K. Atua and B.M. Ayyub, Ph.D., P.E.

Reliability Analysis of Transverse Stability of Surface Ships

ABSTRACT An introduction to a rational development of transverse stability criteria for surface ships is presented in this paper. The development is based on a probabilistic analysis of the uncertainties associated with the design parameters involved in determining the transverse stability. Reliability methods are used for assessing and determining transverse stability of ships. These reliability methods address the uncertainties associated with the basic design variables that are involved in stability assessment and design. The sources of uncertainty in stability variables are presented, uncertainties associated with these basic random variables are summarized, different reliability assessment methods are described, a case study that shows the computation procedure using some of these methods is discussed for illustration purposes, and finally, conclusions and recommendations for further work in this area are presented.

Introduction

Transverse stability is a vital factor in determining the overall design quality of a ship. An over-designed transverse stability may result in stiff-motioned ships which are uncomfortable; whereas an under-designed transverse stability can result in poor ship stability and perhaps the entire loss of a ship by capsizing. Ship instability and capsizing can result from one of many factors as shown in Figure 1 (Atua 1992).

Reliability theory is used for the rational treatment of uncertainties associated with design variables and manufacturing processes of engineering systems and is used for assessing the reliability in strength and serviceability for these systems. Researchers adopted reliability methods to account for uncertainties, including randomness in design variables, by modeling them in terms of their probabilistic characteristics (such as mean, variance, and their probability distributions). Also, statistical and modeling uncertainties were considered using these methods. As a result, reliability theory has evolved into procedures that were used in a wide range of practical applications, especially design codes for structures that include buildings, bridges and recently, marine structures (Ang and Tang 1984, and Sundararajan 1995). Therefore, it is logical to adopt the same theory to the transverse stability design of ships.

The uncertainties associated with the transverse stability of ships originate from design process, manufacturing and fabrication, operation, loading, alterations and modifications in the hull in the case of intact stability, and accidents and collision in the case of damage stability. Each process or stage has its own variables that need to be considered and modeled with their associated uncertainties.

A ship operates in seas which are totally random in nature, causing the ship to roll in a manner that results in random load water line (LWL) characteristics (waterline area, A_W, transverse moment of inertia of the waterline, I_T,..., etc). Therefore, a random metacentric radius, $\overline{BM}$, results. Also, a random vertical distribution of buoyancy produces a random vertical position of the center of buoyancy, $\overline{KB}$. The random nature of loading conditions and the vertical distribution of loads and cargo, especially in commercial ships, can result in significant variability in the vertical position of center of gravity of the ship, $\overline{KG}$. Variability in $\overline{KG}$ may occur from voyage to voyage and even during the same voyage due to different angles of the ship's inclination, θ, for different displacements, Δ. A ship is also susceptible to grounding and flooding. This damage produces sources of uncertainty in ship stability in its damaged state, stemming from the difficulty in predicting the size and location of flooding and grounding damage. Alterations and modifications in the hull structures, equipment and machinery, or cargo arrangement affect both weight and buoyancy distribution, hence, $\overline{KB}$, $\overline{BM}$, and $\overline{KG}$.

Uncertainty in determining the metacentric height, $\overline{GM}$, can be attributed to the difference between the predicted or design values of $\overline{KG}$, $\overline{KB}$, and $\overline{BM}$, and

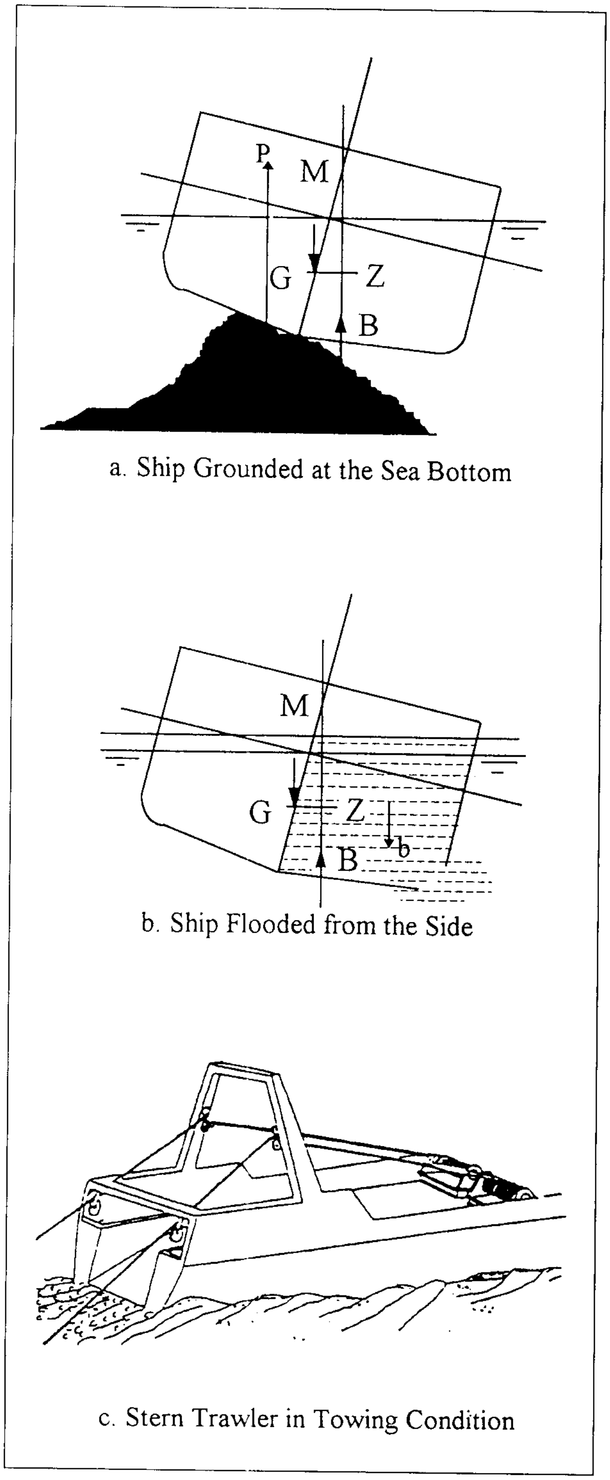

FIGURE 1. Different Hazards for Transverse Stability of Surface Ships

the measured values of these parameters after launching.

Shama (1976) discussed uncertainties in the main variables that affect the reserve dynamic stability of ships with a focus on calculating the probability of capsizing. The probability calculation in this study was based on assuming the following performance function:

$$S_D = M_D - M_H \qquad (1)$$

where M_D is the dynamic stability of a ship, M_H is the work done by the heeling moment on the ship, and S_D = performance or safety margin. Also, he assumed that all relevant stability variables are normally distributed to simplify the calculations. Based on these assumptions, Shama (1976) concluded that both a ship's initial stability and the shape of its static stability curve have a great influence on its capsizing probability.

Lockerby (1993) discussed the development of a reliability analysis approach to damage stability criteria for surface ships to comply with the changes of weapons, environments, and requirements. The results of the analysis can enhance the stability of ships.

The objectives of this paper are (1) to investigate the sources of uncertainty associated with design variables involved in transverse stability of ships; (2) to apply reliability theory in assessing transverse stability; (3) to perform reliability calculations on an example ship to assess its capsizing probability; and (4) to provide recommendations for future work for the development of reliability-based transverse stability criteria of ships.

