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From dissecting ignorance to solving algebraic problems

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Abstract

Engineers and scientists are increasingly required to design, test, and validate new complex systems in simulation environments and/or with limited experimental results due to international and/or budgetary restrictions. Dealing with complex systems requires assessing knowledge and information by critically evaluating them in terms relevance, completeness, non-distortion, coherence, and other key measures. Using the concepts and definitions from evolutionary knowledge and epistemology, ignorance is examined and classified in the paper. Two ignorance states for a knowledge agent are identified: (1) *non-reflective* (or *blind*) *state*, i.e. the person does not know of self-ignorance, a case of ignorance of ignorance; and (2) *reflective state*, i.e. the person knows and recognizes self-ignorance. Ignorance can be viewed to have a hierarchal classification based on its sources and nature as provided in the paper. The paper also explores limits on knowledge construction, closed and open world assumptions, and fundamentals of evidential reasoning using belief revision and diagnostics within the framework of ignorance analysis for knowledge construction. The paper also examines an algebraic problem set as identified by Sandia National Laboratories to be a basic building block for uncertainty propagation in computational mechanics. Solution algorithms are provided for the problem set for various assumptions about the state of knowledge about its parameters.

Keywords: Diagnostics; Evidence; Ignorance; Information; Knowledge; Model; Opinion; Prediction; Uncertainty

1. Introduction

Engineers and scientists are increasingly required to design, test, and validate new complex systems in simulation environments and/or with limited experimental results due to international and/or budgetary restrictions. Examples include space missions, certifications of missile stockpiles, and economic forecasting. These new trends require new practices of rigorous analyses of knowledge, information, uncertainty, and ignorance. This paper deals with knowledge construction by emphasizing both available information and ignorance. Knowledge can be constructed based on ignorance analysis. Ignorance analysis for knowledge construction ensures that our models and simulations do not assume and utilize implicitly or blindly more information than what is available, and accounts for confusion and conflict in available information.

This paper proposes a breakdown of ignorance within a knowledge philosophical framework. The breakdown provides a meaningful context for modeling and analyzing complex systems. This ignorance construct is discussed, and some suitable analytical methods for modeling ignorance components are briefly described in this paper.

The paper also examines an algebraic problem set as identified by Sandia National Laboratories to be a basic building block for uncertainty propagation in computational mechanics [15]. Solution algorithms are provided for the problem set for various assumptions about the state of knowledge about its parameters.

2. Knowledge and ignorance

Systems of the future will require engineers to design them, test their performances, and assess their robustness and vulnerability in a simulated environment. Simulation requires validated building blocks for materials behavior, physical laws, environment–system interaction, unit performance, and system performance. At every level and stage of the simulation process, verification and validation are needed that should include ignorance analysis, and uncertainty analysis and modeling. Example systems include our nuclear weapon stockpile, space stations, satellites, space missions, etc. The processes of qualification, verification and validation are shown in Fig. 1. The verification process

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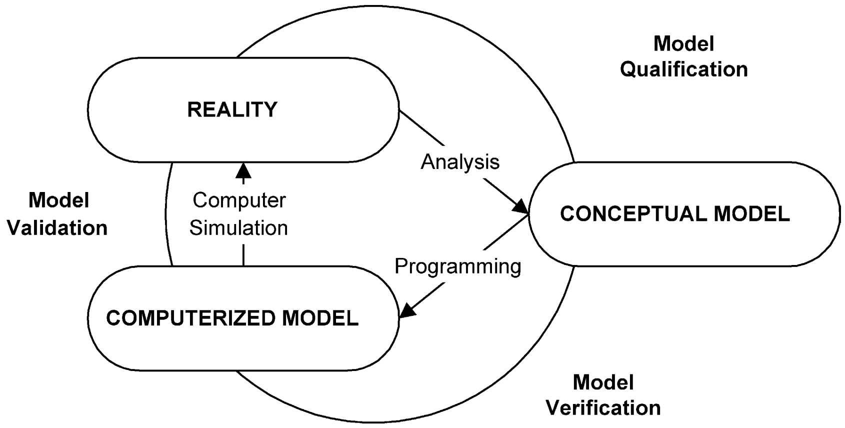


Fig. 1. Qualification, verification and validation [2].

consists of three stages: conceptual model verification, design verification, and code verification. The verification can be done by comparison and test of agreement between the computational model and solution, and results from benchmark (analytical or very accurate numerical solutions) of simplified model problems. The validation consists of two stages: conceptual model validation, and results validation that can be done by expert opinion solicitation [3].

Generally, engineers and scientists, and even almost most humans, tend to focus on what is known and not on the unknowns. Even the English language lends itself for this emphasis. For example, we can easily state that Expert A *informed* Expert B, whereas we cannot *directly* state the contrary. We can only state it by using the negation of the earlier statement in the form of "Expert A *did not inform* Expert B". Statements such as "Expert A *misinformed* Expert B", or "Expert A *ignored* Expert B" do not convey the same (intended) meaning. Another example is "John *knows* David", for which a meaningful *direct* contrary statement does not exist. The emphasis on knowledge and not on ignorance can also be noted in sociology by having a field of study called the *sociology of knowledge* and not having *sociology of ignorance*, although Weinstein and Weinstein [27] introduced the *sociology of non-knowledge*, and Smithson [23] introduced the *theory of ignorance* only in the last two decades.

Engineers and scientists tend to emphasize knowledge and information, and sometimes intentionally or unintentionally brush aside ignorance. In addition, information (or knowledge) can be misleading in some situations because it does not have the truth content that was assigned to it leading potentially to overconfidence. In general, knowledge and ignorance can be classified as shown in Fig. 2 using squares with crisp boundaries for the purpose of illustration. The shapes and boundaries can be made multi-dimensional, irregular and/or fuzzy. The evolutionary

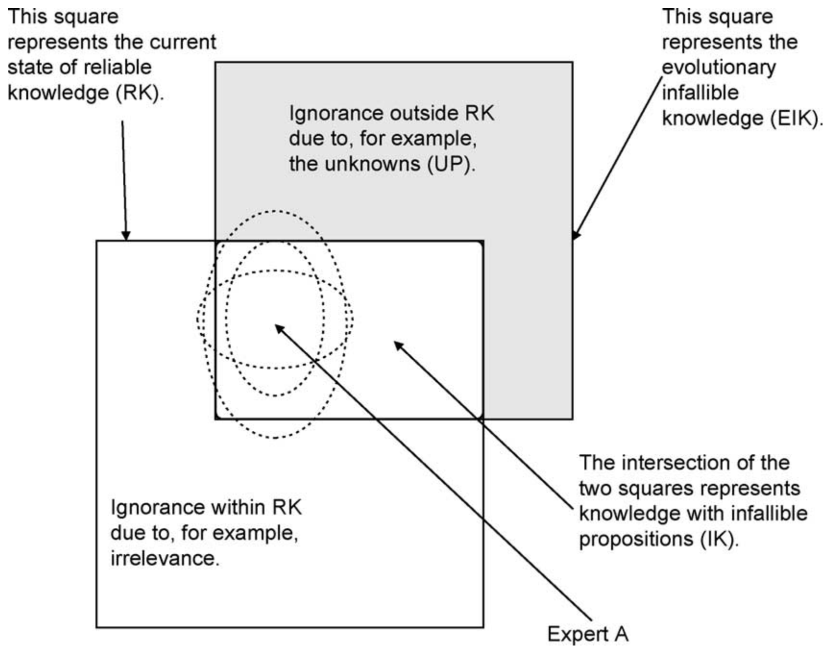


Fig. 2. Human knowledge and ignorance [3].

infallible knowledge (EIK) about a system is shown as the top-right square in the figure, and can be intrinsically unattainable due to the fallacy of humans and the evolutionary nature of knowledge. The state of reliable knowledge (RK) is shown using another square, i.e. the bottom left square, for illustration purpose. The RK represents the present state of knowledge in an evolutionary process, i.e. a snapshot of knowledge as a set of know-how, object and prepositions that meet justifiable true beliefs (JTB) within reasonable reliability levels. At any stage of human knowledge development, this knowledge base about the system is a mixture of truth and fallacy. The intersection of EIK and RK represents the knowledge base with the infallible knowledge (IK) components (i.e. know-how, objects, and propositions). Therefore, the following relationship can be stated using the notations of set theory as:

$$\text{Infallible Knowledge (IK)} = (\text{EIK}) \cap (\text{RK}) \tag{1}$$

where $\cap$ means intersection. IK is defined as knowledge that can survive the dialectic processes developing among humans and in societies, and passes the test of time and use. It constitutes *known possible* propositions. This IK can be schematically defined by the intersection of these two squares of EIK and RK. Based on this representation, two primary types of ignorance can be identified: (1) ignorance within the knowledge base RK due to factors such as irrelevance, and (2) ignorance outside the knowledge base due to unknown or unknowable objects, interactions, laws, dynamics, and know-how.

