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POSTTENSIONED TRUSSES: ANALYSIS AND DESIGN

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ABSTRACT: More than 80% of the steel truss bridges inventoried in the United States are structurally deficient and/or functionally obsolete. Posttensioning these bridges using different posttensioned tendon layouts can be a cost-effective method to strengthen them to meet current and future loading and traffic requirements. A method for the structural stiffness analysis of posttensioned trusses is suggested. The stiffness matrices of straight, one-drape, and two-drape tendon layouts are developed. The tendon layout need not coincide with the truss members. However, it can be externally or internally attached to the truss. A closed-form solution for the relationship between the cross-sectional area, posttensioning force of the tendon, and the desired final member stress after posttensioning is derived for a statically determinate truss. Posttensioning enlarges the elastic range, increases the fatigue resistance, increases redundancy, and reduces deflection and member stresses. Thus, the remaining life of a truss bridge can be increased relatively inexpensively.

INTRODUCTION

According to the 1986 Federal Highway Administration (FHWA) statistics, there are 574,729 bridges on the highway system in the United States. Out of this inventory, there are 24,730 steel truss bridges. There are 11,013 steel truss bridges on the interstate, major, and minor highway systems inventoried by the FHWA.

According to these statistics, more than 80% of the truss bridges on the nation's highways are structurally deficient and/or functionally obsolete. The cost of total rehabilitation and replacement of these bridges is about $2.5 billion. Therefore, it is necessary to find simple and cost-effective methods to strengthen truss bridges to meet current and future loading and traffic requirements.

The FHWA recommends that the states, in developing bridge projects, consider the rehabilitation alternative before deciding to replace a structure. An innovative rehabilitation method is posttensioning, which can be used for retrofitting existing truss bridges as well as designing new ones. Posttensioning tendon layouts and force magnitudes should be determined to meet allowable and ultimate strength requirements for all truss members and tendons. Posttensioning truss bridges is a means of strengthening and creating redundancy, i.e., alternate load paths, in the structural system. This study demonstrates the potential of posttensioning in enlarging the elastic range and reducing the member forces. The method is expected to increase the fatigue resistance, reduce deflection, and increase redundancy.

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Previous Work

Little analytical and experimental work on posttensioned trusses is available in reviewed literature. A book, in Chinese, by Shantong (1986) includes one chapter on posttensioned trusses. The chapter includes discussion of types of posttensioned trusses, construction of posttensioned trusses, and several posttensioning tendon layouts.

Jawerth (1959) developed and presented a method of analysis for a special type of cable truss. Schleyer (1966) developed a method of analysis for a cable truss with vertical tension members connecting two main cables curved in opposite directions. The cable truss was, in both cases, treated as a continuous system.

In papers by Poskitt (1967) and Krishna (1968), the authors treated the cable truss as a discrete system. With a method of this type, the equations of equilibrium for all the joints of the structure were formed and the contribution from each member was considered. A full account of the nonlinearities and an iterative method were used for the solution by Mollmann (1970) and Baron (1971).

Objectives

The objective of this study is to develop a general method for the analysis and design of posttensioned plane trusses. The stiffness matrices of the tendon elements that are suggested for use for posttensioning are developed. Three types of tendon elements are considered, i.e., straight, one-drape, and two-drape tendons.

The posttensioning forces needed to strengthen the deficient members are a function of the tendon layout, tendon cross-sectional area, and truss type. In this study, member forces due to variations in these parameters are analyzed and tested for statically determinate and indeterminate trusses, and the efficiency of the tendon layouts used in posttensioning is discussed.

Stiffness of Posttensioning Tendons

The direct stiffness method, as described by Weaver and Gere (1980), is used in the development of the stiffness matrix of a posttensioning tendon based on the following three assumptions. The material of the tendon is assumed to be linearly elastic, all calculations involving the overall dimension of the truss can be based upon the original dimension of the structure, and the axial tendon force is assumed to be constant throughout the length of the tendon, i.e., friction between the tendon and its path is assumed to be negligible.

Three tendon layouts, as shown in Fig. 1, are considered in this study. They are a straight tendon, one-drape tendon, and two-drape tendon. A draped tendon can be constructed by passing the tendon over a pulley attached to a truss joint, where the tendon needs to change its angle, as shown in Fig. 1. The friction between the pulley and the tendon passing over it is assumed to be negligible, and the tendon is in tension only.