Transverse Stability Criteria

The criteria for the transverse stability of ships depend on several factors and conditions. For small angles of inclination for a ship, as shown in Figure 2, a ship is considered to be stable if the metacentric height, $\overline{GM}$, has a positive value, neutral if $\overline{GM} = 0$, and unstable if $\overline{GM}$ has a negative value, where the metacenric height, $\overline{GM}$ is given by

$$\overline{GM} = \overline{KB} + \overline{BM} - \overline{KG} \qquad (2)$$

In Figure 2, M = transverse metacenter, G = center of gravity of the ship, Z = projection of G on the vertical plane, and K = point at the bottom of the keel. When the ship rolls to an angle θ, the righting moment, RM, provides the lever towards returning it to its original position; where the righting moment, RM, is given by

$$RM = \Delta\overline{GM}\,sin\,\theta = \Delta\overline{GZ} \qquad (3)$$

where Δ = displacement of the ship, $\overline{GZ}$ = the righting arm. If the ship has negative $\overline{GM}$, the righting moment will act in the direction of the capsizing moment, causing the ship to heel more until it capsizes.

For large angles of inclinations for a ship, its dynamic stability is the most important characteristic, which is defined as the energy available by virtue of a ship's righting

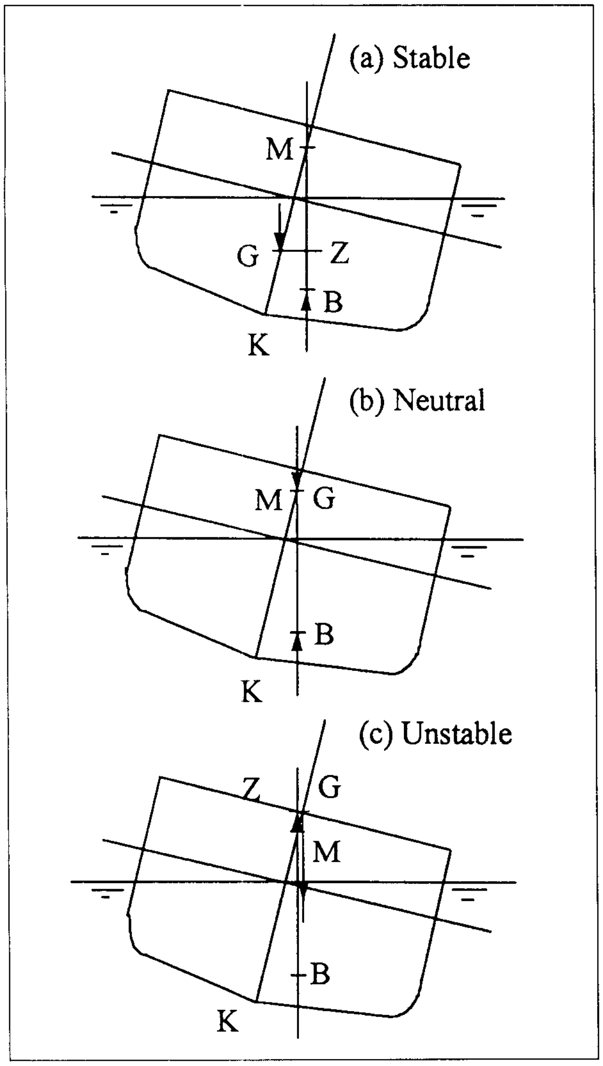

FIGURE 2. Stability Conditions for a Ship According to the Metacentric Height ($\overline{GM}$): (a) $\overline{GM}$ = positive (stable), (b) $\overline{GM}$ = 0 (neutral), (c) $\overline{GM}$ = negative (unstable)

moment to resist any external heeling energy from the position of equilibrium to any inclined position. The dynamic stability at any angle θ is given by

$$E_\theta = \Delta\int_0^\theta GZ\, d\theta \qquad (4)$$

where E_θ = the dynamic stability up to an angle θ. By plotting the heeling moment curve on the same coordinates as the static stability curve, it is possible to integrate both heeling moment and static stability curves and plot them on the same coordinates. The integration of the heeling moment curve up to any angle θ is represented by the vertical coordinate on the heeling dynamic curve which is equal to the heeling work done on the ship up to this angle; whereas the vertical coordinate on the dynamic stability curve is the integration of the static stability curve up to this angle. Both the static and dynamic stability and heeling curves are used in the current transverse stability design criteria in conventional ship design rules that require deterministic values for each ordinate at certain angles, points, and intersections.

Current Design Criteria for Transverse Stability

In this section, the bases behind the U.S. Navy and Coast Guard transverse stability criteria for monohull surface ships are summarized and tabulated in terms of requirements for adequate stability for different operational conditions.

The U.S. Navy transverse stability criteria (USN, DDS 079-1 1975) are for various operational conditions such as combined beam wind with rolling, lifting heavy weights over the side, towing loads for tugs, crowding personnel to one side, high speed turning, and topside icing in case of intact stability. The major transverse stability criteria for surface ships complying with the U.S. Navy requirements are based on the comparison of the righting and heeling arm curves as shown in Figure 3. In addition to

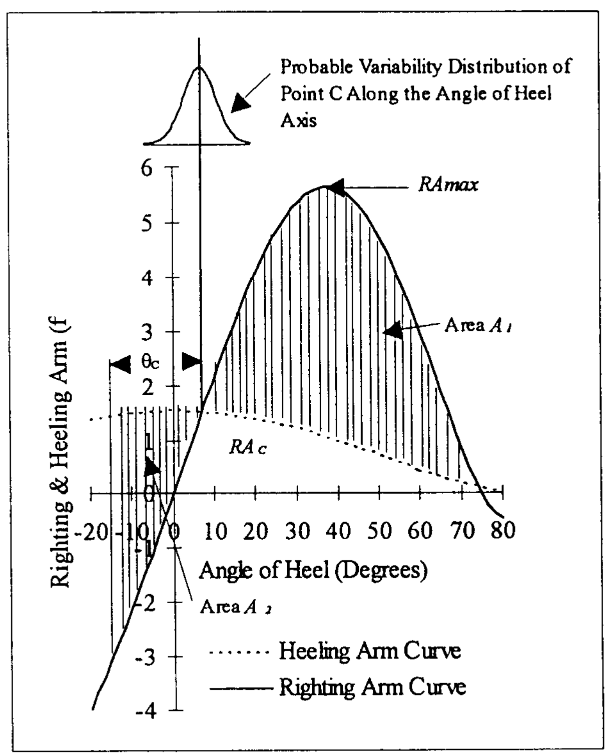

FIGURE 3. Transverse Stability Criteria for an Intact Stability Condition

this, there are special requirements for every design case, such as beam wind speed and topside icing.

The damaged stability criteria are based on limits on the static trimmed-heeled water line, the static heel angle (θ_c), and the adequacy of dynamic stability, which must be available to absorb the energy imported to the ship by moderately rough seas in combination with beam winds. The damage righting arm curve is truncated at 45 degrees or at the angle where unrestricted flooding may occur, whichever is less.