An *Expert A* of some knowledge about a system of interest can be represented as shown in Fig. 2 using ellipses for illustrative purposes. Three types of ellipses can be identified, shown concentric in the figure but can be eccentric: (1) a subset of the EIK that the expert has learned, captured and/or created, (2) self-perceived knowledge by the expert, and (3) perception by others of the expert's knowledge. The EIK of the expert might be smaller than the knowledge self-perceived by the expert, and the difference between the two types is a measure of overconfidence that can be partially related to the expert's ego. Ideally, the three ellipses should be the same, but commonly they are not. They are greatly affected by communication skills of experts and their successes in dialectic processes that with time might lead to evolutionary knowledge marginal advances or quantum leaps; hence the importance of language and linguistics as the primary medium of knowledge archival for humans. Also, their relative sizes and positions within the IK base are unknown. It can be noted from Fig. 2 that the expert's knowledge can extend beyond the RK base into the EIK area as a result of creativity and imagination of the expert. Therefore, the intersection of the expert's knowledge with the ignorance space outside the knowledge base can be viewed as a measure of creativity and imagination. Another expert (i.e. Expert B) would have her/his own ellipses that might overlap with the ellipses of Expert A, and might overlap with other regions by varying magnitudes. Knowledge sources can be identified as shown in Fig. 3.

Using the concepts and definitions from evolutionary knowledge and epistemology, ignorance is classified based on knowledge sources and by sources as described in detail in Section 3. This classification is needed in order to understand and define the limits of our knowledge about a problem, and to appropriately use applicable modeling theory of ignorance.

3. Classification of ignorance

The state of *ignorance* for a person or society can be unintentional or deliberate due to an erroneous cognition state and not knowing relevant information, or ignoring information and deliberate inattention to something for various reasons such as limited resources or cultural opposition, respectively. The latter type is a state of *conscious ignorance* which is not intentional, and once recognized evolutionary species try to correct for that state for survival reasons with varying levels of success. The former ignorance type belongs to the *blind ignorance* category. Therefore, ignoring means that someone can either *unconsciously* or *deliberately* refuse to acknowledge or regard, or leave out an account or consideration for relevant information [7]. These two states should be treated in developing a hierarchal breakdown of ignorance.

Using the concepts and definitions from evolutionary knowledge and epistemology, ignorance can be classified based on the three knowledge sources as follows:

- *Know-how ignorance*: It can be related to the lack of, or having erroneous know-how knowledge. Know-how knowledge requires someone to know how to do a specific activity, function, procedure, etc. such as, riding a bicycle.
- *Object ignorance*: It can be related to the lack of, or having erroneous object knowledge. Object knowledge is based on a direct acquaintance with a person, place or thing, for example, Mr Smith knows the President of the United States.
- *Propositional ignorance*: It can be related to the lack of, or having erroneous propositional knowledge. Propositional knowledge is based on propositions that can be either true or false, for example, Mr Smith knows that the Rockies are in North America.

The above three ignorance types can be cross-classified against two possible states for a knowledge agent, such as a person, of being aware or unaware of their state of ignorance. These two states are

- *Non-reflective* (or *blind*) *state*: The person does not know of self-ignorance, a case of ignorance of ignorance.

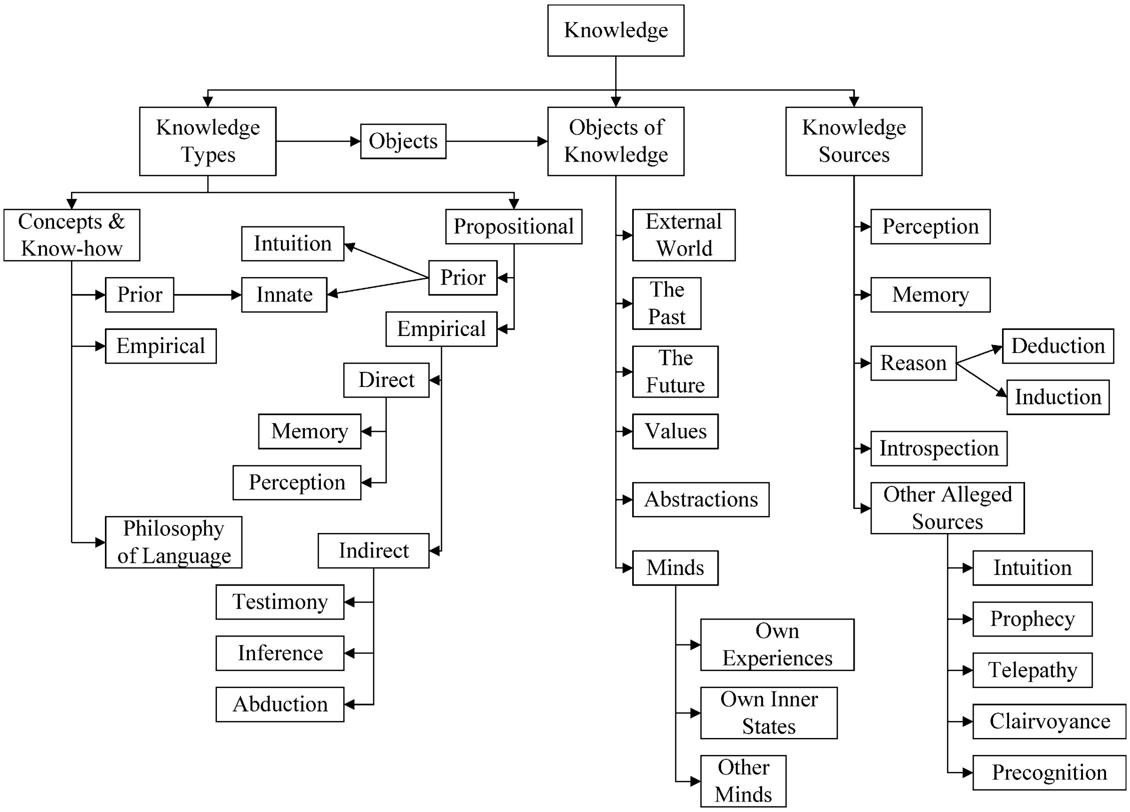


Fig. 3. Knowledge types, objects, and sources [3].

- *Reflective state*: The person knows and recognizes self-ignorance. Smithson (1985) termed this type of ignorance *conscious ignorance*, and the blind ignorance was termed *meta-ignorance*. As a result, in some cases the person might formulate a proposition but still be ignorant of the existence of a proof or disproof. A knowledge agent's response to reflective ignorance can be either passive acceptance or a guided attempt to remedy one's ignorance that can lead four possible outcomes: (1) a successful remedy that is recognized by the knowledge agent to be a success leading to fulfillment, (2) a successful remedy that is not recognized by the knowledge agent to be a success leading to searching for a new remedy, (3) a failed remedy that is recognized by the knowledge agent to be a failure leading to searching for a new remedy, and (4) a failed remedy that is recognized by the knowledge agent to be a success leading to blind ignorance, such as *ignoratio elenchi* (i.e. ignorance of refutation or missing the point or irrelevant conclusion).

The reflective state has a survival value to evolutionary species; otherwise it can be argued that it never would have flourished [6]. Ignorance emerges as a lack of knowledge relative to a particular perspective from which such gaps emerge. Accordingly, the accumulation of beliefs and the emergence of ignorance constitute a dynamic process resulting in old ideas perishing and new ones flourishing [4]. According to Bouissac [4], the process of scientific discovery can be metaphorically described as not only a cumulative sum (positivism) of beliefs, but also an activity geared towards relentless construction of ignorance (negativism), producing architecture of holes, gaps, and *lacunae* so to speak.

Hallden [11] examined the concept of evolutionary ignorance in decision theoretic terms. He introduced the notion of gambling to deal with blind ignorance or lack of knowledge according to which there are times when, in lacking knowledge, gambles must to be taken. Sometimes gambles pay off with success, i.e. continued survival, and sometimes they do not lead to sickness or death.

