Reliability Assessment of Pile Groups in Sands

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Abstract: A probability-based reliability assessment methodology for single flexible piles and pile groups under stochastic lateral loads is developed. The methodology was based on state-of-the-art techniques for the analysis of single piles and pile groups as well as reliability assessment methods. Critical strength and serviceability modes of failure for flexible piles in sandy soil under lateral loads were defined. The reliability of a pile-group system was assessed by accounting for system redundancy with the occurrence of partial failures of the system components. Reliability indices and failure probabilities were used as relative measures for the performance of piles. A case study was presented to illustrate the proposed methodology.

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Introduction

According to the U.S. Army Corps of Engineers' statistics, over 40% of the inland navigation structures are more than 50 years old, and the demand for rehabilitation must be selectively invested to maximize navigation benefits. Evaluation and assessment of existing structures can play a significant role in reducing the likelihood of unexpected failures. However, current evaluation and assessment methods of pile foundation components and pile groups, that are required to maintain system integrity during normal and sever operational conditions, are based on the use of factors of safety or safety margins. For single piles, lateral loading is a problem of soil-structure interaction, in which pile deflection depends on the soil response and soil response depends on pile deflection. For closely spaced pile groups, this behavior is more complicated than single piles due to the following two considerations: (1) the decrease of group efficiency due to close pile spacing; and (2) the distribution of the load from the superstructure to each of the supporting piles in the group. The second of these problems can be solved rationally, if the three nonlinear stiffness coefficients at pile heads for axial loads, lateral loads, and moments can be defined. However, the distribution of the loads from the superstructure to different piles in the group is as accurate as the pile-head stiffness coefficients are determined. Despite the significance of closely spaced pile interaction, there is a lack of knowledge concerning pile-group effect. Full-scale lateral load group tests are few due to their complexity and associated costs. Most lateral load investigations were conducted on isolated single piles, even though piles are most frequently used in groups (Eloseily 1998).

A reliability-based assessment procedure starts with defining performance functions that correspond to limit states for critical modes of failure. Commonly used reliability methods utilize the mean and variance (first and second moments) of basic random variables in calculating reliability measures according to specified performance functions. This reliability study is based on the first-order second-moment (FOSM) methods (Ayyub and McCuen 1997) for assessing failure probabilities according to critical failure modes of piles. FOSM can be used to calculate the failure probabilities for the performance functions.

The proposed reliability assessment methodology for pile groups is applicable to (1) long flexible piles in sandy soil; (2) piles with constant EI, where E=modulus of elasticity and I=moment of inertia of pile material; (3) boundary conditions at the pile top that consists of a static shear force Q_g, a moment M_g, and a constant axial load Q_x; (4) pile groups with three and four rows of piles with all the piles having the same EI and head constraints; (5) pile spacing of $3D$, $5D$, and $7D$; where D=the pile diameter; (6) piles fully embedded in soil; and (7) noncorrelated random variables. These assumptions were set to focus the effort on reliability assessment of pile groups subjected to lateral loads, and to keep related effects and computations tractable.

Appropriate performance functions were identified in this paper for serviceability and strength failure modes for single piles and pile groups. The deterministic limit state models that form the basis for reliability assessment are based on the nondimensional analysis of Matlock and Reese (1960) for single piles and the modified unit load method for pile groups of others.

The development of a probability-based reliability assessment methodology for single flexible piles and pile groups under lateral loads is provided in this paper. Also, the pile-group system reliability is assessed by accounting for system redundancy due to the partial failure of individual piles. The paper shows how to determine pile group probability of failure as a system that can account for partial failure of system components.

Analytical Models

The deterministic models that are used in the limit states for reliability assessment are based on the widely accepted solution of

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piles subjected to lateral loads known as the Winkler approach (1867) or subgrade reaction approach. The subgrade reaction approach utilizes a beam column on an elastic foundation with nonlinear springs to transfer the load from piles to the soil. These springs represent the total soil resistance (P) at a particular depth to the lateral displacement (Y) of a horizontally loaded pile (P-Y curve). The Winkler theory utilizes Hetenyi's beam-column theory that accounts for axial loads. However, the axial loads that act on a pile subjected to lateral loads have a small effect on the bending moment produced by the lateral loads. In almost all cases, the reduction in axial loads from the ground line to the point of maximum moment is negligible. The soil near ground surface principally determines the lateral response, and the soil at depth determines the axial response. The relationship between axial loads and displacements are not affected by the presence of lateral deflection and vice versa (Townsend et al. 1997). The problem of vertical piles subjected to lateral loads cannot be solved by static equilibrium, but can be represented by a fourth-order differential equation for the elastic deflection of a beam as follows:

$$EI\frac{d^4Y}{dx^4}+Q_x\frac{d^2Y}{dx^2}+E_sY=0 \tag{1}$$

where Y=lateral deflection; Q_x=axial load at the pile head; and E_s=soil modulus. The principle of dimensional analysis is commonly applied to physical models; however, Reese (1956) applied the dimensional analysis to mathematical models. They used the principle of dimensional analysis to produce a set of nondimensional coefficients that can be used to solve the governing differential equation. The development of the nondimensional analysis was a result of solving Eq. (1) a few times for each boundary condition using a range of values for each variable. It was found that these solutions could then be applied to many similar problems. The primary advantage of this method is that the nonlinear soil response can be taken into account through successive iterations of solving the differential equation. This paper presents the nondimensional analysis that is based on the simple model of the soil modulus E_s recommended by Terzaghi (1955) as follows:

$$E_s = n_h\, x \tag{2}$$

Eq. (1) was solved using nondimensional analysis and finite difference method for a rotation-free pile head to produce the following equations (Reese 1956):

$$Y_x=Y_A+Y_B=A_y\frac{Q_gT^3}{EI}+B_y\frac{M_gT^2}{EI} \tag{3}$$

$$M_x=M_A+M_B=A_MQ_gT+B_MM_g \tag{4}$$

$$S_x=S_A+S_B=A_s\frac{Q_gT^2}{EI}+B_s\frac{M_gT}{EI} \tag{5}$$

$$V_x=V_A+V_B=A_vQ_g+B_v\frac{M_g}{T} \tag{6}$$

$$P_x=P_A+P_B=A_p\frac{Q_g}{T}+B_p\frac{M_g}{T} \tag{7}$$

where Y_x=deflection along the pile; M_x=moment along the pile; S_x=slope along the pile; V_x=shear along the pile; P_x=soil resistance along the pile; Q_g=lateral applied load at pile head; M_g=applied moment at pile head; T= characteristic length=$(EI/n_h)^{0.2}$; E=pile modulus of elasticity; I=pile moment of inertia; and A_y, A_s, A_M, A_v, A_p, B_y, B_s, B_M, B_v, and B_p are constants that varies with Z, where $Z=x/T$, and x=depth from ground level. Reese (1956) reported that for $Z_{\max}\leqslant 2$, piles behave as rigid body (short piles); while for $Z_{\max}\geqslant 5$, piles behave as a flexible body (long piles).

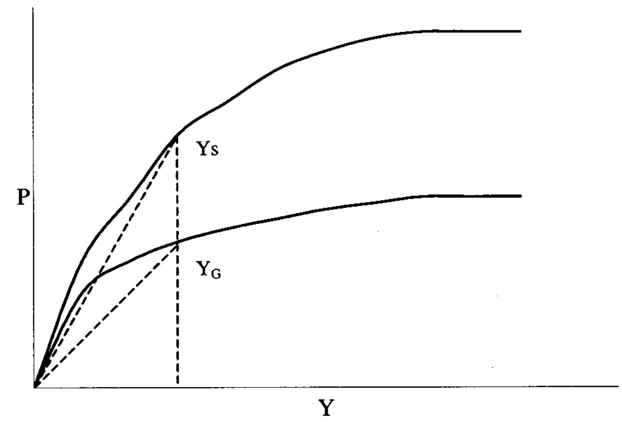

Fig. 1. P-Y curve

Modified Unit Load Transfer Method

The lateral capacity of an individual pile in a pile group is a function of its position in the group and center-to-center pile spacing. Morrison and Reese (1988) proposed a p-multiplier, P_m, to be used to modify a P-Y curve for a single pile to obtain a P-Y curve of an individual pile in the group as shown in Fig. 1. It was suggested that for piles in a given row, the P_m value could be applied to all P-Y curves along the length of the pile. They performed lateral load test on a pile group of 3×3 piles in a very dense sand. Morrison reported that the P_m is 0.8, 0.4, and 0.3 for the leading, middle, and trailing rows, respectively.

McVay et al. (1995) performed centrifuge model tests on a 3×3 pile group having center-to-center pile spacing of $3D$ and $5D$, where D=pile width. Dense and loose sand conditions were simulated in the centrifuge model tests. The centrifuge model test results were similar to Morrison's field results. However, McVay reported that the P_m is affected by position of the pile in the group, pile spacing in the group, and soil density.

Reliability Analysis

The performance function Z in reliability assessment of a structure can be defined as resistance minus the loading (Thoft-Christensen and Baker 1982). When Z is greater than zero, a safe state exits and failure is defined otherwise. Numerous failure functions exist for pile foundations because they may fail in many different ways and because of different effects. A variety of loads such as dead, live, wind, snow, or a combination of each may cause structure failure. Failure does not imply structure collapse in reliability assessment, but it can be defined as exceeding other limits, such as specified deflection values.

Several techniques exist to perform reliability assessment. These techniques include first-order, second moment methods (FOSM), and Monte Carlo simulation methods. A FOSM method can be easily programmed and has been well documented (Ayyub et al. 1996, 1997).