The U.S. Coast Guard stability requirements for all inspected vessels (USCG, 46 CFR 1995) include many aspects such as: (1) approving of plans (general arrangement, lines, curves of forms, capacity plans with vertical, longitudinal, and transverse centers of gravity, and tank soundings), (2) stability instructions for operating personnel such as stability booklets, stability letters, and information for ships engaged in lifting, (3) weather criteria (minimum value of $\overline{GM}$ based on the projected lateral area of the ship above waterline), (4) determination of displacement and center of gravity by stability test performed by the owner, (5) special installations, for example, fixed ballast and foam floatation material, (6) watertight bulkhead doors (types and classes), and (7) consideration of the free surface correction for both intact and damaged stability calculations with or without passive roll stabilization tanks. In addition to the above stability criteria, requirements based on the comparison of the righting and heeling arms for every type of ship and design case are provided.

Since the Coast Guard stability requirements depend on the same parameters as the Navy requirements, the stability criteria presented herein are based on a common examination of both the DDS 079-1 (1975) and the Coast Guard 46 CFR (1995) for the case of combined beam winds and rolling.

INTACT STABILITY CRITERIA

Ships can be classified into two main groups: (1) ships without side protective systems, and (2) ships with side protective systems, such as large carriers. The intact stability criteria under the effect of combined beam winds and rolling require a ship to withstand the effects of beam winds. In this case, adequate stability is assured by comparing a ship's righting arm and heeling arm curves. A ship is considered to have satisfactory stability if

1. the heeling arm at the intersection of the righting arm and heeling arm curves, RA_c (point C in Figure 3) is not greater than six-tenths of the maximum righting arm, RA_{max}; and
2. the area A_1 is not less than 1.4 A_2 where area A_2 extends twenty-five degrees to windward from point C.

DAMAGE STABILITY CRITERIA

An example of damage stability criterion for ships of over 300 feet in length is that a ship must be capable of withstanding flooding from a shell opening equal to fifteen percent of its length. This criterion is satisfied by meeting the following four conditions:

1. The static trimmed-heeled water line after damage does not submerge the margin line at the side.
2. As shown in Figure 4, the static heel angle (θ_c) without wind effects does not exceed fifteen degrees.
3. Adequate dynamic stability must be available to absorb the energy imported to the ship by moderately rough seas in combination with beam winds.
4. The damage righting arm curve is truncated at forty-five degrees or at the angle where unrestricted flooding may occur, whichever is less.

Reliability-Based Stability Criteria

The design parameters of stability ($\overline{GZ}$, righting moment, RM, heeling moment, HM, ..., etc.) are functions of a ship's design characteristics (such as $\overline{KB}$, $\overline{BM}$, $\overline{KG}$), the sea conditions, operating profiles and modes, damage severity, ..., etc. Due to the uncertainties associated with basic design random variables, the stability of a ship is an uncertain condition that needs to be evaluated by several expressions according to existing design rules. The intent behind these rules can be defined in terms of performance functions as follows:

$$\theta_c \leq 15^o \quad \text{for damage stability} \quad (5)$$

$$RA_c \leq 0.6RA_{max} \quad \text{for intact stability} \quad (6)$$

$$A_1 \geq 1.4A_2 \quad \text{for intact and damage stability} \quad (7)$$

Due to the uncertainties associated with the basic design variables, both the righting and heeling arms are uncertain at any angle of inclination. Therefore, they can be represented by probability distributions with mean values and standard deviations estimated according to data collected on the underlying variable. Since the stability design rules for a ship require that θ_c be less than or equal to fifteen degrees, RA_c to be less than or equal to 0.6 RA_{max}, and A_1 to be greater than or equal to 1.4 A_2, the probability of capsizing, P_c, can be stated as follows:

$$P_{c_i} = 1 - P_{s_i} = 1 - (\mathrm{P}((RA_c \leq 0.6RA_{\max})\cup(A_1 \geq 1.4A_2))) \text{ for intact stability} \quad (8)$$

$$P_{cd} = 1 - P_{sd} = 1 - (P((\theta_c \leq 15^o)\cup(A_1 \geq 1.4A_2))) \text{ for damage stability} \quad (9)$$

where P = probability; and P_{ci}, P_{si}, P_{sd}, and P_{sd} are probabilities of capsizing and stability in both intact and damaged conditions, respectively.

The underlying concept in assessing the capsizing probability is to mathematically develop a performance function of the following form:

$$Z = g(X_1, X_2, \ldots, X_n) \quad (10)$$

where the Xs are the basic random variables, $Z<0$ for a failure condition, $Z>0$ for a survival condition, and $Z=0$

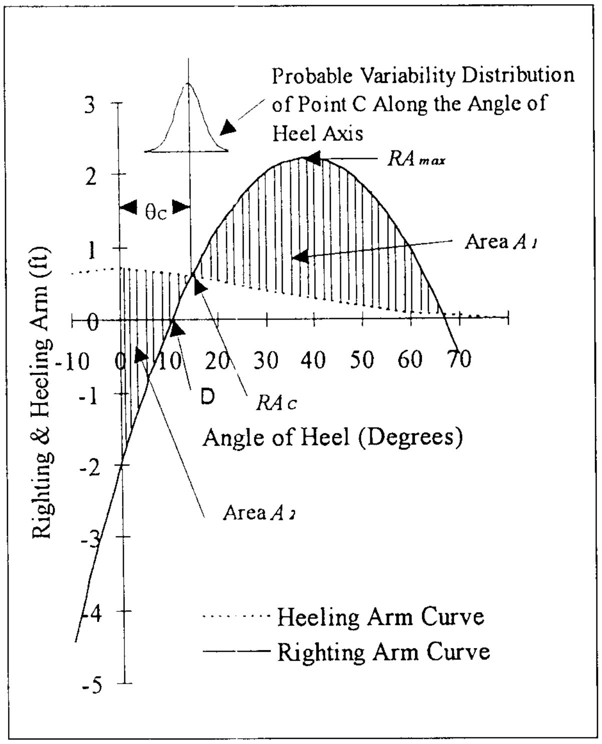

FIGURE 4. Transverse Stability Criteria for a Damaged Stability Condition

for the limit state. The capsizing probability is then defined as the integration of the joint probability density function of X_1, X_2, and X_n over the region where $g<0$. For the case given by Equation 7, let $1.4A_2 = A$, then the performance function can be defined as

$$Z = A_1 - A \tag{11}$$

The capsizing event becomes $A_1 - A < 0$, and the probability of capsizing becomes

$$P_c = p(A_1 < A) = \int_{-\infty}^{\infty} F_{A_1}(x) f_A(x) dx \tag{12}$$

where F_{A_1} is the cumulative distribution function of A_1, and f_A is the probability density function of A. The probability density functions of A_1 and A are represented in Figure 5, where the probability of capsizing as given by Equation 12 is associated by the overlap area between the two functions.

The procedure for development of reliability-based transverse stability criteria can be summarized in the following steps:

1. Analysis of the sources of uncertainties associated stability variables such as ship characteristics, operation profile, loading, and manufacturing and production using probabilistic and statistical analysis.
2. Development of probabilistic characteristics of basic random variables involved in stability calculations (righting moment, heeling moment, $\overline{KB}$, $\overline{BM}$ and $\overline{KG}$) such as mean values or mean/nominal values, coefficients of variation, and distribution types.
3. Determination of limit states for different stability criteria in the current design codes.
4. Selection of target reliability levels based on implied reliability levels in the current design codes or using judgment based on experience for novel designs.
5. Determination of biases between predicted (calculated) values and measured (real) values of stability variables.
6. Computation of partial safety factors to develop a reliability-based code using reliability methods.