According to evolutionary epistemology, ignorance has factitious, i.e. human-made, perspectives. Smithson [24] provided a working definition of ignorance based on "Expert A is ignorant from B's viewpoint, if A fails to agree with or show awareness of ideas which B defines as actually or potentially valid". This definition allows for self-attributed ignorance, and either Expert A or B can be attributer or perpetrator of ignorance. Our ignorance and claimed knowledge depend on our current historical setting which is relative to various natural and cultural factors such as language, logical systems, technologies and standards which have developed and evolved over time. Therefore, humans evolved from blind ignorance through gambles to a state of incomplete knowledge with reflective ignorance recognized through factitious perspectives.

In many scientific fields, the level of reflective ignorance becomes larger as the level of knowledge increases. Duncan and Weston-Smith [9] stated in the *Encyclopedia of Ignorance* that compared to our bond of knowledge, our ignorance remains Atlantic. They invited scientists to state what they would like to know in their respective fields, and noted that the more eminent they were the more readily and generously they described their ignorance. Clearly, before solving a problem, it needs to be articulated.

Ignorance can be viewed to have a hierarchal classification based on its sources and nature as shown in Fig. 3 with the brief definitions provided in Table 1. Ignorance can be classified into two types, blind ignorance (also called meta-ignorance), and conscious ignorance (also called reflective ignorance). Blind ignorance includes *not knowing* relevant know-how, objects-related information, and relevant propositions that can be justified. The unknowable knowledge can be defined as knowledge that cannot be attained by humans based on current evolutionary progressions, or cannot be attained at all due to human limitations, or can only be attained through quantum leaps by humans. Blind ignorance also includes irrelevant knowledge that can be of two types: (1) relevant knowledge that is dismissed as irrelevant or ignored, and (2) irrelevant knowledge that is believed to be relevant through non-reliable or weak justification or as a result of ignoratio elenchi. The irrelevance type can be due to untopicality, taboo, and undecidability. Untopicality can be attributed to intuitions of experts that could not be negotiated with others in terms of cognitive relevance. Taboo is due to socially reinforced irrelevance. Issues that people must not know, deal with, inquire about, or investigate define the domain of taboo. The undecidability type deals with issues that cannot be designated true or false because they are considered insoluble, or solutions that are not verifiable, or as a result of ignoratio elenchi. A third component of blind ignorance is fallacy that can be defined as erroneous beliefs due to misleading notions.

Kurt Gödel (1906–1978) showed that a logical system could not be both consistent and complete; and could not prove itself complete without proving itself inconsistent and vise versa. Also, he showed that there are problems that cannot be solved by any set of rules or procedures; instead for these problems one must always extend the set of

Table 1
Taxonomy of ignorance [3]

Term	Meaning
1. Blind ignorance	Ignorance of self-ignorance or called meta-ignorance
1.1. Unknowable	Knowledge that cannot be attained by humans based on current evolutionary progressions, or cannot be attained at all due to human limitations, or can only be attained through quantum leaps by humans
1.2. Irrelevance	Ignoring something
1.2.1. Untopicality	Intuitions of experts that could not be negotiated with others in terms of cognitive relevance
1.2.2. Taboo	Socially reinforced irrelevance. Issues that people must not know, deal with, inquire about, or investigate
1.2.3. Undecidability	Issues that cannot be designated true or false because they are considered insoluble, or solutions that are not verifiable, or ignoratio elenchi
1.3. Fallacy	Erroneous beliefs due to misleading notions
2. Conscious ignorance	A recognized self-ignorance through reflection
2.1. Inconsistency	Inconsistency in knowledge can be attributed to distorted information as a result of inaccuracy, conflict, contradiction, and/or confusion
2.1.1. Confusion	Wrongful substitutions
2.1.2. Conflict	Conflicting or contradictory assignments or substitutions
2.1.3. Inaccuracy	Bias and distortion in degree
2.2. Incompleteness	Incomplete knowledge due to absence or uncertainty
2.2.1. Absence	Incompleteness in kind
2.2.2. Unknowns	The difference between the *becoming* knowledge state and *current* knowledge state
2.2.3. Uncertainty	Knowledge incompleteness due to inherent deficiencies with acquired knowledge
2.2.3.1. Ambiguity	The possibility of having multi-outcomes for processes or systems
(a) Unspecificity	Outcomes or assignments that are not completely defined
(b) Non-specificity	Outcomes or assignments that are improperly defined
2.2.3.2. Approximations	A process that involves the use of vague semantics in language, approximate reasoning, and dealing with complexity by emphasizing relevance
(a) Vagueness	Non-crispness of belonging and non-belonging of elements to a set or a notion of interest
(b) Coarseness	Approximating a crisp set by subsets of an underlying partition of the set's universe that would bound the set of interest
(c) Simplifications	Assumptions needed to make problems and solutions tractable
2.2.3.3. Likelihood	Defined by its components of randomness, statistical and modeling
(a) Randomness	Non-predictability of outcomes
(b) Sampling	Samples versus populations

axioms. This philosophical view of logic can be used as a basis for classifying the conscious ignorance into two primary branches of *inconsistency* and *incompleteness*.

Inconsistency in knowledge can be attributed to distorted information as a result of inaccuracy, conflict, contradiction, and/or confusion as shown in Fig. 4. Inconsistency can result from assignments and substitutions that are wrong, conflicting or biased producing confusion, conflict or inaccuracy, respectively. The confusion and conflict results from an in-kind inconsistent assignments and substitutions; whereas inaccuracy results from a level bias or error in these assignments and substitutions.

Incompleteness is defined as incomplete knowledge, and can be considered to consist of (1) absence and unknowns as incompleteness in kind, and (2) uncertainty. The unknowns or unknown knowledge can be viewed in evolutionary epistemology as the difference between the *becoming* knowledge state and *current* knowledge state. The knowledge absence component can lead to one of the scenarios: (1) no action and working without the knowledge, (2) unintentionally acquiring irrelevant knowledge leading to blind ignorance, (3) acquiring relevant knowledge that can be with various uncertainties and levels. The fourth possible scenario of deliberately acquiring irrelevant knowledge is not listed since it is not realistic.

Uncertainty can be defined as knowledge incompleteness due to inherent deficiencies with acquired knowledge. Uncertainty can be classified based on its sources into three types: ambiguity, approximations, and likelihood. The ambiguity comes from the possibility of having multi-outcomes for processes or systems. Recognition some of the possible outcomes creates uncertainty. The recognized outcomes might constitute only a partial list of all possible outcomes leading to unspecificity. In this context, unspecificity results from outcomes or assignments that are not completely defined. The incorrect definition of outcomes, i.e. errors in defining outcomes, can be called non-specificity. In this context, non-specificity results from outcomes or assignments that are improperly defined. The unspecificity is a form of knowledge absence and can be treated similar to the absence category under incompleteness. The non-specificity can be viewed as a state of blind ignorance.

The human mind has the ability to perform approximations through reduction and generalizations, i.e. induction and deduction, respectively, in developing knowledge. The process of approximation can involve the use of vague semantics in language, approximate reasoning, and dealing with complexity by emphasizing relevance. Approximations can be viewed to include vagueness, coarseness and simplification. Vagueness results from the non-crisp nature of belonging and non-belonging of elements to a set or a notion of interest; whereas coarseness results from approximating a crisp set by subsets of an underlying partition of the set's universe that would bound the crisp set of interest. Simplifications are assumptions made to make problems and solutions tractable. The likelihood can be defined in the context of chance, odds and gambling. Likelihood has primary components of randomness and sampling. Randomness stems from the non-predictability of outcomes. Engineers and scientists commonly use samples to characterize populations, hence the last type. The ignorance hierarchy of Fig. 4 shows that our knowledge shall be limited by our conscious ignorance and bounded by our blind ignorance.