Stiffness Matrix of Straight Tendon

The stiffness matrix of a straight tendon is the same as the stiffness matrix of a plane truss member, which consists of two degrees of freedom in the

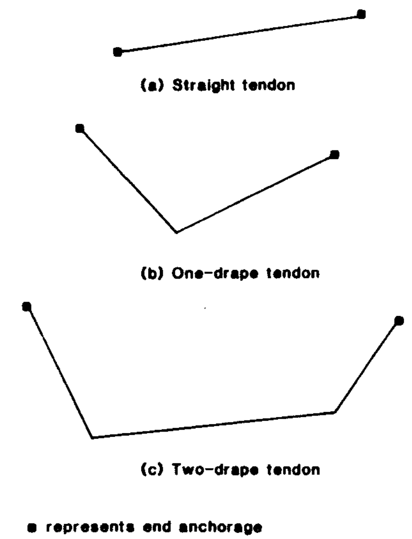


FIG. 1. Posttensioning Tendon Layouts

local degrees of freedom (DOF), and four degrees of freedom in the global (DOF), as shown in Fig. 2.

The stiffness matrix S_G of a straight tendon in the global (DOF) is as follows:

$$S_G = \frac{EA}{L}\begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi & -\cos^2\phi & -\cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi & -\cos\phi\sin\phi & -\sin^2\phi \\ -\cos^2\phi & -\cos\phi\sin\phi & \cos^2\phi & \cos\phi\sin\phi \\ -\cos\phi\sin\phi & -\sin^2\phi & \sin\phi\cos\phi & \sin^2\phi \end{bmatrix} \quad (1)$$

in which EA/L = the axial rigidity of the tendon; E = the modulus of elas-

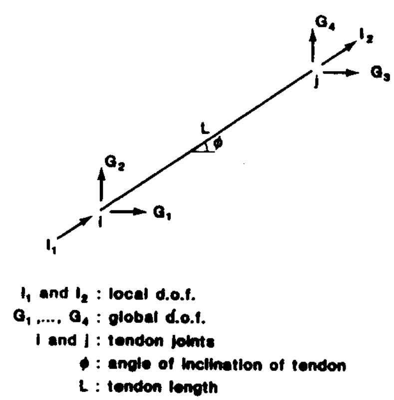
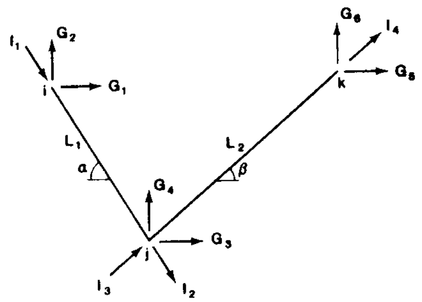


FIG. 2. Straight Tendon Layout



$l_1, \ldots, l_4$: local d.o.f.
$G_1, \ldots, G_6$: global d.o.f.
i, j, k : tendon joints
α, β : tendon angles
$L = L_1 + L_2$: tendon length

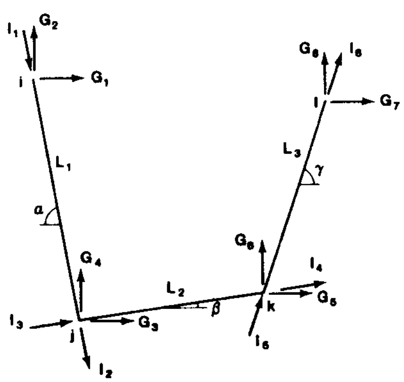
FIG. 3. One-Drape Tendon Layout

ticity of the tendon; A = the cross-sectional area of the tendon; L = its length; and ϕ = the tendon angle with respect to the global X axis.

Stiffness Matrices of One-Drape and Two-Drape Tendons

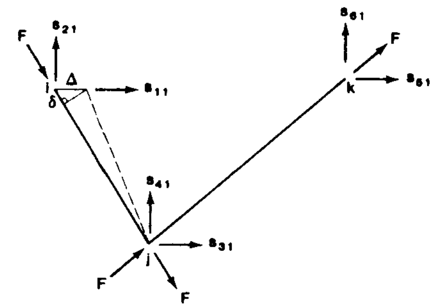
The one-drape tendon is assumed to be anchored at joints i and k, and to pass over a pulley at joint j, as shown in Fig. 3. The two-drape tendon is assumed to be anchored at joints i and l, and to pass over pulleys at joints j and k, as shown in Fig. 4.

The global stiffness matrix $\mathbf{S_G}$ of a tendon is obtained by imposing unit displacements corresponding to the global DOF G_1, G_2, ..., and G_n on the restrained structure where n ranges from 1–6 and from 1–8, as shown in Figs. 3 and 4 for the one-drape tendon and two-drape tendon, respectively. The actions corresponding to the joint displacements are the joint stiffness coefficients S_{n1}, S_{n2}, ..., and S_{nn}. Each joint is moved a unit displacement in the positive global degree of freedom, while all other joint displacements are fixed. As a result, the tendon is lengthened or shortened. The tendon displacement δ is the projected length of the unit displacement along the tendon and the restraining force F is calculated at each joint.