The aforementioned procedure is illustrated in a design flow chart as shown in Figure 6.

Reliability Methods

Commonly used reliability methods utilize the mean and variance (first and second moments) of basic random variables in calculating a reliability measure according to a specified performance function. These methods were used in the reliability assessment of ship capsizing by Shama (1976), and in ship structural reliability by Mansour (1972), Mansour and Faulkner (1973), Faulkner (1981), White and Ayyub (1985, 1987), and Mansour et al (1993). In the following section, different reliability methods are briefly presented and classified according to the manner with which they deal with the probabilistic characteristics of the basic design parameters.

FIRST-ORDER SECOND-MOMENT METHOD

According to this method, an approximate mean and an approximate variance of the performance function of Equation 10 are determined using Taylor's series expansion of g about the mean value of the Xs truncating the series at the linear term. The resulting expressions for the approximate mean and variance are given respectively by

$$\bar{Z} = g(\bar{X}_1, \bar{X}_2, \ldots \bar{X}_n) \tag{13}$$

and

$$\sigma^2_Z \cong \left[\sum_{i=1}^{m} \sum_{j=1}^{n} \left(\frac{\delta g}{\delta X_i}\right)\left(\frac{\delta g}{\delta X_j}\right) Cov(X_i, X_j)\right] \tag{14a}$$

where the partial derivatives are evaluated at the mean values of the random variables, and Cov = covariance. For statistically noncorrelated random variables, the variance is

$$\sigma^2_Z \cong \left[\sum_{i=1}^{n} \sigma^2_{X_i} \left(\frac{\delta g}{\delta X_i}\right)\right] \tag{14b}$$

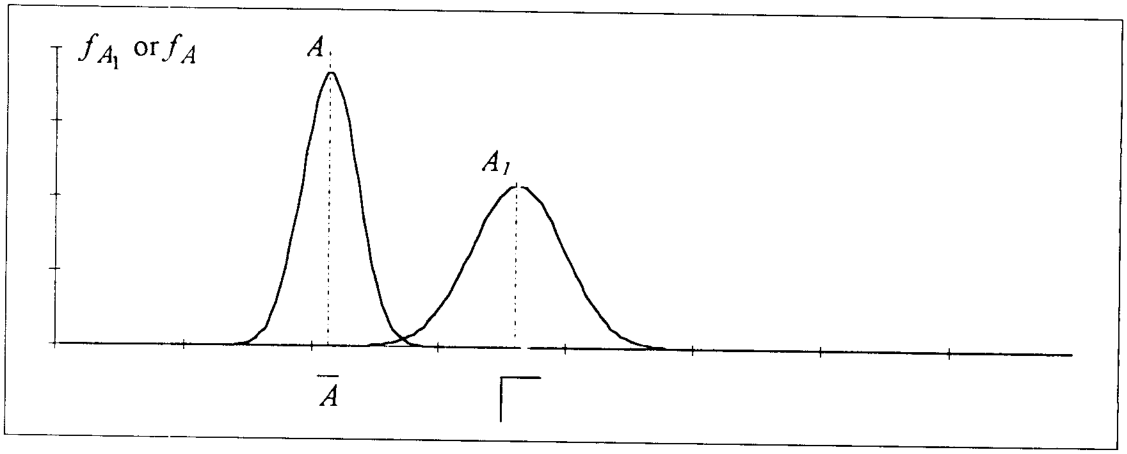

FIGURE 5. Plotting of A_1 and A as Random Variables

where $\bar{Z}$ is the mean value of Z and σ_Z is the standard deviation of Z. The reliability measure here is the reliability index, β, which is given by

$$\beta = \frac{\bar{Z}}{\sigma_Z} \tag{15}$$

where β is the reciprocal of the *Cov* (coefficient of variation) of Z. Assuming Z to be normally distributed, the probability of capsizing, P_c, is given by

$$P_c = 1 - \Phi(\beta) \tag{16}$$

where Φ is the cumulative distribution function of the standard normal variate. The shortcoming of this method is that it produces the exact probability when Z is linear and normally distributed. For lognormally distributed random variables, logarithmic transformation can be used to obtain the exact solution.

ADVANCED SECOND-MOMENT METHOD

The shortcoming of the first-order second-moment method is addressed by this method, which allows us to deal with a non-linear performance function and with non-normal random variables. For this purpose, the performance function can be defined in terms of the following reduced variables:

$$u_i = \frac{X_i - \bar{X}_i}{\sigma_{x_i}} \tag{17}$$

where u_i = reduced variable for X_i, and the limit state g' in the reduced space is given by

$$g' = 0 \tag{18}$$

The safety index, β, in this case is defined as the minimum distance from the origin of the reduced coordinates of the basic random variables to the limit state as shown in Figure 7 for two variables X_1 and X_2. The safety index, β, is determined by iteratively solving the following set of equations:

$$\alpha_i = \frac{\left(\frac{\delta g}{\delta X_i}\right)\sigma_{X_i}}{\left[\sum_{i=1}^{n}\left(\frac{\delta g}{\delta X_i}\right)^2 \sigma_{X_i}^2\right]^{\frac{1}{2}}} \tag{19}$$

$$X_i^* = \bar{X}_i - \alpha_i\beta\sigma_{X_i} \tag{20}$$

$$g(X_1^*, X_2^*, \ldots, X_n^*) = 0 \tag{21}$$

where the derivatives $\delta g/\delta X_i$ are evaluated at the design point or the most probable failure point ($X^{*1}, X^{*2}, \ldots, X^{*3}$), and α_i = the directional cosine of the variable X_i, and β = reliability index. The probability of capsizing in this method is the same as given by Equation 16. As shown in Figure 7, point X^* is called the most probable failure point (MPFP) and corresponds to the shortest distance on the limit state (i.e. failure surface).

This method deals with nonnormal probability distributions for basic random variables by determining equivalent normal distributions at the design point in each iteration in the solution of Equations 19 through 21. The mean value and the standard deviation of an equivalent normal distribution of a random variable are given respectively by

$$\mu_{X_i}^N = X_i^* - \Phi^{-1}(F_X(X_i^*))\sigma_{X_i}^N \tag{22}$$

and

$$\sigma_{Xi}^N = \phi\left[\frac{(\Phi^{-1}(F_X(X_i^*)))}{f_x(X_i)}\right] \tag{23}$$

where Φ = cumulative distribution for the standard normal; ϕ = density function of the standard normal; F =