4. Construction of knowledge

Decision situations commonly require constructing knowledge from information. Knowledge construction starts with data collection and information gathering that

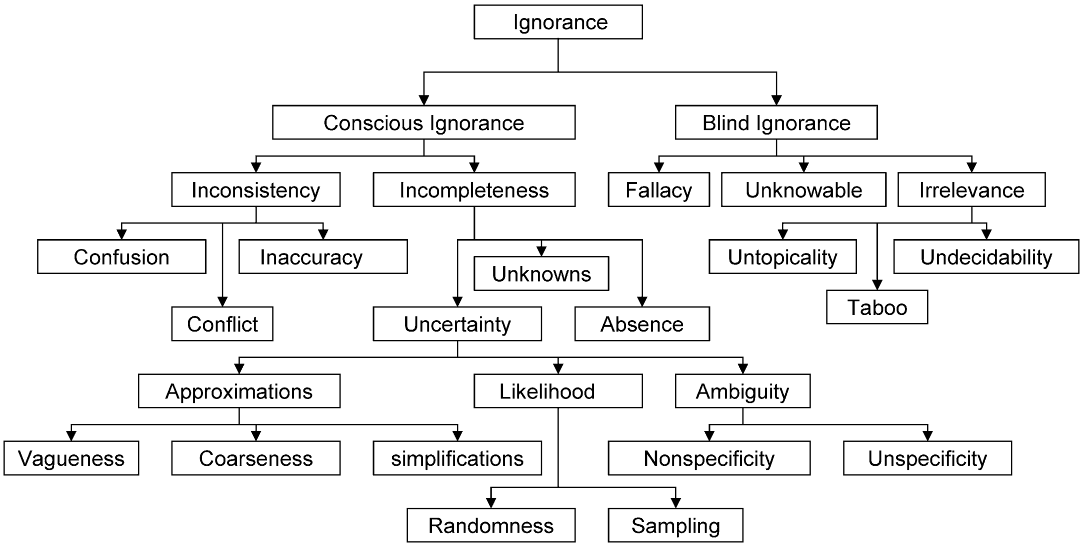


Fig. 4. Ignorance hierarchy [3].

can include various sources and formats as identified in Fig. 3. Ignorance types need to be identified, and their levels should be assessed in order to quantify and qualify their contribution to modeling the decision situation. Klir and Wierman [13] provide analytical methods that can be used for modeling various ignorance types and assessing their magnitudes or levels. Also, uncertainty measures are provided to assess magnitudes or levels of uncertainty. These uncertainty measures can be defined to be non-negative real numbers, and should be inversely proportional to the strength and consistency in evidence as expressed in the theory employed, i.e. the stronger and more consistent the evidence, the smaller the amount of uncertainty [1,3,13]. Such uncertainty measures can be constructed to assess collected information, such as opinions rendered by one expert on some issue of interest, or opinions rendered by several experts on the same issue, or collected data and information.

Ignorance models, uncertainty measures and data collected can be entered into a systematic process to construct knowledge. This knowledge-construction process must have a dialectic nature as schematically demonstrated in Fig. 5 [3]. Information can be defined as sensed objects, things, places, processes, and information and knowledge communicated by language and multimedia. Information can be viewed as a pre-processed input to our intellect system of cognition, and knowledge acquisition and creation. Information can lead to knowledge through investigation, study, and reflection. However, knowledge and information about the system might not constitute the eventual evolutionary knowledge state about the system as a result of not meeting the justification condition in JTB or the ongoing evolutionary process or both. Knowledge is defined in the context of the humankind, evolution, language and communication methods, and social and economic dialectic processes; and cannot be removed from them. As a result, knowledge would always reflect the imperfect and evolutionary nature of humans that can be attributed to their reliance on their senses for information acquisition; their dialectic processes; and their mind for extrapolation, creativity, reflection and imagination with associated biases as a result of preconceived notions due to time asymmetry, specialization and other factors. An important dimension in defining the state of knowledge and truth about a system is non-knowledge or ignorance.

Opinions rendered by experts, that are based on information and exiting knowledge, can be defined as preliminary propositions with claims that are not fully justified or justified with adequate reliability but are not necessarily infallible. Expert opinions are seeds of propositional knowledge that do not meet one or more of the conditions required for the JTB with the reliability theory of knowledge. They are valuable as they might lead to knowledge expansion, but decisions made based on them sometimes might be risky propositions since their preliminary nature might lead to proving them false by others or in the future.

The relationships among knowledge, information, opinions, and evolutionary epistemology are schematically shown in Fig. 5. The dialectic processes include communication methods such as languages, visual and audio formats, and other forms. Also, they include economic, class, schools of thought, political and social dialectic processes within peers, groups, colonies, societies, and the world.

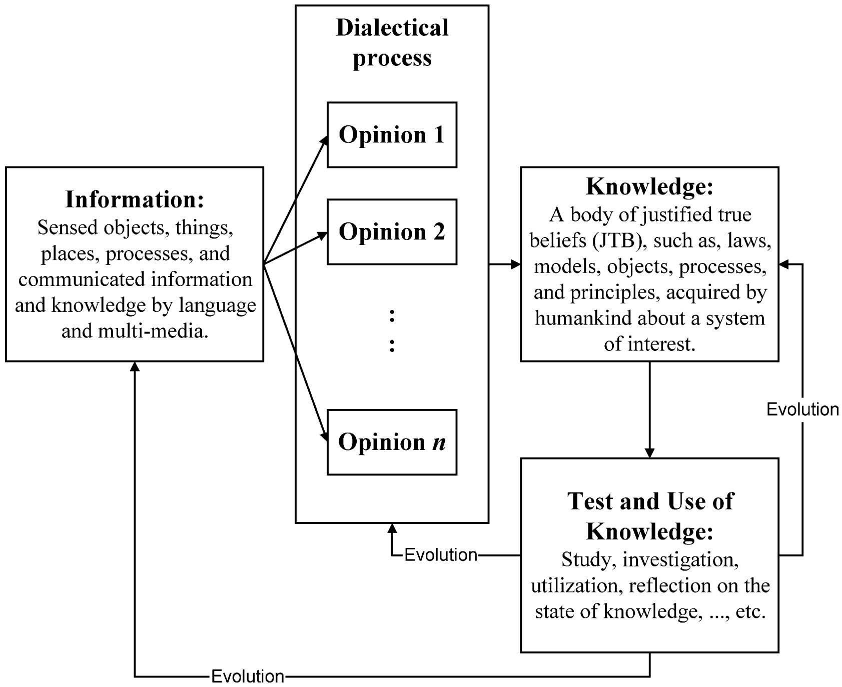


Fig. 5. Knowledge, information, opinions, and evolutionary epistemology [3].

4.1. Limits on knowledge construction

Complex decision situations can challenge human ability to construct knowledge from information. Humans as complex, intelligent systems have the ability to anticipate the future, and learn and adapt in ways that are not yet fully understood. Engineers and scientists, who study or design systems, have to deal with complexity more often than ever; hence the interest in the field of complexity. The study of complexity led to developing theories, such as chaos and catastrophe theories. Even if complexity theories would not produce solutions to problems, they can still help us to understand complex systems and perhaps direct experimental studies. Theory and experiment go hand in glove, therefore providing opportunities to make major contributions.

Complexity can be classified into two broad categories [25]: (1) complexity with structure, (2) complexity without structure. The complexity with structure was termed *organized complexity* [26]. Organized complexity can be observed in a system that involve non-linear differential equations with a lot of interactions among a large number of components and variables that define the system, such as in life, behavioral, social and environmental sciences. Such systems are usually non-deterministic in their nature. Problem solutions related to such models of organized complexity tend to converge to statistically meaningful averages [13]. Advancements in computer technology and numerical methods have enhanced our ability to obtain such solutions effectively and inexpensively. As a result, engineers design complex systems in simulated environments and operations, such as a space mission to a distant planet, and scientists can conduct numerical experiments involving, for example, nuclear blasts. In the area of simulation-based design, engineers are using parallel computing and physics-based modeling to simulate fire propagation in engineering systems, or the turbulent flow of a jet engine using molecular motion and modeling. These computer and numerical advancement are not limitless, as the increasing computational requirements lead to what is termed *transcomputational problems* capped by the *Bremermann's limit* [5]. The nature of such transcomputational problems is studied by the theory of computational complexity [10]. The Bremermann's limit was estimated based on quantum theory using the following proposition [5]:

> "No data processing systems, whether artificial or living, can process more than 2×10^{47} bits per second per gram of its mass,"where data processing is defined as transmitting bits over one or several of a system's channels. Klir and Folger [12] provide additional information on the theoretical basis for this proposition showing that the maximum processing value to be 1.36×10^{47} *bits per second per gram of its mass*. Considering a hypothetical computer that has the entire mass of the Earth operating for a time period equals to an estimated age of the Earth, i.e. 6×10^{27} g and 10^{10} years, respectively, with each year containing 3.15×10^{7} s, this imaginary computer would be able to process 2.57×10^{92} bits, or rounded to the nearest power of ten, 10^{93} bits, defining the Bremermann's limit. Many scientific and engineering problems defined with a lot of details can exceed this limit. Klir and Folger [12] provide the examples of pattern recognition and human vision that can easily reach transcomputational levels. In pattern recognition, consider a square $q \times q$ spatial array defining $n = q^2$ cells that partition the recognition space. Pattern recognition often involves color. Using k colors, as an example, the number of possible color patterns within the space is k^n. In order to stay within the Bremermann's limit, the following inequality must be met:
>
> $$k^n \leq 10^{93} \tag{2a}$$
>
> Fig. 6 shows a plot of this inequality for values of $k = 2\text{–}10$ colors. For example using only two colors, a transcomputational state is reached at $q > 18$ colors. These computations in pattern recognition can be directly related to human vision and the complexity associated with processing information by the retina of a human eye. If we consider a retina of about one million cells with each cell having only two states of *active* and *inactive* in recognizing an object, modeling the retina in its entirety would require the processing of
>
> $$2^{1,000,000} = 10^{300} \tag{2b}$$
>
> bits of information, far beyond the Bremermann's limit [12].