$l_1, \ldots, l_6$: local d.o.f.
$G_1, \ldots, G_8$: global d.o.f.
i, j, k, l : tendon joints
α, β, γ : tendon angles
$L = L_1 + L_2 + L_3$: tendon length

FIG. 4. Two-Drape Tendon Layout



$s_{11}, \ldots, s_{61}$: stiffness coefficients
F : axial tendon force
Δ : unite displacement
δ : axial tendon displacement

FIG. 5. Stiffness Coefficients of One-Drape Tendon due to Unit Displacement in (DOF) G_1

For example, introducing a unit displacement Δ in the global DOF G_1 of the one-drape tendon, the elongation of the tendon is determined from the displacement occurring at joint i, as shown in Fig. 5. From the triangle in Fig. 5, the elongation δ of the tendon is given by

$$\delta = \cos\alpha \qquad (2)$$

Therefore, the axial force in the tendon F, which is assumed to be constant throughout the length of the tendon, is given by

$$F = \frac{EA}{L}\cos\alpha \qquad (3)$$

Due to the unit displacement in the global DOF G_1, the components of this axial force in the global DOF are the global tendon stiffnesses. Therefore, the global stiffness matrix $\mathbf{S_G}$ of the one-drape tendon can be shown as

$$\mathbf{S_G} = \frac{EA}{L}\begin{bmatrix} C^2\alpha & -C\alpha S\alpha & C\alpha(C\beta - C\alpha) & C\alpha(S\alpha + S\beta) & -C\alpha C\beta & -C\alpha S\beta \\ & S^2\alpha & S\alpha(C\alpha - C\beta) & -S\alpha(S\alpha + S\beta) & S\alpha C\beta & S\alpha S\beta \\ & & (C\alpha - C\beta)^2 & (S\alpha + S\beta)(C\beta - C\alpha) & C\beta(C\alpha - C\beta) & S\beta(C\alpha - C\beta) \\ & & & (S\alpha + S\beta)^2 & -C\beta(S\beta + S\alpha) & -S\beta(S\alpha + S\beta) \\ \text{symmetric} & & & & C^2\beta & C\beta S\beta \\ & & & & & S^2\beta \end{bmatrix} \qquad (4)$$

where C = cosine of an angle and S = sine of an angle.

Similarly to the derivation of the one-drape tendon, the global stiffness of the two-drape tendon can be found. The stiffness matrix of the two-drape tendon in the global DOF can be shown as

$$\mathbf{S}_G = \frac{EA}{L}\begin{bmatrix} c_1^2 & -c_1s_1 & c_1(c_2-c_1) & c_1(s_1+s_2) & c_1(c_3-c_2) & c_1(s_3-s_2) & -c_1c_3 & -c_1s_3 \\ & s_1^2 & s_1(c_1-c_2) & -s_1(s_1+s_2) & s_1(c_2-c_3) & s_1(s_2-s_3) & s_1c_3 & s_1s_3 \\ & & (c_1-c_2)^2 & (s_1+s_2)(c_2-c_1) & (c_1-c_2)(c_2-c_3) & (c_2-c_1)(s_3-s_2) & c_3(c_1-c_2) & s_3(c_1-c_2) \\ & & & (s_1+s_2)^2 & (s_1+s_2)(c_3-c_2) & (s_1+s_2)(s_3-s_2) & -c_3(s_1+s_2) & -s_3(s_1+s_2) \\ & & & & (c_2-c_3)^2 & (c_2-c_3)(s_2-s_3) & c_3(c_2-c_3) & s_3(c_2-c_3) \\ & & & & & (s_2-s_3)^2 & c_3(s_2-s_3) & s_3(s_2-s_3) \\ \text{symmetric} & & & & & & c_3^2 & c_3s_3 \\ & & & & & & & s_3^2 \end{bmatrix} \quad (5)$$

where $c_1 = \cos\alpha$; $c_2 = \cos\beta$; $c_3 = \cos\gamma$; $s_1 = \sin\alpha$; $s_2 = \sin\beta$; and $s_3 = \sin\gamma$.

Methodology of Analysis and Design of Posttensioned Trusses

In this paper, the analysis of posttensioned trusses is divided into three stages. In the first stage, an analysis is performed using the dead load. The second stage of analysis is performed using the dead and the posttensioning loads as applied to the truss joints. In the third stage, an analysis is performed using the live, impact, and any other loads. The stiffness of the tendons is considered only in the third analysis stage. The final solution is achieved by superimposing the solutions of the second and third stages.

