspect ratio of gross panel (Huber 1929) and it is given by

$$\rho = \frac{L_a}{L_b}\sqrt[4]{\frac{D_x}{D_y}} \qquad (63)$$

η = torsion stiffness parameter (Schade 1938, 1940, 1941) and it is given by

$$\eta = \sqrt{\frac{I_{px} I_{py}}{I_x I_y}} \qquad (64)$$

in which I_x = moment of inertia of the stiffener with effective breadth of plating extending in the x-direction, I_y = moment of inertia of the stiffener with effective breadth of plating extending in the y-direction, I_{px} = moment of inertia of the effective plating associated with stiffeners extending in the x-direction, I_{py} = moment of inertia of the effective plating associated with stiffeners extending in the y-direction, S_x = spacing of the stiffeners extending in the x-direction, and S_y = spacing of the stiffeners extending in the y-direction.

API Bulletin 2V (1993). *Uniaxial compression.* The serviceability limit state is given by the following two cases:

1. For elastic range ($F_s < F_{pr}$):

$$F_s = k\frac{\pi^2(D_x D_y)^{0.5}}{t_x L_b^2} \qquad (65)$$

2. For plastic range ($F_s \geq F_{pr}$):

$$F_s = \begin{cases} \dfrac{C_e F_y}{C_e + 1} & \text{for } \dfrac{L_a}{L_b} \geq 1.0 \\[2ex] F_y - \dfrac{1}{C_s} & \text{for } \dfrac{L_a}{L_b} < 1.0 \end{cases} \tag{66}$$

where D_x, D_y, ρ, and η as given by equations (61), (62), (63), and (64), respectively, and

$$C_e = \frac{f_e^{\,2}}{F_{pr}\,(F_y - F_{pr})} \tag{67}$$

$$C_s = \frac{f_s}{F_{pr}\,(F_y - F_{pr})} \tag{68}$$

in which

$$f_e = \frac{4\pi^2 (D_x D_y)^{0.5}}{t_x L_b^{\,2}} \tag{69}$$

and

$$f_s = \left(\frac{1}{\rho^2} + 2\eta + \rho^2\right) \frac{f_e}{4} \tag{70}$$

Biaxial compression. The serviceability limit state is given by the following two cases (API 1993):

1. For elastic range ($F_{sx}^2 - F_{sx}F_{sy} + F_{sy}^2 < F_{pr}^2$):

$$\frac{F_{sx}}{F_x{*}} m^2 + \frac{F_{sy}}{F_y{*}} n^2 = \frac{m^4}{\rho^2} + 2\eta m^2 n^2 + \rho^2 n^4 \qquad \text{for } F_{sx} \leq F_{pr} \tag{71}$$

$$F_x{*} = \frac{\pi^2 (D_x D_y)^{0.5}}{t_x L_b^{\,2}} \tag{72}$$

$$F_y{*} = \frac{\pi^2 (D_x D_y)^{0.5}}{t_y L_a^{\,2}} \tag{73}$$

The serviceability limit state is elastic if the stresses F_{sx} and F_{sy} computed from the above expressions satisfy the following criterion:

$$F_{sx}^2 - F_{sx}F_{sy} + F_{sy}^2 < F_{pr}^2 \tag{74}$$

A trial procedure for equation (71) is required to determine the values of F_{sx}, F_{sy}, m, and n. The symbols m and n appearing in equation (71) represent the integer number of half waves in which the panel buckles in the x and y directions, respectively.

2. For plastic range ($F_{sx}^2 - F_{sx}F_{sy} + F_{sy}^2 \geq F_{pr}^2$):

$$\left(\frac{F_{sx}}{F_{sxp}}\right)^2 + \left(\frac{F_{sy}}{F_{syp}}\right)^2 = 1 \tag{75}$$

where F_{sxp} and F_{syp} can be obtained from the procedure outlined in the previous section ("Uniaxial compression") for the plastic case, namely equation (66).

Uniform lateral pressure. *Elastic buckling serviceability limit state (Mansour 1986).* To avoid elastic buckling of the gross panel under uniform lateral pressure, the bending stress in the x and y direction should not exceed the values given by the following expression:

$$F_{sx} \text{ or } F_{sy} \leq \gamma_s F_y \tag{76}$$

where γ_s = safety factor associated with the serviceability limit of the bending stress, and the bending stress in the x and y direction, F_{sx} and F_{sy}, respectively, are determined as follows (Mansour 1976):

$$F_{sx} = \frac{(M_x S_y) y_a}{I_x} \tag{77}$$

and

$$F_{sy} = \frac{(M_y S_x) y_b}{I_y} \tag{78}$$

where y_a = distance from the neutral axis of the longitudinal stiffener to the outer fiber of the flange or to the plate, y_b = distance from the neutral axis of the transverse stiffener to the outer fiber of the flange or to the plate, M_x = bending moment in the transverse direction, and M_y = bending moment in the longitudinal direction, which are determined as follows:

1. Stiffener's free flange:

$$M_x = a_o \sqrt{\frac{D_x}{D_y}}\, p L_b^{\,2} \tag{79}$$

$$M_y = \beta_o p L_b^{\,2} \tag{80}$$

2. Plate field:

$$M_x = a_1 \sqrt{\frac{D_x}{D_y}}\, p L_b^{\,2} \tag{81}$$

$$M_y = \beta_1 p L_b^{\,2} \tag{82}$$

where p = lateral pressure on the panel, α_o and β_o = parameters given by Mansour (1976) based on the virtual aspect ratio, ρ, and α_1 and β_1 are determined as follows:

$$\alpha_1 = \alpha_o + 0.3 \sqrt{\frac{D_x}{D_y}}\, \beta_o \tag{83}$$

and

$$\beta_1 = \beta_o + 0.3 \sqrt{\frac{D_y}{D_x}}\, \alpha_o \tag{84}$$

According to Mansour (1976), the deflection at the panel center is given by

$$w = \delta_o \frac{p L_b^{\,2}}{D_y} \tag{85}$$

where δ_o = nondimensional parameter as given by Mansour (1976).

Ultimate strength (Mansour 1976). Excessive deformation of a stiffened panel (or gross panel) under uniform lateral pressure can be a limiting factor in some design situations. A procedure is presented herein to allow expressing the requirement to avoid excessive plastic deformation as a limit state function.

The strength is the stiffness of the gross panel under lateral pressure. An expression for ultimate strength can be given by

$$F_u = \frac{P_c(n_r + 1)}{a} \quad (86)$$

where P_c = parameter representing stiffness, and it can be computed according to the following two cases:

1. For n_s even:

$$P_c = \frac{\delta\,(n_s + 1)^2}{n_s\,(n_s + 2)\,B^2} M_t + \frac{(n_s + 1)}{B} R_c \quad (87)$$

2. For n_s odd:

$$P_c = \frac{\delta}{B^2} M_c + \frac{(n_s + 1)}{B} R_c \quad (88)$$

where δ = length of the transverse stiffener. The value for R_c is found according to:

1. For n_r even:

$$R_c = \frac{\delta\,(n_r + 1)}{n_r\,(n_r + 2)a} M_t \quad (89)$$

2. For n_r odd:

$$R_c = \frac{\delta}{(n_r + 1)a} M_t \quad (90)$$

where M_t = plastic moment of transverse stiffener at center, M_l = plastic moment of longitudinal stiffener at center.

Recommended design criteria for gross panels

The collapse of a gross stiffened panel can be prevented by choosing the size of the transverse stiffeners so that they provide sufficient flexural rigidity to enforce nodes at the location of the transverse stiffeners. The collapse of the stiffened panel then is controlled by the strength of the longitudinally stiffened subpanel.

The design philosophy in this context is to design the gross panel so that its overall failure will be prevented and reduced to the level of the failure of the longitudinally stiffened subpanel. In the following section, one failure mode will be checked to fulfill the goal of this design approach.

Minimum transverse rigidity to prevent gross panel buckling (Hughes 1988). This limit state given herein is used to solve for the minimum required flexural rigidity of the transverse stiffeners. A minimum required moment of inertia and section modulus could be found based on this limit state. As long as the moment of inertia and section modulus of the transverse stiffeners are larger than the prescribed value, the stiffened (or gross) panel failure will be controlled by the strength of the longitudinally stiffened subpanel. According to Mansour et al (1996), the limit state g for minimum transverse rigidity to prevent gross panel buckling is given by

$$g = \text{Strength} - \text{Load Effect} \quad (91)$$

or

$$g = \frac{I_y a}{I_x b} - \frac{{L_b}^4}{\pi^2 C a^4}\left(1 + \frac{1}{N}\right) \quad (92)$$

where a = the length or span of the panel between transverse webs, L_b = breadth of the panel, b = distance between longitudinal stiffeners, C = panel stiffness parameter, D = plate flexural rigidity = $Et^3/12(1-\nu^2)$, E = Young's modulus, I_x,I_y = the moment of inertia of the plate-stiffener combination, longitudinal and transverse, N = number of longitudinal subpanels in overall (or gross) panel, γ_x, γ_y = flexural rigidity of the longitudinal and transverse stiffeners, respectively.

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