Generally an engineering system needs to be modeled with a portion of its environment that interact significantly with it in order to assess some system attributes of interest. The level of interaction with the environment can only be subjectively assessed. By increasing the size of the environment and level of details in a model of the system, the complexity of the system model increases, possibly in a manner that does not have a recognizable or observable structure. This complexity without structure is more difficult to model and deal with in engineering and sciences. By increasing the complexity of the system model, our

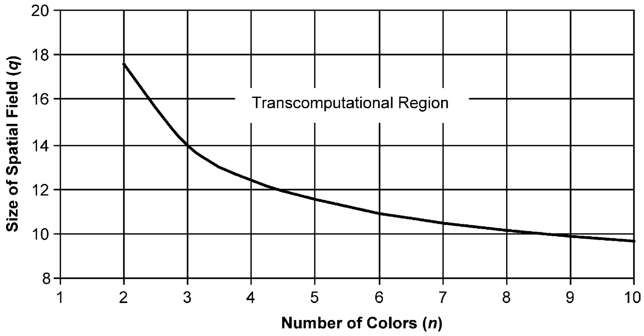


Fig. 6. The Bremermann's limit for pattern recognition.

ability to make *relevant* assessments of the system's attributes can diminish. Therefore, there is a tradeoff between relevance and precision in system modeling in this case. Our goal should be to model a system with a sufficient level of detail that can result into sufficient precision and can lean to relevant decisions in order meet the objective of the system assessment.

Living systems show signs of these tradeoffs between precision and relevance in order to deal with complexity. The survival instincts of living systems have evolved, and manifest themselves as processes to cope with complexity and information overload. The ability of a living system to make relevant assessments diminishes with the increase in information input [14]. Living systems commonly need to process information in a continuous manner in order to survive. For example, a fish needs to process visual information constantly in order to avoid being eaten by another fish. When a school of larger fish rushes towards the fish, presenting it with images of threats and dangers, the fish might not be able to process all the information and images, and becomes confused. Considering the information processing capabilities of living systems as input–output black boxes, the input and output to such systems can be measured and plotted in order to examine such relationships and any non-linear characteristics that they might exhibit. These relationships for living systems can be described using the following hypothesis that was analytically modeled and experimentally validated [14]:

> "As the information input to a single channel of a living system—measured in bits per second—increases, the information output—measured similarly—increases almost identically at first but gradually falls behind as it approaches a certain output rate, the channel capacity, which cannot be exceeded. The output then levels off at that rate, and finally, as the information input rate continues to go up, the output decreases gradually towards zero as breakdown or the confusion state occurs under overload."

The above hypothesis was used to construct families of curves to represent the effects of information input overload as shown schematically in Fig. 7. Once the input overload is removed, most living systems recover instantly from the overload and the process is completely reversible; however, if the energy level of the input is much larger than the channel capacity, a living system might not fully recover from this input overload. Living systems also adjust the way they process information in order to deal with an information input overload using one or more of the following processes by varying degrees depending on the level of a living system in terms of complexity: (1) omission by failing to transmit information, (2) error by transmitting information incorrectly, (3) queuing by delaying transmission, (4) filtering by giving priority in processing, (5) abstracting by processing messages with less than complete details, (6) multiple channel processing by simultaneously transmitting messages over several parallel channels, (7) escape by acting to cut off information input, and (8) chunking by transformation information in meaningful chunks. These actions can also be viewed as simplification means to cope with complexity and/or an information input overload.

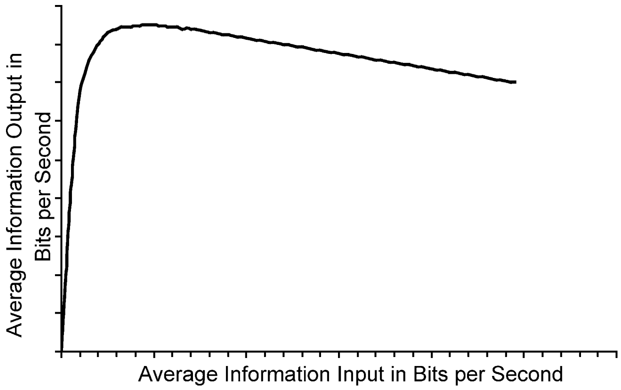


Fig. 7. A schematic relationship of input and output information transmission rates for living systems.

4.2. Closed-world versus open-world assumption

The simulated performance of a system depends heavily upon the information available at hand about the problem under consideration. Complete information is difficult to come by, and is generally not available even for simple applications. For instance, database systems use the *closed world assumptions* and introduce *null values* to deal with incomplete information. In general, an intelligent system must be able to make plausible propositions that may turn out to be incorrect when more information becomes available. The transferable belief model (TBM) provides a basis for a class of methods for making such propositions when faced with incomplete information.

The TBM is a non-probabilistic approach that derives from the Dempster–Shafers mathematical theory of evidence [18]. It is a means for representing quantified degrees of belief. Degrees of belief are obtained from agents providing evidence at a given time within a given frame of discernment. The method is capable of treating inconsistency in data by introducing the 'open-world' assumption. In TBM, a set of all propositions consists of the three subsets: (1) a set of propositions known as possible (PP), (2) a set of propositions known as impossible (IP), (3) and a set of unknown propositions (UP). The content of the subsets depends not only on the given problem, but also on the evidence, which is available at a given time. As evidence becomes available, propositions are redistributed between the three sets as shown in Fig. 2.

The *closed-world assumption* postulates an empty UP set. The *open-world assumption* admits the existence of a non-empty

UP set, and the fact that the truth might be in UP. In this assumption, *unknown* referred to *none of the known propositions*.

The UP set can be considered to be empty where the truth is necessarily in the PP set, and the Ω power set of 2^{Ω} is PP. The selection of the type of the world depends on the problem at hand. The closed-world assumption can be selected for a quality condition problem where the condition cannot be other than one or more of the ratings, e.g. poor, poor or good, etc. For diagnosing the degradation underlying causes, the open-world assumption can be a suitable selection since an analyst trying to solve the problem cannot always consider all the possibilities, i.e. one or more underlying causes might exist that are not known to the analyst.

The degree of conflict between two or more evidence sources, k, implies the existence of a proposition not defined in the frame of discernment. In the idealized closed-world assumption that amount of conflict is redistributed among the known propositions. In the open-world assumption, the degree of conflict corresponds to the amount of belief allocated to the proposition that *none of the known* propositions has the truth. One must keep in mind that the actual underlying physics might be something else other than the causes considered, i.e. the solution is in the set $\Theta = \mathrm{UP}$ and not in the set $\Omega = \mathrm{PP}$.

4.3. Open-world assumption mathematical framework

The known possible propositions set PP is based on Ω, a finite set of elementary propositions. The set ϕ is defined as the null or impossible event. In the Dempster–Shafer's framework, the mass of the null set, $m(\phi)$ is defined as zero when belief functions are normalized and correspondingly $\mathrm{Bel}(\Omega) = 1$. In contrast, under an open-world assumption, the mass of the null set may be non-zero if the frame of discernment Ω does not contain the truth [19, 22]. A set of focal elements can be defined based on $m(A) > 0$ as follows:

$$F(m) = \{A \subseteq \Omega | m(A) > 0\} \tag{3}$$

The elements of $F(m)$ are called the *focal elements* of m. Shafer [18] initially imposed a normality condition for belief structures, i.e. $\phi \notin F(m)$. Smets [22] proposed to relax this condition, and to interpret $m(\phi)$ as the part of belief committed to the assumption that none of the hypotheses in Ω might be true to allow for an open-world assumption. If, however, the truth is known with absolute certainty to lie in Ω, i.e. closed-world assumption, then the normality condition can be justified.