The derivation of the stiffness matrices of the tendons, as discussed in the previous section, is based on the direct-stiffness method. Every tendon layout is treated as a separate member like any other truss member. The tendon force is assumed to be constant throughout the length of the tendon, regardless of whether the tendon is straight or draped. A tendon layout need not coincide with truss members. Tendon ends are to be anchored to truss joints, and in the case of a draped tendon, where a pulley is used, the pulley must be attached to a truss joint.

The effect of posttensioning on truss bridges is a function of the truss type, tendon layout, and magnitude of the posttensioning force. For a statically determinate truss, if the tendon layout coincides with one or more truss members, then these members are the only ones affected by posttensioning; all other members are unaffected. On the other hand, for a statically indeterminate truss, no matter how the tendons are arranged, a group of redundant members is affected by posttensioning if the tendon passes within that group. The effect of the posttensioning force on the members is dependent on the truss type, connectivity of the members, and tendon layout within the group of members.

A closed-form solution for the relationship between the cross-sectional area, the posttensioning force of the tendon, and the desired final member stress, after posttensioning, is derived for a statically determinate truss. The final truss member stress f_m is given by

$$f_m = \frac{T_D}{A_m} - f_{ci}\frac{A_c}{A_m} + \frac{T_{L+I}}{A_m + A_c} \quad (6)$$

where T_D = the truss member force due to dead load; A_m = the cross-sectional area of the truss member; f_{ci} = the applied posttension stress in the tendon; A_c = the cross-sectional area of the tendon; and T_{L+I} = the truss member force due to live and impact loads. The final tendon stress is

$$f_c = f_{ci} + \frac{T_{L+I}}{A_m + A_c} \le f_t \quad (7)$$

where f_t = the allowable tendon stress. Eq. 7 can be rewritten as an inequality in the following form

$$f_{ci} \le f_t - \frac{T_{L+I}}{A_m + A_c} \quad (8)$$

In Eqs. 6 and 8, two design parameters are identified, which are A_c and f_{ci}. By solving Eqs. 6 and 8, the required cross-sectional area of the tendon is

$$A_c \ge \frac{T_D + T_{L+I} - f_m A_m}{f_t} \quad (9)$$

Alternatively, Eqs. 6 and 8 can be solved to determine the required posttensioning stress for the tendon as follows

$$f_{ci} \le f_t\left[\frac{T_D + A_m(f_t - f_m)}{T_D + T_{L+I} + A_m(f_t - f_m)}\right] \quad (10)$$

Since the posttensioning force of the tendon can be calculated as

$$P_F = f_{ci}A_c \quad (11)$$

for A_c and f_{ci} that satisfy Eqs. 9 and 10, respectively, the required posttensioning force is given by

$$P_F = (T_D + T_{L+I} - f_m A_m)\left[\frac{T_D + A_m(f_t - f_m)}{T_D + T_{L+I} + A_m(f_t - f_m)}\right] \quad (12)$$

For statically indeterminate trusses, the stiffness analysis can be based on the three-stage solution, as discussed earlier. However, the design, which involves the selection of the magnitude of the posttensioning force for a specified tendon profile, requires an iterative trial-and-error solution. Eqs. 9 and 12 can be used as a guide to start the iterative solution scheme.

Other design considerations requiring special attention include posttensioning losses, detailing end anchorages, pulleys for draped tendons, buckling of compression elements, members' stress levels before and after posttensioning, initial and final fatigue conditions, corrosion, and construction feasibility.

Posttensioning losses include tendon relaxation, structural steel creep, and anchorage set. Creep of structural steel is relatively small and therefore can be neglected. Losses due to tendon relaxation and anchorage set can be determined with the currently used method in posttensioned concrete elements. End anchorages for posttensioned trusses can be of the same type as those used in posttensioned concrete elements. These end anchorages have been successfully used to prestress steel girders in real bridges, and by several researchers for bridge testing. The use of pulleys at the joints of gusset-plated trusses for draped-tendon layouts can be relatively difficult in construction and requires additional investigation.

The effect of the sequence of posttensioning on the stress level and the instability of all truss members need to be evaluated and checked. Adequate safety against yielding of tension and compression members, and buckling of compression members at the end of each posttensioning stage should be provided. The posttensioning tendons in trusses can be run along the shielded surface of the truss elements, wherever possible, for damage and corrosion protection purposes. Epoxy-coated tendons can be used for additional corrosion protection.