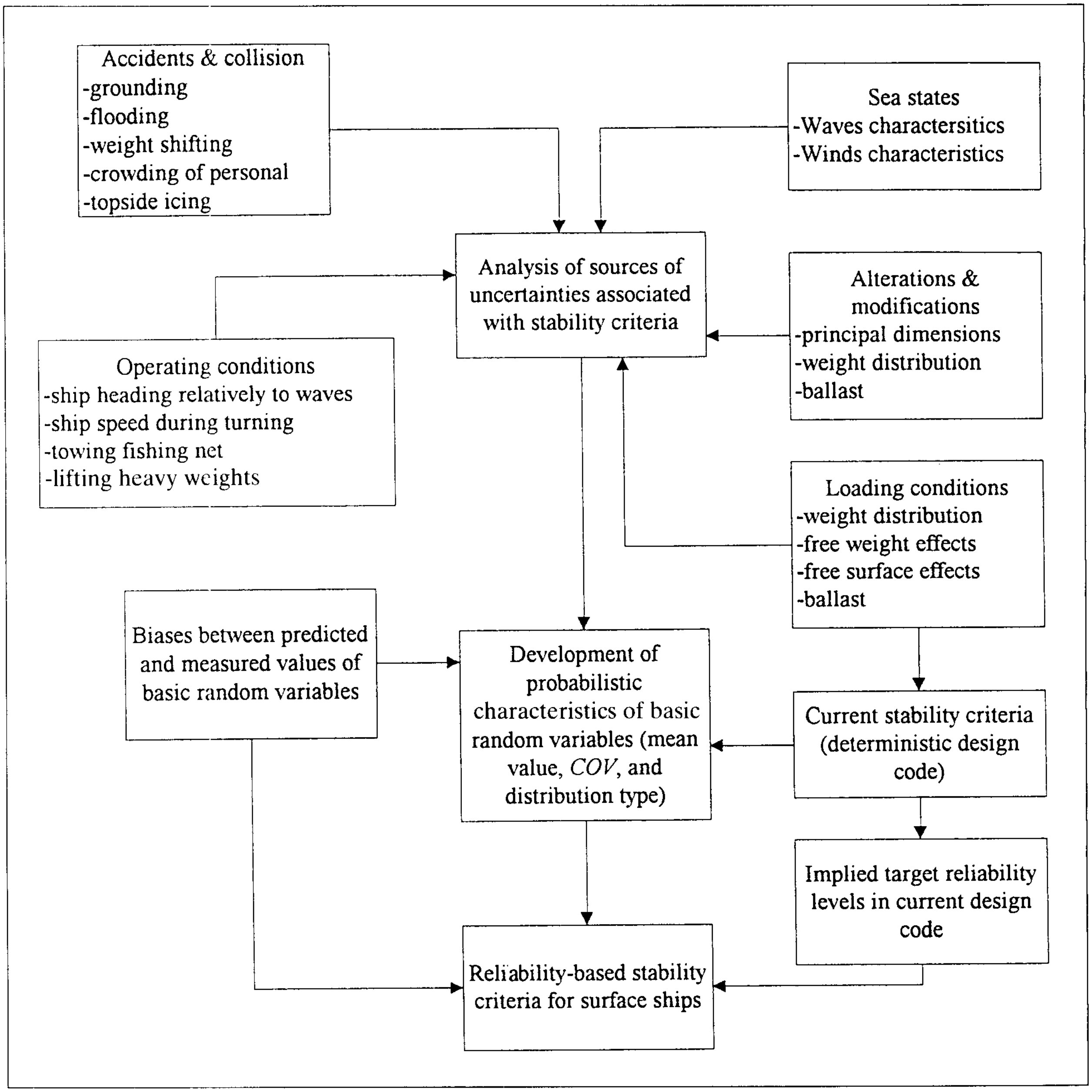

FIGURE 6. Development of Reliability-based Design Criteria for Transverse Stability of Surface Ships

commutative distribution of the random variable; f = density function of the random variable; and μ_{Xi}^{N}, and σ_{Xi}^{N} = mean value and standard deviation of the equivalent normal distribution, respectively.

SIMULATION METHODS

Direct Simulation

The direct simulation technique is basically a process of drawing samples of the basic variables according to their probabilistic characteristics and then feeding them into the performance function. Assuming N_c to be the number of simulation cycles for which $g < 0$ in a total N simulation cycles, then an estimate of the mean capsizing probability $\overline{P}_c$ can be expressed as

$$\overline{P}_c = \frac{N_c}{N} \tag{24}$$

The estimated $\overline{P}_c$ should approach the true value for the

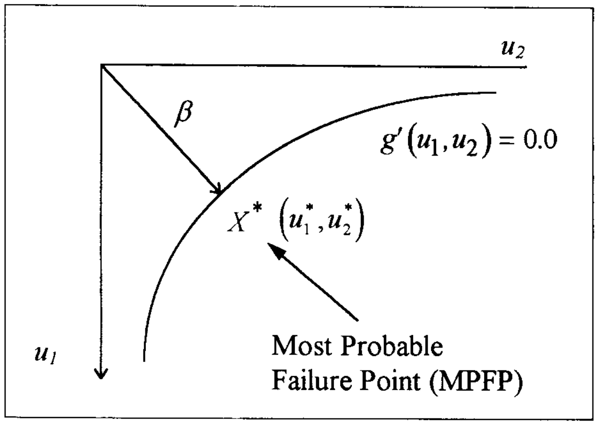

FIGURE 7. Limit State in Reduced Coordinates

population when N approaches infinity. The variance of the $\bar{P}_c$ can be approximately computed as

$$Var(\bar{P}_c) = \frac{(1-\bar{P}_c)\bar{P}_c}{N} \qquad (25)$$

Therefore, the coefficient of variation of the estimated capsizing probability is

$$Cov(\bar{P}_c) = \frac{1}{\bar{P}_c}\sqrt{\frac{(1-\bar{P}_c)\bar{P}_c}{N}} \qquad (26)$$

The direct simulation method is straightforward, but it might require a large number of cycles that depend on the magnitude of the capsizing probability; the smaller the capsizing probability, the larger the needed number of simulation cycles. In order to achieve the desired accuracy of simulation and reduce the sample error without increasing sample size, variance reduction techniques (VRT) are used (Ayyub and Haldar 1984).

Conditional Expectation Variance Reduction Technique

The conditional expectation (CE) variance reduction technique (VRT) reduces the variance of an estimated value by conditioning all random variables except one or more random variables with relatively large variability, therefore removing the effect of their variability on the sampling procedure. The theory behind this technique is that the total variance of X can be given in terms of conditional means and variances of X conditioned on Y as follows (Ang and Tang 1975):

$$Var(X) = E_Y[Var(X|Y)] + Var_Y[E(X|Y)] \qquad (27)$$

The subscript Y on E and Var shows that the expectation and variance are with respect to Y. Rearranging the above equation produces the following:

$$Var_Y[E(X|Y)] = Var(X) - E_Y[Var(X|Y)] \le Var(X) \qquad (28)$$

Equation 28 indicates that computing $E(X|Y)$ analytically from the random variable Y results in a smaller variance than directly computing X (Ayyub and Haldar 1984). In simulation, this concept can be utilized by not generating random numbers for those random variables with large variability. For instance, consider a performance function with three random variables as follows:

$$Z = g(X_1, X_2, X_3) = X_1 - X_2 - X_3 \qquad (29)$$

If X_3 has the largest variability, then the capsizing probability P_c of Equation 29 can be written as

$$P_c = 1 - P_s = 1 - P(X_1 - X_2 - X_3 > 0) \qquad (30)$$

For the ith simulation cycle,

$$P_{ci} = 1 - P(X_3 < X_1 - X_2 \mid X_1 = x_{1i}, X_2 = x_{2i}) = 1 - F_{X_3}(x_{1i} - x_{2i}) \qquad (31)$$