System Reliability

Systems that are connected in parallel are called "*parallel systems.*" The failure of such systems requires the failure of all the systems' components. Systems that are connected in a series are called "*series systems.*" The failures of one or more of the systems' components constitute to a systems failure; such systems have no redundancy and are defined by "*weakest-link*" systems.

Engineering systems are usually a mixture of parallel and series systems, the failure and survival of such systems can be represented as a combination of failure or survival events in a series (union) and/or in parallel (intersection). The calculation of the failure probabilities of combined systems using the exact solution is generally difficult due to correlation and load distribution. Therefore, approximations are necessary. Lower and upper bounds of the corresponding failure probabilities could be useful and form approximate solutions. The failure probabilities bounds for N performance modes connected in a series are

$$\max_{i=1}^{N} P_{fi} \leq P_f \leq \sum_{i=1}^{N} P_{fi} \tag{8}$$

where P_{fi}=the failure probability of the ith mode, and the lower and upper limits correspond to fully correlated and noncorrelated modes, respectively. Similarly, the failure probabilities bounds for N performance modes connected in parallel are

$$\prod_{i=1}^{N} P_{fi} \leq P_f \leq \min_{i=1}^{N} P_{fi} \tag{9}$$

Reliability Assessment Methodology for Single Piles

Three performance functions influence the response of single-piles subjected lateral loads that must be considered for reliability assessment: (1) lateral deflection performance function; (2) flexure strength performance function for the pile material; and (3) soil strength performance function.

Lateral Deflection Performance Function

The lateral deflection performance function at the pile head can be defined as follows:

$$Z_Y = Y_u - \left(A_y \frac{Q_g T^3}{EI} + B_y \frac{M_g T^2}{EI} \right) \tag{10}$$

where Y_u=maximum deflection at pile head defined by the type of structure or a serviceability limit; A_y and B_y are constants; Q_g=applied lateral loads at pile head; M_g=applied moment at pile head; $T=(EI/n_h)^{0.2}$=characteristic length; E=pile section modulus of elasticity; I=pile moment of inertia; and n_h=constant of horizontal subgrade reaction.

Moment Strength Performance Function

The moment strength performance function for pile material can be defined as follows:

$$Z_M = M_u - (A_M Q_g T + B_M M_g) \tag{11}$$

where A_M and B_M=constants; M_u=ultimate moment.

Soil Resistance Performance Function

The soil strength performance function can be defined as follows:

$$Z_p = P_u - P_x \tag{12}$$

where P_u=ultimate soil resistance and P_x=soil resistance along the pile length (e.g., lb/in.).

Reliability Assessment Methodology for Pile Groups Using Modified Nondimensional Method

The concept of using the P multiplier (P_m) for modifying the P-Y curve of a single pile to obtain a P-Y curve for a pile in a group is shown in Fig. 1. This approach is based on "squashing" the P-Y curve to account for the "shadowing effect." The overlapping of shear zones for the piles in the front row can be described by the simple wedge-type failure that might reduce the soil resistance for closely spaced pile groups. At greater depth, the flow-type failure might not result in a reduction for the soil resistance because the near-surface soil clearly dominates pile behavior under lateral loads. Overlapping shear zones for trailing row piles occur due to the "shadowing effect" as the piles in the leading rows push the soil away from active areas in providing soil resistance. Densification during pile driving may reduce the "shadowing effect." However, the soil within the upper five-to-ten pile diameters clearly dominates lateral-load response. Therefore, it is less likely for the soil near ground surface to be densified enough by vibration from pile driving to reduce the "shadowing effect."

The concept of P_m shown in Fig. 1 is based on reducing the ultimate soil resistance or the soil resistance at any point for single P-Y curve by a factor, at the same deflection, to produce the P-Y curve for any pile in the group. The P_m methodology can be modified for the purpose of obtaining the performance functions to represent the failure modes of each pile in the group as follows:

$$P_G = P_s P_m \tag{13}$$

where

$$P_s = -Y_s E_{ss} \tag{14}$$

and

$$P_G = -Y_G E_{sG} \tag{15}$$

where P_s and P_G=soil resistance for a single pile and a pile in the group expressed as force per unit length of the pile, respectively; E_{ss} and E_{sG}=soil modulus for the single pile and the pile in the group, respectively; and Y_s and Y_G=lateral deflection for the single pile and the pile in the group, respectively. The negative sign indicates that the direction of the soil resistance is opposite to the direction of the pile deflection. A typical relationship between P and Y is nonlinear as shown in Fig. 1. The linear soil modulus E_s presented by Eqs. (14) and (15) is the slope of secant modules drawn from the origin to any point along the P-Y curve. From Fig. 1, the following condition can be stated:

$$Y_G = Y_s \tag{16}$$

Therefore, from Eqs. (14) and (15)

$$\frac{P_G}{P_s} = \frac{E_{sG}}{E_{ss}} = \frac{n_{\mathrm{hG}} x}{n_{\mathrm{hs}} x} \tag{17}$$