Two types of posttensioning tendons are considered, i.e., internal and external tendon profiles. Internal tendon profiles can have either straight or draped layout. For straight internal tendons, the tendon profile is constructed by connecting, then posttensioning any two joints of a truss. Therefore, the tendon layout is enclosed within the truss system. For draped internal tendons, the tendon profile connects two truss joints and passes over one or more other joints. However, the tendon layout is contained within the truss system. On the other hand, external tendons can only be draped. An external tendon profile connects two truss joints and passes over one or more new additional joints. These new joints are commonly provided below the bottom chord of the truss by utilizing additional members, which need to be attached to existing truss joints at the bottom chord. Therefore, this type of profile falls below and outside the truss system, i.e., external to the truss. Examples of both types of profile are provided in the next sections. By posttensioning with an internal tendon layout, the forces in tension members can be alleviated, while the magnitudes of the forces in compression members do not change, or slightly increase. Posttensioning with an external tendon layout is most effective in upgrading the tension and the compression truss members; however, it has a limited use where highway or river clearances are geometric considerations.

Other considerations in the design of posttensioned truss bridges that are closely related to posttensioning include corrosion protection of the tendons, tendon anchorages, and the effect of posttensioning on the fatigue strength of the truss. The corrosion protection of the tendons can easily be achieved by using epoxy-coated cables or high-strength bars. Epoxy-coated tendons are commercially available and used by other industries. The end anchorages for straight tendons should be kept simple. They can be constructed using plates attached to the ends of the members at the edges of the truss joints. Due to the relative flexural flexibility of the posttensioning tendons, the posttensioning force is carried along the centroidal axis of the tendons. Therefore, there is no need to provide a pulley at the ends of the tendons. A pulley is needed at the joint where the tendon layout is draped, i.e., angular change in layout. This might result in relatively complex structural details at this joint. The writers are currently studying end anchorages for posttensioned bridges at the University of Maryland.

Examples

In this section, three examples are discussed. In the first two, statically determinate and indeterminate trusses are internally posttensioned. In the third, a statically determinate truss is externally posttensioned. Different cable lay-

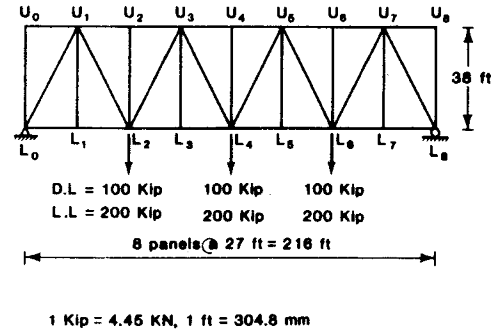


FIG. 6. Statically Determinate Symmetrical Truss One Configuration

outs are used in posttensioning the trusses, and the effect of the cable layout, as well as truss type, on the member forces is discussed.

Example 1

A statically determinate symmetrical truss called truss one, as shown in Fig. 6, is analyzed with three different internal cable layouts. Figs. 7(*a–c*) show the truss with a straight cable connected between nodes L_0 and L_8, the truss with a one-drape cable running from node U_0 through node L_4 to node U_8, and the truss with a two-drape cable starting from node U_1, passing through nodes L_2 and L_6, and ending at node U_7, respectively.

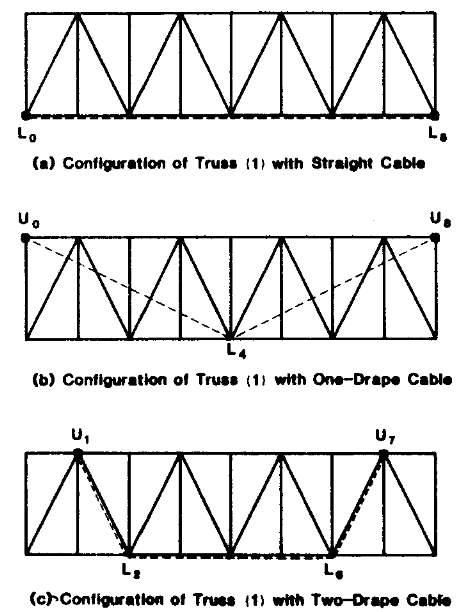


FIG. 7. Posttensioning of Truss One Using Three Internal Cable Layouts

In all the cases of cable layouts in Fig. 7, the cables consist of three strands of 15.24 mm (0.6 in.) diameter, and an ultimate tensile strength of 1,860 MPa (270 ksi). The cross-sectional area of the cables is 548 mm^2 (0.85 sq in.). The cables are initially posttensioned by a force equal to 916 kN (206 kips), such that the initial cable posttensioning stress is equal to 1,674 MPa (243 ksi). This stress corresponds to 85% of the ultimate strength of the strands. The effect of the posttensioning on the truss member forces using these cable layouts is summarized in Table 1. The second column of the table gives the member forces using the dead, live, and impact loads, and before posttensioning. The third, fourth, and fifth columns show the final truss member forces after posttensioning, using the cable layouts shown in Fig. 7, respectively.