Therefore, the capsizing probability can be calculated by not generating a random number for X_3. The average capsizing probability can then be computed for N simulation cycles as follows:

$$\bar{P}_c = \frac{1}{N}\sum_{i=1}^{N} P_{ci} \qquad (32)$$

This sample average is an unbiased estimation of the population mean. The uncertainty associated with this estimation can be expressed in terms of its variance as

$$Var(\bar{P}_c) = \frac{Var(P_c)}{N} = \frac{1}{N}\left[\frac{1}{N-1}\sum_{i=1}^{N}(P_{ci} - \bar{P}_c)^2\right] \qquad (33)$$

Antithetic Variates Variance Reduction Technique

The antithetic variates (AV) reduce the variance of an estimated mean value by introducing a negative correlation between two sets of samples. Considering two unbiased estimates $X_i^{(1)}$ and $X_i^{(2)}$ of a mean $\bar{X}$ from two separate samples, these two estimates can be combined to form another estimate by taking the average as

$$X_i = \frac{X_i^{(1)} + X_i^{(2)}}{2} \qquad (34)$$

The expected value of X_i is

$$E(X_i) = E\left[\frac{1}{2}(X_i^{(1)} + X_i^{(2)})\right] = \frac{1}{2}[E(X_i^{(1)}) + E(X_i^{(2)})] = \frac{1}{2}(\bar{X}+\bar{X}) = \bar{X} \qquad (35)$$

which means that X_i is an unbiased estimate of $\bar{X}$. It also can be shown that the corresponding variance is

$$Var(X_i) = \frac{1}{4}[Var(X_i^{(1)}) + Var(X_i^{(2)}) + 2Cov(X_i^{(1)},X_i^{(2)})] \quad (36)$$

Therefore if $X_i^{(1)}$ and $X_i^{(2)}$ are negatively correlated, i.e., $Cov(X_i^{(1)}, X_i^{(2)}) < 0$, the variance of the estimate X_i can be reduced. Thus if $X_i^{(1)}$ is a random variable uniformly distributed in (0, 1), then $X_i^{(2)} = 1-X_i^{(1)}$ is also uniformly distributed in (0, 1) and the covariance of $X_i^{(1)}$ and $X_i^{(2)}$ is negative. Consequently, the variance of X_i can be reduced.

The variance of an estimated quantity using simulation can be reduced even more by combining the conditional expectation with antithetic variates. The result of this combination reduces the variance of the estimated probability by about fifty percent (Ayyub and Haldar 1984).

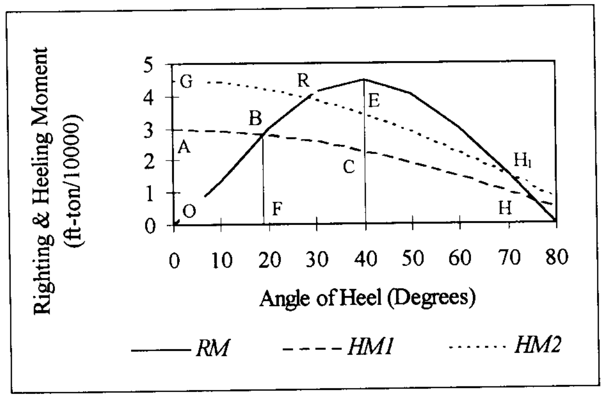

FIGURE 8. Righting and Healing Moment Curves

Example: Capsizing Probability Assessment

The goal of this example to demonstrate the computation of capsizing probability for a ship that meets stability criteria according to some design rules. The capsizing probability computations were performed according to selected reliability methods. The stability performance function for the ship was kept to its simplest form in order to illustrate the presented concepts. The function has two random variables that correspond to the dynamic stability and heeling for a ship as represented by the respective two areas, A_1 and A_2. The stability characteristics of the example ship are shown in Tables 1 and 2. As shown in Figure 8, the area BEH (excess in stability energy) should be greater than the area OAB (excess in heeling energy) so that the ship can restore its original upright position. However, uncertainties in both the righting and heeling moments at each angle of inclination result in uncertainties in both A_1 and A_2. These uncertainties can be represented by a coefficient of variation of 0.10 for each area. By representing the heeling moment curve by the curve GRH_1, the area REH_1 becomes less than the area OGR. As a result, the ship continues to heel through the range of stability and capsizes. The ship chosen for this numerical example is after Gillmer (1959) with the following stability characteristics:

Displacement Δ = 15,000 tons
Heeling moment (in flooding) = 30,000 cos(θ) foot tons
The stability criterion of Equation 7 considers the ship to be stable with an unlikely capsizing occurrence. The stability design variables can be considered to be random for the purpose of illustrating computing capsizing probability as shown in Table 3. These probabilistic characteristics can be used to perform a reliability analysis for the transverse stability of a ship.

TABLE 2

Probabilistic Characteristics of A_1 and A_2

Random Variables	Mean Values	Coefficient of Variation	Distribution Type
A_1	91000	0.10	Normal
A_2	54000	0.10	Normal

TABLE 3

Capsizing Probability Using Different Reliability Methods

Method	Results
Deterministic Approach	The ship is totally stable, and capsizing is unlikely to occur.
Factor of Safety	Factor of Safety = 1.20
First Order Second Moment	β = 1.301707, P_c = 0.0965083
Advanced Second Moment	β = 1.301707, P_c = 0.0965083
Direct Simulation	N = 2000, P_c = 0.0988, and $Cov(P_c)$ = 0.067
Conditional Expectation	N = 900, P_c = 0.09647, $Cov(P_c)$ = 0.062

TABLE 1

Stability Characteristics for the Example Ship

θ°	−5	0	10	20	30	40	50	60	70	80
Righting Arm (ft) *GZ*	−0.45	.0	.9	2.0	2.75	3.0	2.7	2.0	1.05	0
Righting Moment (ft-ton) *RM*	6750	.0	13500	30000	41250	45000	40500	30000	15750	.0
Heeling Moment (ft/ton) *HM*	29885	30000	29544	28190	25980	22980	19283	15000	10260	5210

A_1 = 91000 ft-ton, A_2 = 64000 ft-ton

FACTOR OF SAFETY

The mean values were used herein to calculate the factor of safety (FS) as follows:

$$FS = \frac{\mu_{A_1}}{\mu_{A_2}} = \frac{91000}{1.4(54000)} = 1.20 \qquad (37)$$

The factor of safety in this case does not give any indication of the probability of capsizing.

FIRST-ORDER SECOND-MOMENT METHOD

The reliability index, β, in this case is calculated using Equations 13 to 15 as

$$\beta = \frac{\bar{Z}}{\sigma_Z} = \frac{\mu_{A_1} - 1.4\mu_{A_2}}{\sqrt{\sigma_{A_1}^2 + (1.4)^2\,\sigma_{A_2}^2}} = 1.301707 \qquad (38)$$

Then, the probability of capsizing, P_c, is computed using Equation 16 as

$$P_c = 1 - \Phi(\beta) = 0.0965083 \qquad (39)$$

Therefore, the probability of capsizing is 9.7 percent, although both the deterministic and the safety factor approaches show that the ship is safe and stable with a margin of twenty percent.

ADVANCED SECOND MOMENT

The resulting reliability index according to this method is the same as the first-order second-moment method because the performance function is linear and the random variables are normal. The reliability index is shown in Table 3.