Given a mass function m for A, for all $B \subseteq \Omega$, the belief and the plausibility of B are defined respectively as:

$$\mathrm{Bel}(B \vee \Theta) = \sum_{A \subseteq B,\ A \neq \phi} m(A) \tag{4a}$$

$$\mathrm{Pl}(B \vee \Theta) = \sum_{A \cap B \neq \phi} m(A) \tag{4b}$$

for all subsets B of Ω, where the sums range over all the focal elements A of m. The set UP is denoted by Θ. The value $\mathrm{Bel}(B \vee \Theta)$ quantifies the belief that the true value of the frame of discernment is contained in B or Θ.

The belief and plausibility functions satisfy the following rules: (1) $\mathrm{Bel}(\phi) = 0$, (2) $\mathrm{Bel}(\Omega) = 1 - m(\phi) \leq 1$, and (3) $\mathrm{Bel}(\mathrm{B}) \leq \mathrm{Pl}(B)$. By definition, $\mathrm{Bel}(\phi) = 0$, even though $m(\phi)$ might be positive. If the frame of discernment Ω is defined such that it included the unknown propositions set Θ, then this would lead to the same belief function as with the open-world assumption if one takes care to never allocate any masses to propositions of Ω that did not include Θ.

4.4. Evidential reasoning mechanism

In evidence-hypothesis reasoning, an *evidence space E* is a set of mutually exclusive and collectively exhaustive *evidential elements* that can arise from a source of evidence, e.g. the set of all possible results of a laboratory test. A *hypothesis space H* is a set containing all the mutually exclusive and collectively exhaustive hypotheses possible in the situation under consideration. Evidence-hypothesis reasoning is a mapping from an *evidence space E* to a *hypothesis space H*, which describes the relationships between evidence and hypothesis subsets.

Evidence usually exists in two forms either as a linguistic observation such as 'high rusted member' or a measured parameter such as 'chloride ion concentration rate equals to 0.6 kg Cl^-/m^3 (1 lb Cl^-/yd)'. Accordingly, the handling of evidence-hypothesis reasoning differs.

The evidence-hypothesis reasoning mechanism is the task of inferring the belief in some hypotheses by collecting relevant evidence for or against these hypotheses. The inexact relationships among hypotheses and evidence are classified depending on the nature of evidence, i.e. measurement or observation. Linguistic hypothesis–evidence reasoning manipulates if–then rules to manifest the uncertainty associated with hypothesis–evidence relationships. Numerical hypothesis–evidence reasoning deals with computations based on measurements, where the inexact relationships between evidence and hypotheses are presented by two-dimensional plots.

4.5. Belief revision

Information is subject to change due to inherent uncertainty in information, or because the various ignorance types, or due to an environment that is volatile and dynamic. Current non-monotonic reasoning systems cannot adequately treat changes in information. Once a change in the knowledge base, however minor, is performed, one must begin from scratch to deal with a problem at hand as result of evidence fusion being computationally non-monotonic with perhaps consequentially changing system architecture. Belief revision methods can be used to deal with changing information [8].

4.6. Diagnostics

Diagnostics in general is the task of inferring plausible explanations for set of evidence, or to decide which explanation accounts for given evidence. Observations of distress and results of laboratory tests can be considered as evidence for possible degradation underlying causes. The problem is then to infer the belief in the possible underlying causes producing the observed evidence. A classical method of diagnostic analysis is based on Bayesian analysis. In this case, the relations between evidence and underlying causes are described by conditional probabilities. Since mechanical, physical, and chemical processes of degradation can act in a synergistic manner, assigning a degradation cause might not be a clear-cut case. Since distresses might bear on a set of causes rather than on an individual cause and that evidences are not infallible, one can concludes that Bayesian theorem is not an appropriate tool for diagnostic problem. In addition, Bayesian theorem postulates an exhaustive frame of discernment that constitutes a complete set of well-defined causes. The reality is that the actual underlying causes might be something else other than the defined or identified causes. An open-world assumption might more appropriate for such cases as described in Section 4.3. In addition, the Bayesian theorem can be generalized within the framework of evidence theory where conditional probabilities are replaced by belief functions [16,17,20,21]. Another generalization is obtained by extending the generalized Bayesian theorem to handle all types of belief functions, i.e. precise, interval, and fuzzy.

5. Solving an algebraic problem set

This section examines an algebraic problem set as identified by Sandia National Laboratories to be a basic building block for uncertainty propagation in computational mechanics [15]. The problem set is based on a model structure that is know with certainty and provided as follows:

$$Y = (A + B)^A \tag{5}$$

where A and B are the parameters that are independent, and positive and real numbers. This model represents the response Y of a system. Six problem types that reflect various uncertainty representation of A and B are examined and solved in subsequent sections. The solutions presented in this section are based on methods that propagate uncertainties using endpoints of the input intervals to demonstrate the propagation processes. In order to obtain the output interval endpoints, all possible combinations of all values in the input intervals should be propagated using the proposed methods, and solutions as output interval endpoints can be obtained through incremental numerical evaluations throughout the input intervals and using max or min operators. In some of the problems, the endpoints of the output intervals might not correspond to the input interval endpoint evaluations. This step of obtaining output interval endpoints was not performed in the paper.

5.1. Interval parameters

The parameters in this case are provided in the form of intervals as follows:

$$A = [a_1, a_2] \tag{6a}$$

$$B = [b_1, b_2] \tag{6b}$$

The interval arithmetic definition of the power of a positive real-valued interval $[b_1, b_2]$ using a positive real-valued power (a) can be defined as:

$$[b_1, b_2]^a = [b_1^a, b_2^a] \tag{7}$$

Using an interval, positive real-valued power $[a_1, a_2]$, the interval arithmetic definition of the power of a positive real-valued interval $[b_1, b_2]$ is

$$[b_1, b_2]^{[a_1,a_2]} = [b_1^{a_1}, b_2^{a_2}] \tag{8}$$

Based on Eqs. (7) and (8), the response Y can be computed utilizing interval addition as follows:

$$Y = [[a_1, a_2] + [b_1, b_2]]^{[a_1,a_2]} = [y_1, y_2] \tag{9}$$

where

$$y_1 = [a_1 + b_1]^{a_1} \tag{10a}$$

$$y_2 = [a_2 + b_2]^{a_2} \tag{10b}$$

Example 5.1. This problem is illustrated using the following values for the parameters A and B :

$$A = [0.1, 1.0]$$

$$B = [0.0, 1.0]$$

The response can be computed as

$$Y = [[0.1, 1.0] + [0.0, 1.0]]^{[0.1,1.0]} = [y_1, y_2]$$

$$= [0.7943282, 1.0]$$

where $y_1 = [0.1 + 0.0]^{0.1} = 0.7943282$, and $y_2 = [1.0 + 1.0]^{1.0} = 2.0$.

5.2. An interval power and a set of intervals

The parameters in this case are provided as follows:

$$A = [a_1, a_2] \tag{11a}$$

$$B_i = [b_{i1}, b_{i2}] \qquad \text{for } i = 1, 2, \ldots, n \tag{11b}$$

The information on B is provided based on n independent sources. The universal set of B is defined as the union of the n intervals. Three cases are considered herein based on specific additional information on B.

5.2.1. A consonant or nested set of intervals

The B_i intervals are nested according to the following structure:

$$B_i \subseteq B_{i+1} \qquad \text{for } i = 1, 2, \ldots, n-1 \tag{12}$$

Since the B_i intervals are equally credible, they can be given a basic assignment $m = 1/n$. The belief and plausibility measures, i.e. necessity and possibility, respectively, can be computed as follows:

$$\mathrm{Bel}(B_i) = \sum_{\text{all } B_j \subseteq B_i} m(B_j) \tag{13}$$

$$\mathrm{Pl}(B_i) = \sum_{\text{all } B_j \cap B_i \neq \varnothing} m(B_j) \tag{14}$$

Eqs. (13) and (14) can be evaluated as follows:

i	B_i	$\mathrm{Bel}(B_i)$	$\mathrm{Pl}(B_i)$
1	B_1	$1/n$	1
2	B_2	$2/n$	1
3	B_3	$3/n$	1
⋮	⋮	⋮	⋮
n	B_n	1	1

(15)

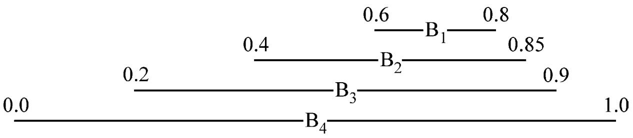
Eq. (9) can now be used to compute the response according to each B_i, and resulting interval should be associated with the corresponding Bel and Pl.