Thus

$$P_m = \frac{P_G}{P_s} = \frac{n_{\mathrm{hG}}}{n_{\mathrm{hs}}} \tag{18}$$

The P_m=ratio of the soil resistance of any pile in the group (P_G) to the soil resistance of the single pile (P_s). The P_m=also the ratio of the constant of horizontal subgrade reaction of any pile in the group (n_{hG}) to the constant of horizontal subgrade reaction of the single pile (n_{hs}). In the proposed methodology for reliability assessment, it is recommended to reduce the value of n_{hs} using P_m to obtain n_{hG} to account for the group effect. The advantage of using n_{hG} over P_G is that n_{hG} represents the soil characteristic in

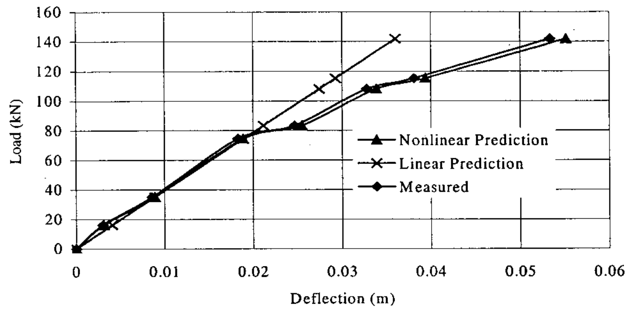

Fig. 2. Load deflection curve

all the performance functions. Eq. (18) shows that reducing n_{hs} by P_m has the same affect on the performance functions as reducing P_s by the same P_m.

In the proposed methodology for reliability assessment of pile groups, the total lateral loads applied on pile groups are assumed to be equally distributed among the piles within an individual row, but each row carries a different portion of the loads. Three curves are needed for the average pile of respective rows in the group to perform reliability assessment. These curves are (1) the load-deflection curves; (2) the maximum moment-load curves; and (3) the *P*-*Y* curves. If test results for a full-scale pile group subjected to lateral loads is available. The following procedure can be used:

1. Average the pile head load for each row at the given deflection for each load increment.
2. Obtain the constant of horizontal subgrade reaction n_h for each row in the group using the average pile head load at the given deflection as follows:

$$n_h = \frac{C_n\left(\frac{Q_g}{Y_g}\right)^{1.67} + D_n\left(\frac{M_g}{Y_g}\right)^{1.67}}{(EI)^{0.67}} \tag{19}$$

3. Determine the lateral deflection using the average load in step (1), n_h in step (2), and Eq. (3).
4. Compare the measured and predicted deflections for each load increment, using values from step (1) and step (3), respectively. Plot the measured and predicted load-deflection curves for each row in the group.
5. Obtain the average measured maximum moment along the piles in each row for each load increment.
6. Calculate the moment on each row using the average load in step (1), n_h in step (2), and Eq. (4).
7. Compare the measured and predicted moments for each load increment, using values in step (5) and step (6), respectively. Plot the measured and predicted maximum moment-load curves for each row in the group.
8. Determine the soil resistance (*P*) and the lateral deflection (*Y*) at any specified depth using n_h in step (2), and Eqs. (3) and (7), respectively. Then, plot the *P*-*Y* curve.

The pile-group analysis proposed for reliability assessment is based on the average pile representing respective rows in the group. Therefore, the performance functions for the average pile of respective rows in the group are the same as the performance functions considered for the single pile. The three performance functions for reliability assessment for the average pile of respective rows are: (1) lateral deflection performance function; (2) moment strength performance function for pile material; and (3) soil resistance performance function. The performance functions for the average pile of respective rows in the group consider the reduction of the soil stiffness to account for group effect.

The reliability assessment for the single pile and the average piles in the group is performed using the performance functions for the failure modes and the first-order reliability method in a form of a computer program (Ayyub et al. 1996).

Case Study

A load test result was used to provide a better understanding for the proposed reliability assessment methodology presented in this paper. Morrison and Reese (1988) performed the load test on a large-scale, well- instrumented group of piles in sand, and a similar single pile. A 3×3 group of piles and a single pile, installed in Houston. The native clay soil was removed from the upper portion of the piles and replaced with clean sand. The sand was placed in a dry state and compacted in six sections to have a relative density of 50% using a Dyna-pac EY 15 vibrating-plate compactor. The average dry density after compaction was 13.83 kN/m^3 (98 lb/ ft^3). The sand had a uniform gradation, of medium density, and classified SP by the unified soil classification system. The majority of particle sizes fell between the No. 30 and No. 50.