SIMULATION METHODS

The results for direct simulation are shown in Figure 9 and Table 3. Figure 9 shows the capsizing probability as a function of simulation cycles. Also shown in the figure is the capsizing probability according to the moment methods. Figure 10 shows the coefficient of variation of the estimated capsizing probability using direct simulation as a function of simulation cycles. Figures 11 and 12 show an estimated capsizing probability and its coefficient of variation as functions of simulation cycles based on conditional expectation. In this example, the capsizing probability can be estimated using a relatively small number of simulation cycles, because the capsizing probability is relatively large and the performance function is relatively simple with normal random variables.

Conclusion

The primary purpose of this paper is to introduce and demonstrate reliability methods in assessing the transverse stability of ships. Different reliability methods were used in computation of the capsizing probability of ships. This effort can be viewed as the first step toward the development of reliability-based design rules for ship stability. Based on this study, several conclusions and recommendations for further research are made.

The absolute stability of a ship can neither be achieved nor guaranteed due to the presence of uncertainties in basic random variables that define the stability of a ship. Therefore, a capsizing probability exists for all ships. Reliability methods should be adopted to assess the transverse stability of ships to account for these uncertainties. This effort can be viewed as the first step toward the ultimate goal of developing reliability-based design rules for ship stability. The basic random variables that appear in design stability calculations need to be probabilistically characterized. Also, any correlation and dependency among them needs to be assessed. This type of information is essential for performing reliability studies of ship stability. The selection of a reliability method for a ship stability study depends on available information and complexity of the performance function. In general, either the

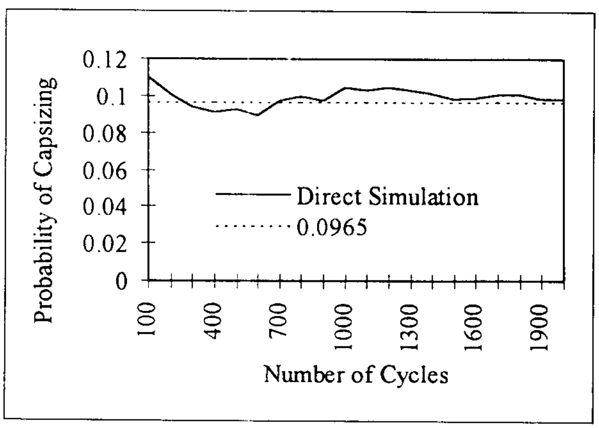

FIGURE 9. Capsizing Probability based on Direct Simulation

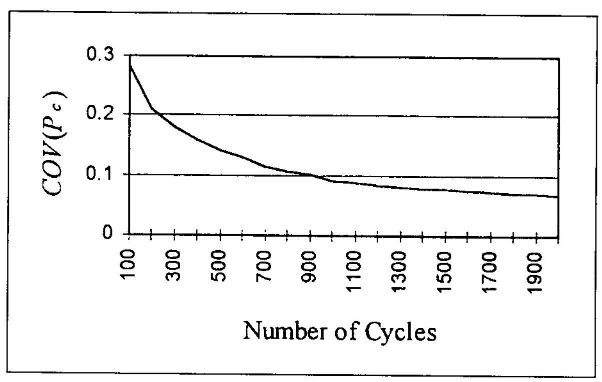

FIGURE 10. Coefficient of Variation of Estimated Capsizing Probability based on Direct Simulation

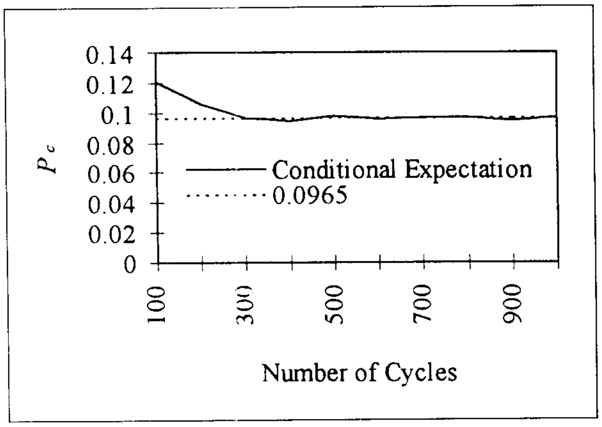

FIGURE 11. Capsizing Probability based on Conditional Expectation

advanced second moment or simulation with variance reduction techniques can be used for this purpose.

The various modes of capsizing need to be clearly defined in order to facilitate reliability studies. Also, operational stability requirements need to be defined in order to achieve a comfort level for passengers and crews. Combinations of the requirements dictated by design rules can result in complex performance functions and complicated calculations. Attention should be paid to the mathematical expressions used in the calculation of capsizing probability. Systems reliability concepts need to be used to address these combinations.

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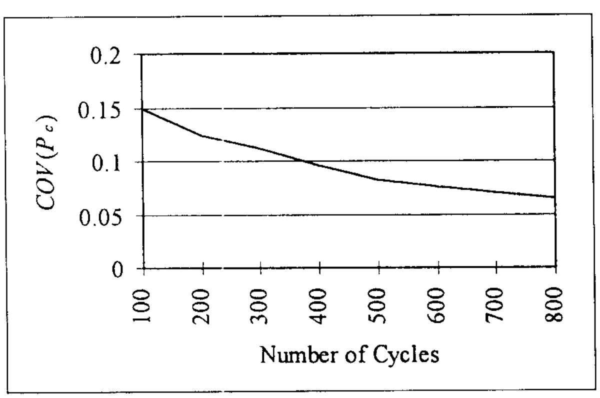

FIGURE 12. Coefficient of Variation of Estimated Capsizing Probability based on Conditional Expectation

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Khaled I. Atua *is a graduate student at the Reliability Engineering Program at the University of Maryland at College Park. He completed his B.S. degree in naval architecture and marine engineering in 1988, and completed his M.S. degree in naval* *architecture and marine engineering in 1992 at Alexandria University. He is also a visiting scholar in the Civil Engineering Department at the University of Maryland at College Park. He is conducting research with Dr. Bilal M. Ayyub in the area of development of reliability-based design of ship structures and systems. He is a member of ASNE and SNAME. He has authored or coauthored several papers and reports.*

Bilal M. Ayyub, Ph.D., P.E. *is a professor of civil engineering at the University of Maryland at College Park. He is also a researcher and consultant in the areas of structural engineering, inspection methods and practices, reliability and risk analysis. Dr. Ayyub completed his B.S. degree in civil engineering in 1980, and completed both his M.S. (1981) and Ph.D. (1983) degrees in civil engineering at the Georgia Institute of Technology. Dr. Ayyub has an extensive background in uncertainty modeling and analysis, risk-based analysis and design, simulation, and marine structural reliability. He is engaged in research work involving structural reliability, marine structures, uncertainty modeling and analysis, and mathematical modeling using the theories of probability, statistics, and fuzzy sets. He has completed several research projects that were funded by the National Science Foundation, the U.S. Coast Guard, the U.S. Navy, the U.S. Department of Defense, the U.S. Army Corps of Engineers, the Maryland State Highway Administration, the American Society of Mechanical Engineers, and several engineering companies. Dr. Ayyub served the engineering community in various capacities through societies that include ASNE, ASCE, ASME, SNAME, IEEE-CS, and NAFIPS. He is a member of the ASME Research Committee of Risk Technology, and the ASME Committee on Human Factors. Currently, he is the chairman of the ASCE Committee on the Reliability of Offshore Structures. He also was the general chairman of the first and second International Symposia on Uncertainty Modeling and Analysis that were held in 1990 and 1993. He is the author and coauthor of about 250 publications in journals, conference proceedings, and reports. His publications include edited books, textbooks, and book chapters. Dr. Ayyub has been awarded ASNE's "Jimmie" Hamilton Award for the best paper published in* Naval Engineers Journal *in 1985 and 1992. Also, he received the ASCE "Outstanding Research Oriented Paper" in the* Journal of Water Resources Planning and Management *for 1987, the ASCE Edmund Friedman Award in 1989, and the NAFIPS K.S. Fu Award for distinguished service. He is a registered professional engineer (PE) with the State of Maryland.*