From the results shown in Table 1, a great force reduction in only the tension members can be achieved by using internal posttensioning cables. If the cable coincides with the truss members, then those are the only members affected by the posttensioning force, as shown in columns three and five of Table 1. If the cable does not coincide with the truss members as shown in Fig. 7(*b*), then most of the truss members are affected by posttensioning, i.e., column four of Table 1. Using the internal cable layout, which does not coincide with the truss members, is not very effective compared to the

TABLE 1. Member Forces of Statically Determinate Symmetrical Truss One before and after Posttensioning Using Internal Cable Layouts

Truss member (1)	TENSION (COMPRESSION) Force due to dead load + live load + impact load (kips) (2)	Force due to Dead Load + Live Load + Impact Load + Posttensioned Load (kips) Straight cable (3)	One-draped cable (4)	Two-draped cable (5)
$L_0 L_1$	320	106	270	320
$L_1 L_2$	320	106	270	320
$L_2 L_3$	746	533	597	529
$L_3 L_4$	746	533	597	529
$U_0 U_1$	(0)	(0)	(200)	(0)
$U_1 U_2$	(640)	(640)	(740)	(640)
$U_2 U_3$	(640)	(640)	(740)	(640)
$U_3 U_4$	(853)	(853)	(853)	(853)
$L_0 U_0$	(0)	(0)	(70)	(0)
$L_0 U_1$	(552)	(552)	(466)	(552)
$L_1 U_1$	(0)	(0)	(0)	(0)
$L_2 U_1$	552	552	466	380
$L_2 U_2$	(0)	(0)	(0)	(0)
$L_2 U_3$	(184)	(184)	(98)	(184)
$L_3 U_3$	(0)	(0)	(0)	(0)
$L_4 U_3$	184	184	98	184
$L_4 U_4$	(0)	(0)	(0)	(0)
Cable	—	214	212	216

Note: 1 kip = 4.45 kN.

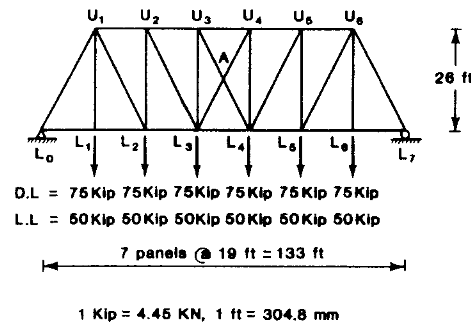


FIG. 8. Statically Indeterminate Symmetrical Truss Two Configuration

cable that coincides with the truss members; especially where the forces in the compression members are increased due to posttensioning.

Example 2

A statically indeterminate symmetrical truss, called truss two, as shown in Fig. 8, is analyzed using three different internal cable layouts. Fig. 9(*a*) shows the case where two straight cables and a two-drape cable are used in posttensioning truss two. The first straight cable is connected between nodes L_0 and L_2. The second straight cable is connected between nodes L_5 and L_7. The two-drape cable is anchored to the truss at nodes U_1 and U_6, passing over two pulleys attached to nodes L_2 and L_5.

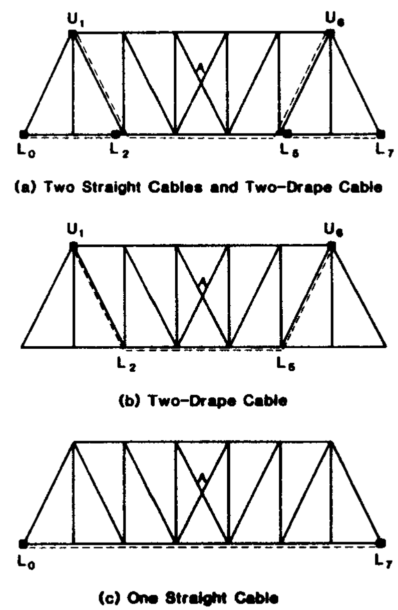


FIG. 9. Posttensioning of Truss Two Using Combinations of Internal Cable Layouts

TABLE 2. Member Forces of Statically Indeterminate Symmetrical Truss Two before and after Posttensioning Using Internal Cable Layouts