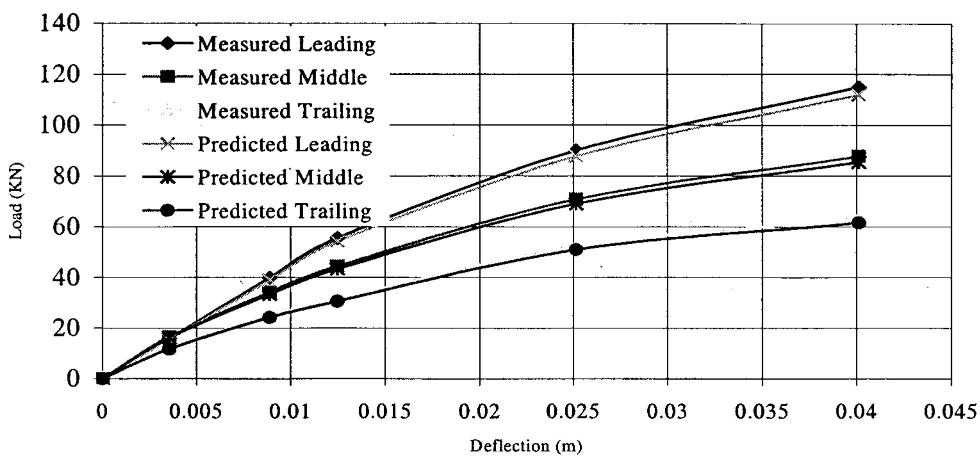

Fig. 3. Load-deflection curve for pile group

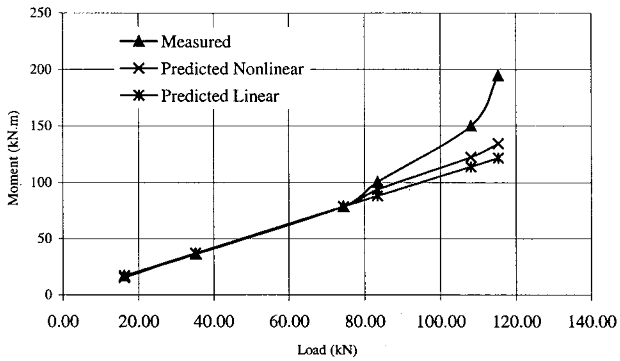

Fig. 4. Maximum moment-load curves

The sand had an angle of internal friction of 38.5° using the direct shear test. After compaction was completed, the sand was slowly saturated from below using perforated pipes that had been placed at the surface of the clay. The site was flooded until the test was over.

The piles were instrumented for the measurement of load, deflection, slopes at the top of the piles, and bending moments along the piles. Lateral loading was applied to the piles, and the response of the instrumentation was recorded. The results of the load tests were used for both the single pile and piles within the group to generate load-deflection curves at the top of the piles, the maximum moment-load curves, and the *P-Y* curves along the piles.

Load Deflection Curves

The measured load-deflection curve, nonlinear predicted load-deflection curve, and linear predicted load-deflection curve for the single pile are shown in Fig. 2. The predicted nonlinear load-deflection curve is developed by changing n_h with the load and deflection values to obtain the secant modulus at each load increment using Eq. (19). The linear load-deflection curve is developed by using $n_h=0.0054$ kN/cm^3 (0.02 kip/in.^{3}). The predicted nonlinear load-deflection curve shows good agreement with the measured load-deflection test data. The linear load-deflection curve can be considered as the lower boundary for the deflection because it underestimates the deflection at higher loads as shown in Fig. 2. The lower deflection at higher load in the linear analysis is because of using higher soil stiffness.

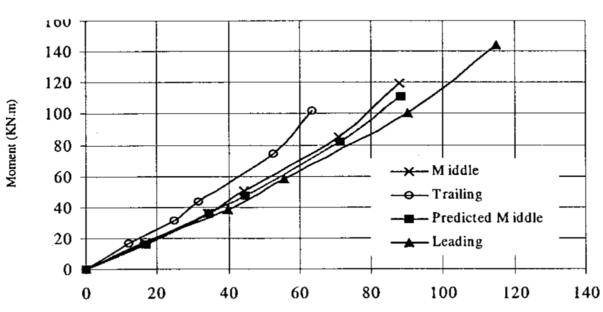

Fig. 5. Measured moment-load curves for pile group

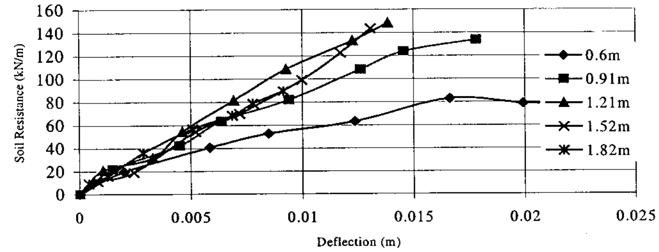

Fig. 6. Measured *P-Y* curves for single piles

The average pile-head load versus deflection for the average pile of respective rows in the pile group is shown in Fig. 3. Comparison between the measured and predicted data for leading, middle, and trailing rows is also shown in Fig. 3, respectively. It is believed that the model prediction for load-deflection curves provides good agreement for reliability assessment.