COMMENTS BY

Professor G.J. White

I would first like to thank the authors for venturing into an area of naval architecture where there has not been a lot of development recently. There has been a sort of tacit general acceptance of the current measures of transverse stability proposed by Sarchin and Goldberg (1962). The problem is that the criteria for determining acceptable transverse stability was empirically based. There has been discussion among practicing naval architects about the validity of the required area ratios and the magnitude of the maximum righting arm. The use of the 100-knot beam wind, how it is applied, and what heeling moment is generated, is an area where there is considerable room for variation in results.

The application of reliability-based methods to the problem of ship transverse stability seems to be an excellent way to provide better measures of ship stability. The authors have done a good job of defining the basic problem and evaluating reliability methods, which will be useful for this area. I understand and appreciate this paper reports on the first efforts by the authors in this area, and I hope they will continue the investigation. When they do, I would like to recommend they consider including the following additional items.

- The authors correctly point out in the beginning of their paper that all of the properties associated with waterplane area are going to vary as the ship rolls and pitches. However, the principle measure of dynamic stability is the righting arm curve, which only depends on the relative positions of the center of gravity (G) and the center of buoyancy (B). A sensitivity analysis of the effects of variation of the positions of B and G with roll angle on righting moment energy would be very useful.
- One of the key items in the authors' flow chart (Figure 6) is the uncertainty analysis. I would expect this to be a very difficult task. I suggest concentrating on the position of the center of gravity as this will have the largest level of uncertainty.
- Of even greater difficulty is the important task of evaluating the uncertainty associated with the limit state formulation. This gets back to the premise that the current criteria does not provide a good measure of stability. Why should the ratio of Area 1 to Area 2 be 1.4? There is an ongoing project being conducted at he U.S. Naval Academy Hydromechanics Laboratory under USCG sponsorship which is using model experiments in wind and waves to look at re-evaluating the Sarchin and Goldberg criteria. This sort of information could provide a basis for establishing the modeling uncertainty.

Again, I would like to thank the authors for taking the first steps in this new application of reliability methods. Their paper has provided a clear description of the problem and has pointed a direction for further research, which can ultimately benefit the entire marine industry.

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AUTHORS' RESPONSE

We gratefully appreciate the valuable discussion of Professor White. His comments and recommendations are insightful and can be used to revise the proposed methodology in future work. In the following, we provide some discussion on his comments.

1). It is true that the principal measure of dynamic stability is the righting arm curve; however, in this paper, we were trying to allocate the different sources of uncertainties in both the center of gravity, G, and the center of buoyancy, B. Since the waterline characteristics and the ship rolls affect the vertical position of the center of buoyancy, the suggested sensitivity analysis by Professor White could be of great value in that domain.

2). The vertical location of the center of gravity, KG, should have the greatest uncertainty, making it eligible for the focus of future study. This could be achieved by the following:

- Building up a databank of different cargo distributions from ship owners for different ship types. The data should also include their estimate of KG for every situation.
- Making use of the available software that calculate stability criteria to produce results based on randomly generated input to the software in the form of Monte Carlo simulation. This simulation process can be used to examine different hazardous situations.

The above-mentioned methods can help develop population distribution for both KB and KG.

3). Considering the complexity of modeling and assessing stability uncertainties, researchers characterized factors affecting ship stability that can be used in future work in this area, such as prediction of steep and high waves in deep water (Myrhaug and Kjeldsen 1987), probability of capsizing in steep and high waves (Dahle et al 1988), developing a two-dimensional Weibull distribution for ship roll (Myrhaug et al 1996), probability of ship capsize in breaking waves (Myrhaug and Dahle 1994), and developing a joint distribution of successive wave periods (Myrhaug and Rue 1993).

4). The example calculations performed in the limit state were based on $A_1 \geq 1.4A_2$ and are cited only for the sake of illustration; however, reformulation of the limit state is encouraged to put it in the form of supply and demand factor design, i.e., $\phi A_1 \geq \gamma A_2$ where ϕ and γ are partial safety factors.

5). The limit state can be defined either as shown in item 4 above, or based on basic random variables at a lower level as shown in the stability criteria breakdown in Figure 1A.

We would like to thank Professor White for his valuable suggestions. The use of reliability methods in developing design criteria for transverse stability is expected to produce future rules in this area, giving them the same basis as the anticipated design rules of structural systems that are currently being developed.

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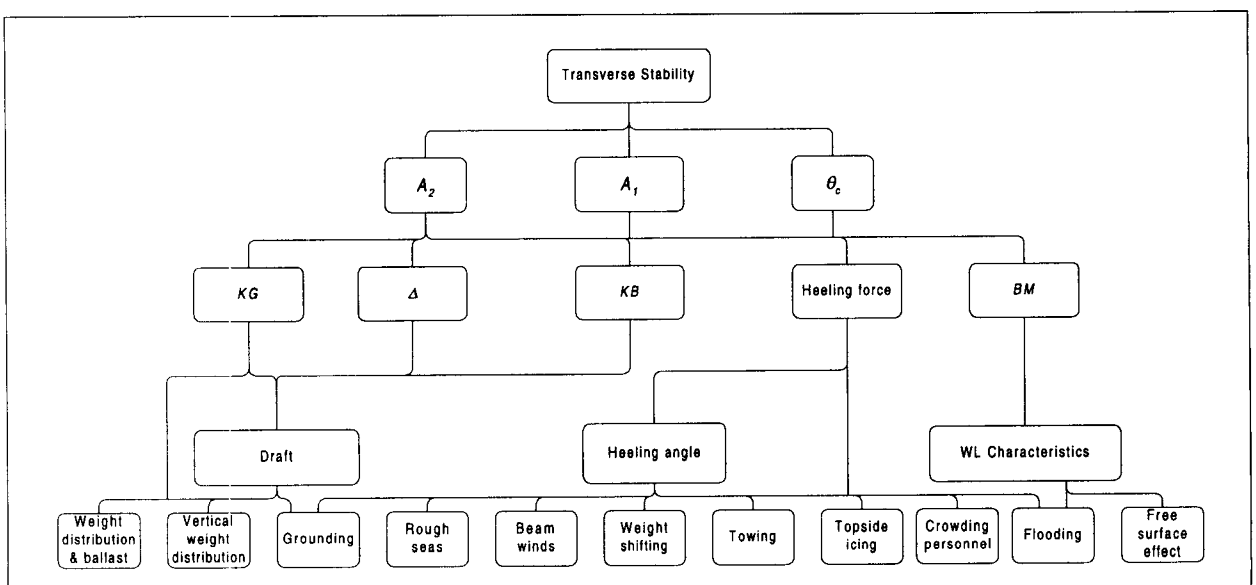

FIGURE 1A. Basic Random Variables for Stability Criteria