Truss member (1)	TENSION (COMPRESSION)			
	Force due to dead load + live load + impact load (kips) (2)	Force due to Dead Load + Live Load + Impact Load + Posttensioned Load (kips)		
		Fig. 10(*a*) (3)	Fig. 10(*b*) (4)	Fig. 10(*c*) (5)
(*a*) Truss member				
$L_0 L_1$	274	136	274	135
$L_1 L_2$	274	136	274	135
$L_2 L_3$	456	318	318	318
$L_3 L_4$	546	410	410	411
$U_1 U_2$	(456)	(456)	(456)	(456)
$U_2 U_3$	(548)	(548)	(548)	(548)
$U_3 U_4$	(550)	(547)	(547)	(547)
$L_0 U_1$	(464)	(464)	(464)	(464)
$L_1 U_1$	125	125	125	125
$L_2 U_1$	310	171	171	310
$L_2 U_2$	(125)	(125)	(125)	(125)
$L_3 U_2$	155	155	155	155
$L_3 U_3$	(3.22)	1.58	1.55	1.6
$L_3 A$	4	(1.96)	(1.96)	(1.95)
$U_3 A$	4	(1.96)	(1.96)	(1.95)
(*b*) Cables				
$L_0 L_2$	—	138	—	—
$U_0 L_2 L_5 U_6$	—	139	139	—
$L_0 L_7$	—	—	—	138

Note: 1 kip = 4.45 kN.

Fig. 9(*b*) shows truss two with a two-drape cable. The cable is anchored at nodes U_1 and U_6, passing over nodes L_2 and L_5 as in case (*a*) of Fig. 9. Fig. 9(*c*) shows truss two with one straight cable used to posttension the entire bottom chord of the truss. The straight cable is anchored at nodes L_0 and L_7.

In all the cases in Fig. 9, the cables consist of three strands of 12.7 mm (0.5 in.) diameter, and an ultimate tensile strength of 1,860 MPa (270 ksi). The cross-sectional area of each cable is 380 mm^2 (0.59 sq in.). The cables are initially posttensioned by a force equal to 600 kN (135 kips), such that the initial cable posttension stress is 1,584.7 MPa (230 ksi). This stress corresponds to 85% of the ultimate strength of the strands.

The effect of posttensioning on truss two member forces, using the cases in Fig. 9, is summarized in Table 2. The second column of Table 2 shows truss two member forces using only the dead, live, and impact loads. The third, fourth, and fifth columns show the truss member forces using the dead, live, and impact loads, and posttensioning force, according to the cable layouts shown in Fig. 9, respectively.

The results in Table 2 show that a great force reduction in only the tension members can be achieved by using internal posttensioning cable layouts. All

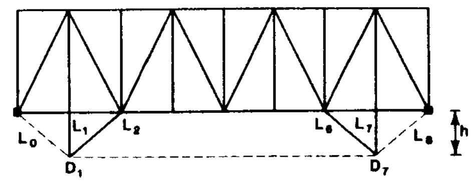


FIG. 10. Statically Determinate Symmetrical Truss One Posttensioned by External Two-Drape Cable Layout

the redundant members in the panel through which the cable passes are affected by posttensioning. For example, in the fourth panel, the vertical member (L_3-U_3) has a compression force, and the diagonal members (U_3-A and L_3-A) have a tension force in the original truss due to the dead, live, and impact loads. After posttensioning, the vertical member goes into tension and the diagonal members go into compression. This reversal of stresses must be accounted for in design.

Example 3

Fig. 10 shows an external two-drape cable connected to truss one, which is shown in Fig. 6. The cable is anchored at nodes L_0 and L_8, and passes over two pulleys at nodes D_1 and D_7 by the addition of two vertical members, (L_1,D_1) and (L_7,D_7), to truss one. The additional two diagonal members,

TABLE 3. Member Forces of Statically Determinate Symmetrical Truss One due to Posttensioning Using External Two-Drape Cable Layouts

Truss member (1)	TENSION (COMPRESSION)			
	Force due to dead load + live load + impact load (kips) (2)	Force due to Dead Load + Live Load + Impact Load + Posttensioned (kips)		
		For *h* = 10 ft (3)	For *h* = 20 ft (4)	For *h* = 30 ft (5)
$L_0 L_1$	320	192	184	185
$L_1 L_2$	320	192	184	185
$L_2 L_3$	746	610	577	545
$L_3 L_4$	746	610	577	545
$U_0 U_1$	(0)	(0)	(0)	(0)
$U_1 U_2$	(640)	(611)	(581)	(551)
$U_2 U_3$	(640)	(611)	(581)	(551)
$U_3 U_4$	(853)	(824)	(794)	(764)
$L_0 U_0$	(0)	(0)	(0)	(0)
$L_0 U_1$	(552)	(505)	(471)	(450)
$L_1 U_1$	(0)	(34)	(49)	(41)
$L_2 U_1$	552	549	532	501
$L_2 U_2$	(0)	(0)	(0)	(0)
$L_2 U_3$	(184)	(184)	(184)	(184)
$L_3 U_3$	(0)	(0)	(0)	(0)
$L_4 U_3$	184	184	184	184
$L_4 U_4$	(0)	(0)	(0)	(0)