Maximum Moment-Load Curve

The maximum measured moment-load curve, maximum nonlinear predicted moment-load curve, and maximum linear predicted moment-load curve are shown in Fig. 4. The model predication shows good agreement at lower-load levels or up to 44.48 kN (100 kips) of the pile-head load. At higher-load level, the model prediction underestimates the maximum moment occurred in the pile material.

Maximum bending moment as a function of the average pile-head load is shown in Fig. 5 for the group by row. Fig. 5 also indicates that for a given pile head load, piles in the middle and back rows sustained larger bending moment. This trend reflects the softening of soil resistance due to ‘‘shadowing effect.’’ A good curve fitting between the measured and the predicted data is also shown in Fig. 5 for the middle row.

The P-Y Curve

Using a Winkler soil model, polynomial curves were fitted to the bending moment-depth data using the same approach described by Morrison and Reese (1988). The measured *P-Y* curve for levels 0.6, 0.91, 1.21, 1.52, and 1.82 m (24, 36, 48, 60, and 72 in.) are shown in Fig. 6. It can be observed from Fig. 6 that the deeper the *P-Y* curve the higher the soil response up to 1.21 m (48 in.) then the soil response starts to decrease. The principle reasons for this behavior are (Matlock and Reese 1962): (1) sandy soils frequently increases in strength characteristics with depth as a results of overburden pressure and of natural deposition; and (2) pile deflection decreases with depth for a given loading. Thus, for long flexible piles, piles behave as a beam on an elastic foundation fixed at some point under the ground level. Therefore, at some depth below the ground surface, the soil response to the lateral loads at the pile head is negligible. The ultimate lateral resistance of the pile group subjected to lateral loads is determined either by the excessive lateral deflection of the pile cap or the yielding moment of the piles' material in the group. The yielding moment of the piles' material will be reached before the full mobilization of the ultimate soil resistance along the length of the piles.

Reliability Assessment for Case Study

Reliability assessment for single piles as well as pile groups require the probabilistic characteristics of the basic random vari-

Table 1. Probabilistic Characteristics of Random Variables Used in Limit States

Random variables	Coefficient of variation	Distribution type
Y_u = Ultimate lateral deflection	0.10	Normal
Q_g = Applied lateral loads	0.10	Normal
M_u = Ultimate moment	0.12	Lognormal
n_h = Constant of subgrade reaction	0.20	Normal
E = Modulus of elasticity	0.02	Normal
I = Moment of inertia of pile material	0.05	Normal

ables used in the performance functions. A summary for the probabilistic characteristics of the basic random variables is provided in Table 1. The summary is based on different studies and sources such as: (1) a series of tests in October 1978 on Ellis Island, near Alton, Ill.; and (2) the analysis of the pile-supported fixed-crest dam at locks and dam No. 2 on Monongahela River.

Lateral Deflection Performance Function

Four cases for reliability assessment of the single pile were considered in the case study: (1) reliability indices (β_1) and (β_2) for nonlinear analysis by considering nonlinear spring constant (n_h) with constant and variable ultimate deflection, respectively; and (2) reliability indices (β_3) and (β_4) for linear analysis by considering linear spring constant n_h=0.0054 kN/cm^3 (0.02 kip/in.^{3}) with constant and variable ultimate deflection, respectively.

The reliability indices for the four cases as well as the measured and predicted load and deflection results are presented in Table 2. The reliability indices versus lateral deflection for the four cases are shown in Fig. 7. The observations that can be drawn from Fig. 7 are: (1) at the same applied lateral loads, the linear analysis obtains higher-reliability indices than the nonlinear analysis; and (2) the constant ultimate deflection produces higher-reliability indices than the variable ultimate deflection for both linear and nonlinear cases. The ultimate deflection was taken as 0.0508 m (2.0 in.), which is the upper limit for lateral deflection allowed by AASHTO.