Example 5.2.1. This case is illustrated herein using the following values for the parameters A and B :

$A = [0.1, 1.0]$

$B_1 = [0.6, 0.8],\ B_2 = [0.4, 0.85],\ B_3 = [0.2, 0.9]$, and $B_4 = [0.0, 1.0]$

These intervals are nested as provided below:



The response can be computed using Eqs. (9) and (15) as

i	B_i	$\mathrm{Bel}(B_i)$	$\mathrm{Pl}(B_i)$	y_1	y_2
1	[0.6, 0.8]	0.25	1.00	0.9649611	1.80
2	[0.4, 0.85]	0.50	1.00	0.9330329	1.85
3	[0.2, 0.9]	0.75	1.00	0.8865681	1.90
4	[0.0, 1.0]	1.00	1.00	0.7943282	2.00

5.2.2. A consistent set of intervals

The B intervals are structured such that

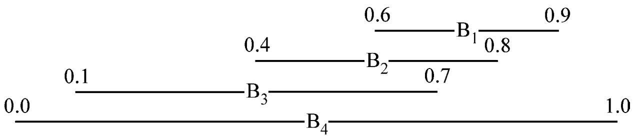
$$B_i \cap B_j \neq \Phi \qquad \text{for } i = 1, 2, \ldots, n \text{ and } j = 1, 2, \ldots, n \tag{16}$$

Similar to the previous case, since the B intervals are equally credible they can be given a basic assignment $m = 1/n$. The belief and plausibility measures can be computed using Eqs. (13) and (14). Then, Eq. (9) can be used to compute the response according to each B intervals, and resulting interval should be associated with corresponding Bel and Pl.

Example 5.2.2. This case is illustrated using the following values for the parameters A and B :
$A = [0.1, 1.0]$

$B_1 = [0.6, 0.9],\ B_2 = [0.4, 0.8],\ B_3 = [0.1, 0.7]$, and $B_4 = [0.0, 1.0]$

These intervals have a common range as follows:



The response can be computed using Eqs. (9), (13) and (14) as follows for all the B intervals:

i	B_i	$\mathrm{Bel}(B_i)$	$\mathrm{Pl}(B_i)$	y_1	y_2
1	[0.6, 0.9]	0.25	1.00	0.9649611	1.90
2	[0.4, 0.8]	0.25	1.00	0.9330329	1.80
3	[0.1, 0.7]	0.25	1.00	0.8513399	1.70
4	[0.0, 1.0]	1.00	1.00	0.7943282	2.00

The common range (B_c) among all the B intervals might be of special interest, and its response can be assessed as follows:

i	B_i	$\mathrm{Bel}(B_i)$	$\mathrm{Pl}(B_i)$	y_1	y_2
c	[0.6, 0.7]	0.25	1.00	0.9649611	1.70

The belief and plausibility of the common range (B_c) were computed based on extension from possibility theory concepts since B_c is common to all B intervals as follows:

$$\mathrm{Bel}(B_c) = \min[B_i] \tag{17a}$$

$$\mathrm{Pl}(B_c) = \max[B_i] \tag{17b}$$

The proof of Eqs. (17a) and (17b) is not available and not provided herein. These equations should be qualified before their use.

5.2.3. An arbitrary set of intervals

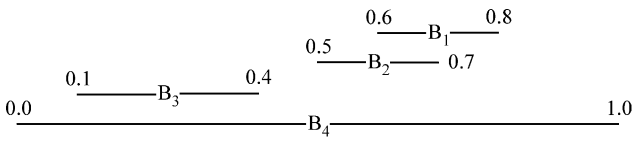
In this case, the B intervals are provided in any arbitrary structure. Similar to the previous case, since the B_i intervals are equally credible they can be given a basic assignment $m = 1/n$. The belief and plausibility measures can be computed using Eqs. (13) and (14). Then, Eq. (9) can be used to compute the response according to each B_i, and resulting interval should be associated with corresponding Bel and Pl.

Example 5.2.3. This case is illustrated using the following values for the parameters A and B :

$A = [0.1, 1.0]$

$B_1 = [0.6, 0.8],\ B_2 = [0.5, 0.7],\ B_3 = [0.1, 0.4],$ and $B_4 = [0.0, 1.0]$

These intervals do not have a common range and can be represented as follows:



The response can be assessed using Eqs. (9), (13) and (14) as follows:

i	B_i	$\mathrm{Bel}(B_i)$	$\mathrm{Pl}(B_i)$	y_1	y_2
1	$[0.6, 0.8]$	0.25	0.75	0.9649611	1.80
2	$[0.5, 0.7]$	0.25	0.75	0.9779327	1.70
3	$[0.1, 0.4]$	0.25	0.50	0.9330329	1.40
4	$[0.0, 1.0]$	1.00	1.00	0.7943282	2.00

5.3. Sets of intervals

In this case, the parameters are provided as follows:

$$A_i = [a_{i1}, a_{i2}] \quad \text{for } i = 1, 2, \ldots, k \tag{18a}$$

$$B_i = [b_{i1}, b_{i2}] \quad \text{for } i = 1, 2, \ldots, n \tag{18b}$$

The information on A and B is provided based on k and n independent sources, respectively. The universal sets of A and B are defined as the union of the k and n respective intervals. Three cases are considered herein based on specific additional information on A and B.

5.3.1. Consonant or nested sets of intervals

The A_i and B_i intervals are nested according to the following structure:

$$A_i \subseteq A_{i+1} \quad \text{for } i = 1, 2, \ldots, k-1 \tag{19a}$$

$$B_i \subseteq B_{i+1} \quad \text{for } i = 1, 2, \ldots, n-1 \tag{19b}$$

Since the A_i and B_i intervals are equally credible, they can be given a basic assignment $m_A = 1/k$ and $m_B = 1/n$, respectively. The belief and plausibility measures, i.e. necessity and possibility, respectively, can be computed according to Eqs. (13) and (14) as follows:

i	A_i	$\mathrm{Bel}(A_i)$	$\mathrm{Pl}(A_i)$
1	A_1	$1/k$	1
2	A_2	$2/k$	1
3	A_3	$3/k$	1
$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	A_n	1	1

(20a)

and

i	B_i	$\mathrm{Bel}(B_i)$	$\mathrm{Pl}(B_i)$
1	B_1	$1/n$	1
2	B_2	$2/n$	1
3	B_3	$3/n$	1
$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	B_n	1	1

(20b)

Eq. (9) can now be used to compute the response according to each combination of A_i and B_i, and resulting interval should be associated with corresponding Bel and Pl using the intersection relationships from the following rules:

$$\mathrm{Bel}(A \cap B) = \min[\mathrm{Bel}(A), \mathrm{Bel}(B)] \tag{21a}$$

$$\mathrm{Bel}(A \cup B) \geq \max[\mathrm{Bel}(A), \mathrm{Bel}(B)] \tag{21b}$$

$$\mathrm{Pl}(A \cap B) \leq \min[\mathrm{Pl}(A), \mathrm{Pl}(B)] \tag{21c}$$

$$\mathrm{Pl}(A \cup B) = \max[\mathrm{Pl}(A), \mathrm{Pl}(B)] \tag{21d}$$

Example 5.3.1. This case is illustrated using the following values for the parameters A and B :

$A_1 = [0.5, 0.7],\ A_2 = [0.3, 0.8],$ and $A_3 = [0.1, 1.0]$

$B_1 = [0.6, 0.8],\ B_2 = [0.4, 0.85],\ B_3 = [0.2, 0.9],$ and $B_4 = [0.0, 1.0]$

The B_i intervals are the same as the previous corresponding case. The response can be computed as follows with the Pl for the resulting interval is an upper bound according to Eq. (21c):

	(Bel,Pl) =	$B_1 = [0.6, 0.8]$ (0.25,1.00)	$B_2 = [0.4, 0.85]$ (0.50,1.00)	$B_3 = [0.2, 0.9]$ (0.75,1.00)	$B_4 = [0.0, 1.0]$ (1.00,1.00)
$A_1 = [0.5, 0.7]$	(0.33,1.00)	$y_1 = 1.048809$, $y_2 = 1.328201$ (0.25,1.00)	$y_1 = 0.948683$, $y_2 = 1.359040$ (0.33,1.00)	$y_1 = 0.836666$, $y_2 = 1.389581$ (0.33,1.00)	$y_1 = 0.707107$, $y_2 = 1.449821$ (0.33,1.00)
$A_2 = [0.3, 0.8]$	(0.67,1.00)	$y_1 = 0.968886$, $y_2 = 1.456451$ (0.25,1.00)	$y_1 = 0.898523$, $y_2 = 1.492750$ (0.50,1.00)	$y_1 = 0.812252$, $y_2 = 1.528830$ (0.67,1.00)	$y_1 = 0.696845$, $y_2 = 1.600361$ (0.67,1.00)
$A_3 = [0.1, 1.0]$	(1.00,1.00)	$y_1 = 0.964961$, $y_2 = 1.80$ (0.25,1.00)	$y_1 = 0.933033$, $y_2 = 1.85$ (0.50,1.00)	$y_1 = 0.886568$, $y_2 = 1.90$ (0.75,1.00)	$y_1 = 0.794328$, $y_2 = 2.00$ (1.00,1.00)

The common ranges (A_c and B_c) can be treated similar to the case presented in Section 5.2.2 if needed.