Note: 1 kip = 4.45 kN, and 1 ft = 304.8 mm.

(D_1, L_2) and (D_7, L_6), are established to accomplish the stability of the truss. The cable consists of two strands of 15 mm (0.6 in.) diameter, and an ultimate tensile strength of 1,722.5 MPa (250 ksi). The cross-sectional area of the cable is 361 mm^2 (0.56 sq in.). The cable is initially posttensioned by a force equal to 445 kN (100 kips), such that the initial cable posttensioning stress is equal to 1,226.4 MPa (178 ksi). This stress corresponds to 70% of the ultimate strength of the strands. The effect of posttensioning on the truss member forces is summarized in Table 3. The second column of the table gives the member forces before posttensioning using the dead, live, and impact loads. The third, fourth, and fifth columns show the final truss member forces after posttensioning using the cable layout in Fig. 10, where the height h of the additional vertical members is varied as 10, 20, and 30 ft (3.05, 6.10, and 9.14 m), respectively.

The results from Table 3 show that compression members can be strengthened, as well as the tension members, by posttensioning the truss using external cable layouts. The effect of posttensioning the truss on the member forces using the external cable depends on many factors, such as truss type, cable layout, cable cross-sectional area, and posttensioning force, as in the case of internal cables. In addition to these factors, the position of the cable under the lower chord of the truss as well as the length of the truss panel have major effect. In general, as the position of the cable measured in terms of the height h is increased or the length of the truss panel is decreased, a larger reduction in member forces can be achieved.

Summary and Conclusions

Posttensioning deficient truss bridges using different tendon layouts is presented as a means of strengthening these bridges to meet current and future loading and traffic requirements. This study theoretically demonstrates the potential of posttensioning the deficient truss bridges using different tendon layouts, i.e., straight tendon, one-drape tendon, and two-drape tendon, attached internally or externally to the truss bridge. The stiffness matrices for these tendon layouts have been developed. Every tendon layout is treated as a separate member, like any other truss member. The axial tendon force is assumed to be constant throughout its length, and the tendon can be in tension only. Tendon ends are anchored to truss joints, and in the case of a draped tendon where a pulley is used, the pulley is attached to a truss joint.

The analysis of posttensioned trusses is divided into three stages. In the first and second stages, the analysis is performed using the dead load alone, and the dead load in addition to the posttensioning load, respectively; without considering the stiffness of the tendons. The third analysis stage is based on the live, impact, and any other loads considering the stiffness matrices of the tendons to be in effect. The final solution is achieved by superimposing the solutions of the third stage to the second stage.

The posttensioning forces designed to strengthen the deficient members are a function of tendon layouts, tendon cross-sectional areas, and truss types. Member forces due to variations in these parameters were determined for statically determinate and indeterminate trusses using internal and external tendons. A closed-form solution for the relationship between the cross-sectional area and posttensioning force of the tendons, and the desired final member stress after posttensioning is derived for a statically determinate truss.

For statically indeterminate trusses, a non-closed-form solution for the same relationship is described.

Posttensioning statically determinate trusses using internal posttensioned tendons results in a great force reduction in only the tension members. If the tendon coincides with a group of truss members, then these members are the only members affected by the posttensioning force. If the tendon does not coincide with the truss members, then most of the truss members are affected by posttensioning. Using the internal tendon layout, which does not coincide with the truss members, is not very effective compared with the tendon that coincides with the members; especially where the forces in the compression members are increased due to posttensioning. Posttensioning statically indeterminate trusses using internal posttensioning tendon layouts results in a great force reduction in the tension members only. Compression as well as tension members can be strengthened by posttensioning a truss using external tendon layouts.

The effect of posttensioning on the forces of truss members depends on many factors. For internal tendon layouts, the truss type, tendon layout, tendon cross-sectional area, and posttensioning force are the main factors affecting member forces. For external tendon layouts, the position of the tendon under the lower chord of the truss, the length of the truss panel, as well as all previous factors, have a great effect on member forces.

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