The reliability assessment for the lateral deflection performance function for the pile group is based on the assumption that all the piles within the group do not move relative to each other and the piles have the same head constraints. Therefore, reliability assessment for lateral deflection performance function of the group can be based on any row results. The reliability indices for the lateral deflection performance function of the pile group are presented in Fig. 8. It can be observed from Fig. 8 that the leading row carries more loads than the middle and trailing rows at the

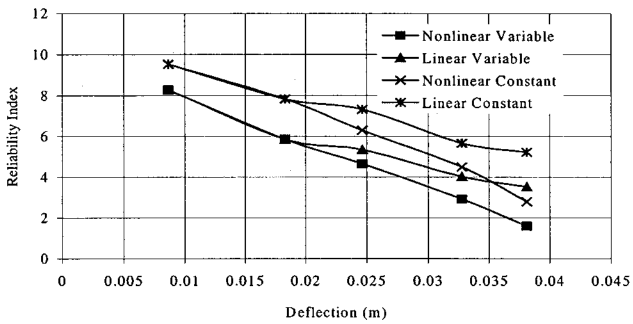

Fig. 7. Reliability assessment for lateral deflection performance function

same reliability index. As an example, at reliability index=4.5, the leading, middle, and trailing rows carry loads=90.72, 70.90, and 52.39 kN (20.25, 15.94, and 11.78 kips), respectively.

Moment Strength Performance Function

The reliability assessment for the moment strength performance function (MSPF) for the single pile under lateral loads are considered for: (1) nonlinear analysis by considering variable spring constant n_h; and (2) linear analysis by considering linear spring constant n_h=0.0054 kN/cm^3 (0.02 kip/in.^{3}). The reliability index curve is shown in Fig. 9. The reliability assessment results, as shown in Fig. 9, indicate that there is no significant difference between the linear and nonlinear analysis.

Reliability assessment for the moment strength performance function for the pile group is determined for the average pile of respective rows. Measured and predicted moment, as well as the reliability indices for the average pile of respective rows are presented in Fig. 10. Fig. 10 shows the maximum moment-deflection curves, where at a certain deflection for the pile group, the leading row carries more loads and in turn has more bending moment than the middle and trailing rows. Therefore, the reliability of the leading row is less than the reliability of the middle and trailing rows because it is related to the bending capacity of the pile material. As an example, at a deflection=0.0254 m (1 in.) the reliability indices= 4.7, 5.8, and 7.23 for leading, middle, and trailing rows, respectively.

Reliability Assessment of Pile Groups

The reliability assessment for the pile groups composed of long flexible piles as a system depends on the reliability of the lateral deflection performance function of the pile cap, and the moment strength performance function of the average pile of respective

Table 2. Reliability Assessment for Lateral Deflection Performance Function for Single Piles

Measured deflection m (in.)	Measured load kN (kips)	Reliability Indices β_1 (Nonlinear analysis with constant deflection)	β_2 (Nonlinear analysis with variable deflection)	β_3 (Linear analysis with constant deflection)	β_4 (Linear analysis with variable deflection)
0.0086 (0.34)	35.13 (7.90)	9.53	8.279	9.53	9.53
0.0182 (0.72)	74.41 (16.7)	7.83	5.86	7.83	5.86
0.0246 (0.97)	83.40 (18.7)	6.30	4.66	7.31	5.35
0.0320 (1.29)	108.0 (24.2)	4.50	2.92	5.67	4.03
0.0381 (1.50)	115.3 (25.9)	2.80	1.60	5.22	3.53

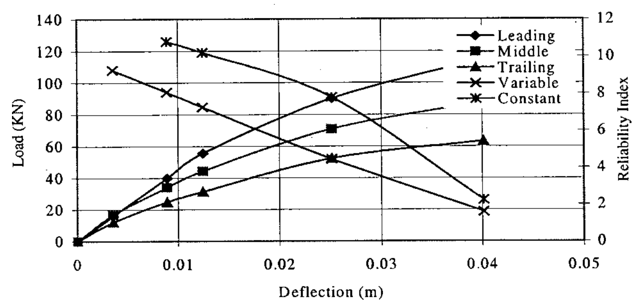

Fig. 8. Reliability assessment for lateral deflection performance function

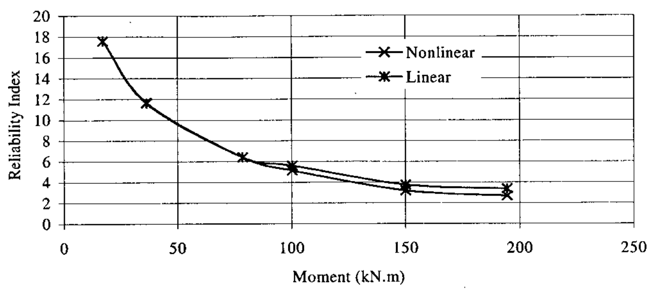

Fig. 9. Reliability assessment for moment strength performance function

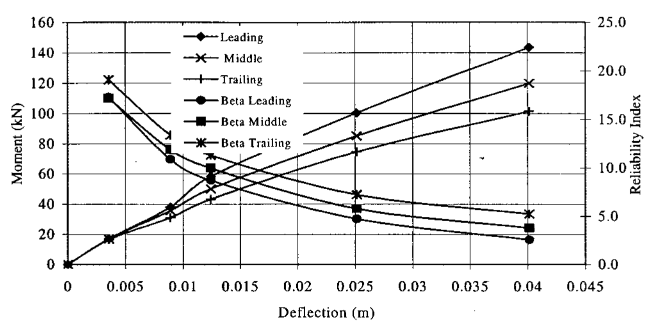

Fig. 10. Reliability assessment for moment strength performance function of pile group