5.3.2. Consistent sets of intervals

The A_i and B_i intervals are structured such that

$$A_i \cap A_j \neq \Phi \qquad \text{for } i = 1, 2, \ldots, k \text{ and } j = 1, 2, \ldots, k \quad (22a)$$

$$B_i \cap B_j \neq \Phi \qquad \text{for } i = 1, 2, \ldots, n \text{ and } j = 1, 2, \ldots, n \quad (22b)$$

Similar to the previous case, since the A_i and B_i intervals are equally credible they can be given a basic assignment $m_A = 1/k$ and $m_B = 1/n$, respectively. The belief and plausibility measures can be computed using Eqs. (13) and (14). Then, Eq. (9) can be used to compute the response according to pair of A_i and B_i, and resulting interval should be associated with corresponding Bel and Pl using Eqs. (21a) and (21c).

Example 5.3.2. This case is illustrated using the following values for the parameters A and B :

$A_1 = [0.5, 1.0]$, $A_2 = [0.2, 0.7]$, and $A_3 = [0.1, 0.6]$

$B_1 = [0.6, 0.9]$, $B_2 = [0.4, 0.8]$,
$B_3 = [0.1, 0.7]$, and $B_3 = [0.0, 1.0]$

The response can be computed using Eqs. (9), (13) and (14) as follows:

	(Bel,Pl) =	$B_1 = [0.6, 0.9]$ (0.25,1.00)	$B_2 = [0.4, 0.8]$ (0.50,1.00)	$B_3 = [0.1, 0.7]$ (0.75,1.00)	$B_4 = [0.0, 1.0]$ (1.00,1.00)
$A_1 = [0.5, 1.0]$	(0.33,1.00)	$y_1 = 1.048809$, $y_2 = 1.90$ (0.25,1.00)	$y_1 = 0.948683$, $y_2 = 1.80$ (0.33,1.00)	$y_1 = 0.774597$, $y_2 = 1.70$ (0.33,1.00)	$y_1 = 0.707107$, $y_2 = 2.00$ (0.33,1.00)
$A_2 = [0.2, 0.7]$	(0.67,1.00)	$y_1 = 0.956353$, $y_2 = 1.389581$ (0.25,1.00)	$y_1 = 0.902880$, $y_2 = 1.328201$ (0.50,1.00)	$y_1 = 0.786003$, $y_2 = 1.265580$ (0.67,1.00)	$y_1 = 0.724780$, $y_2 = 1.449821$ (0.67,1.00)
$A_3 = [0.1, 0.6]$	(1.00,1.00)	$y_1 = 0.964961$, $y_2 = 1.275426$ (0.25,1.00)	$y_1 = 0.933033$, $y_2 = 1.223705$ (0.50,1.00)	$y_1 = 0.851340$, $y_2 = 1.170485$ (0.75,1.00)	$y_1 = 0.794328$, $y_2 = 1.325782$ (1.00,1.00)

5.3.3. An arbitrary set of intervals

In this case, the A_i and B_i intervals are provided in any arbitrary structures. Similar to the previous case, since the A_i and B_i intervals are equally credible they can be given a basic assignment $m_A = 1/k$ and $m_B = 1/n$, respectively. The belief and plausibility measures can be computed using Eqs. (13) and (14). Then, Eq. (9) can be used to compute the response according to pair of A_i and B_i, and resulting interval should be associated with corresponding Bel and Pl using Eqs. (21a) and (21c).

5.4. An interval power and a lognormally distributed parameter

In this case, the parameters are provided as follows:

$$A = [a_1, a_2] \quad (23)$$

$$\ln(B) \sim N(\mu, \sigma) \quad (24)$$

Monte Carlo simulation can be used to evaluate the response according to the following steps:

- Randomly generate B to obtain b values according to its probability distribution as provided in Eq. (24).
- Compute the response interval as follows:

$$Y = [[a_1, a_2] + b]^{[a_1,a_2]} = [y_1, y_2] \tag{25}$$

where

$$y_1 = [a_1 + b]^{a_1} \tag{26a}$$

$$y_2 = [a_2 + b]^{a_2} \tag{26b}$$

- Repeat the simulation process N times and compute the moments and distribution types of y_1 and y_2.

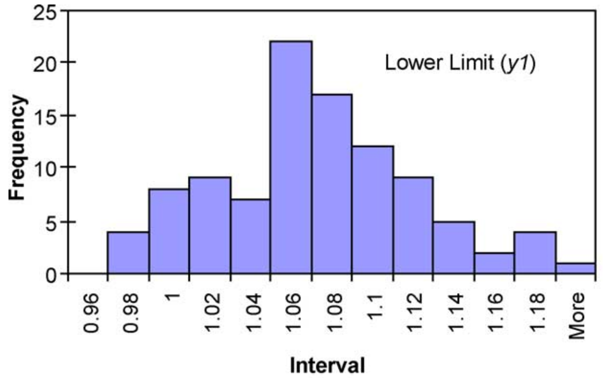
Example. For the following parameters:

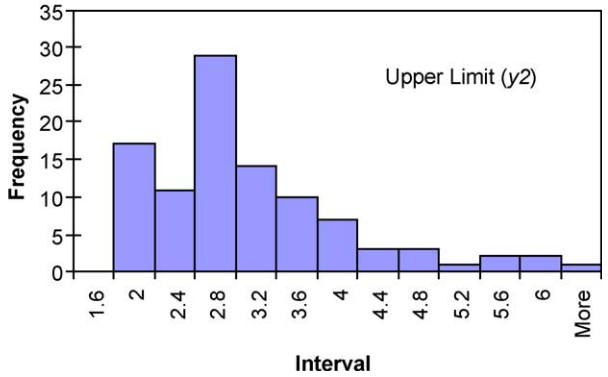
$A = [0.1, 1.0]$

$\ln(B) \sim N(0.5, 0.5)$

simulation was used to compute the response. A total of 100 simulation cycles produced the following response moments and histograms that show bimodal characteristics:

Moment	B	y_1	y_2
Mean	1.923245	1.062593	2.923245
Standard deviation	1.009403	0.049313	1.009403
Coefficient of variation	0.5248	0.0464	0.3453





5.5. An interval power and an uncertain lognormally distributed parameter

In this case, the parameters are provided as follows:

$$A = [a_1, a_2] \tag{27}$$

$$\ln(B) \sim N([\mu_1, \mu_2], [\sigma_1, \sigma_2]) \tag{28}$$

The second order uncertainty provided in characterizing the lognormal parameter can be rolled into the parameters using Monte Carlo simulation to obtain $\ln(B) \sim N(\mu, \sigma)$. Then, the computational procedure presented in Section 5.4 can be used to solve the problem.

5.6. A set of power intervals and a set of an uncertain lognormally distributed parameter

The parameters, in this case, are provided as follows:

$$A_i = [a_{i1}, a_{i2}] \qquad \text{for } i = 1, 2, \ldots, k \tag{29}$$

$$\ln(B_i) \sim N([\mu_{i1}, \mu_{i2}], [\sigma_{i1}, \sigma_{i2}]) \qquad \text{for } i = 1, 2, \ldots, n \tag{30}$$

The information on A and B is provided based on k and n independent sources, respectively. The universal sets of A and B are defined as the union of the k and n respective intervals. Three cases can be developed as combinations of the computational procedures of Sections 5.3 and 5.5.

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