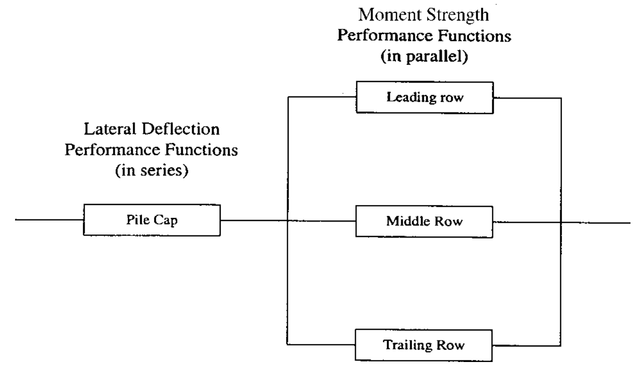

Fig. 11. System reliability model for pile group

rows in the group as shown in Fig. 11. A summary of the failure probabilities for the pile group is presented in Table 3. Column 2 of Table 3 is the failure probability for the lateral deflection performance function (LDPF) at each load increment. Columns 3 to 5 of Table 3 are failure probabilities for the moment strength performance function (MSPF) at each load increment for the leading, middle, and trailing rows, respectively. Columns 6 and 7 of Table 3 are the lower and upper bounds for the failure probabilities for the MSPF for the group using Eq. (9) for the unibound complex system in parallel. The failure probabilities for the whole system are the failure probabilities for the LDPF and MSPF connected in series as shown in Fig. 11. It can be obtained by using Eq. (8) for the unibound complex system in series. The failure probabilities range for the pile group as a system is presented in Columns 8 and 9 of Table 3, respectively.

Conclusions

A probability-based reliability assessment methodology for single piles and pile groups subjected to lateral loads in sandy soil is proposed in this paper. Based on the proposed methodology, the following conclusions can be drawn:

1. The nondimensional method is a reasonable approach for modeling single piles with appropriate single-pile test results or *P-Y* curves for reliability assessment.
2. The modes of failure for flexible piles in sandy soil subjected to lateral loads are the excessive lateral deflection at the pile head and the yielding moment of the pile material. The yielding moment of the pile material is most likely to be reached before the full mobilization of the ultimate soil resistance along the pile length.
3. Applying *P* multipliers on a *P-Y* curve for a single pile is an accurate and easy approach to account for the pile group effect. However, for the proposed methodology, the *P* multipliers should be applied to the soil modulus instead of the soil resistance.

Table 3. Failure Probabilities for Pile Group Using Modified Nondimensional Method

Measured deflection m (in.)	Lateral deflection performance function	Moment Strength Performance Function: Leading row	Middle row	Trailing row	Lower bound	Upper bound	Lower bound	Upper bound
0.025 (0.99)	3.3E−15	1.0E−06	3.2E−08	2.3E−12	7.66E−26	2.3E−12	2.3E−12	2.3E−12
0.040 (1.58)	1.2E−02	5.0E−03	7.4E−04	8.0E−07	2.78E−11	7.4E−07	1.2E−02	1.2E−02

4. The modes of failure for pile groups composed of flexible piles in sandy soil subjected to lateral loads are the excessive lateral deflection of the pile cap and the yielding moment of the average pile of respective rows in the group.
5. The proposed reliability-based assessment methodology is practical for evaluating pile foundations. The methodology considers uncertainties involved in loads, strength variables, and prediction models. The proposed methodology can be an alternate replacement to the currently used safety margin method.

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Notation

The following symbols are used in the paper:

A = cross-sectional area of pile;
D = pile diameter;
E = modulus of elasticity of pile material;
E_j = individual failure event in system reliability;
E_s = soil modulus;
I = moment of inertia of pile section;
L = pile length;
M_g = applied bending moment at ground surface;
M_u = ultimate moment capacity;
M_x = bending moment along pile length;
n_h = constant of horizontal subgrade reaction;
P = lateral soil resistance;
P_f = probability of failure as estimated from analysis;
P_{fi} = probability of failure *i*th mode;
P_u = ultimate lateral soil resistance;
P_x = lateral soil resistance along pile;
Q_g = applied lateral load at ground level;
Q_x = axial load at pile head;
T = characteristic pile length;
V_x = shear along pile;
Y = lateral deflection of pile head;
Y_u = ultimate lateral deflection;
Y_x = lateral deflection along pile;
Z = performance function of interest;
Z = x/T;
Z_P = performance function for soil strength;
Z_y = performance function for lateral deflection;
β = reliability index; and
γ = soil unit weight.

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