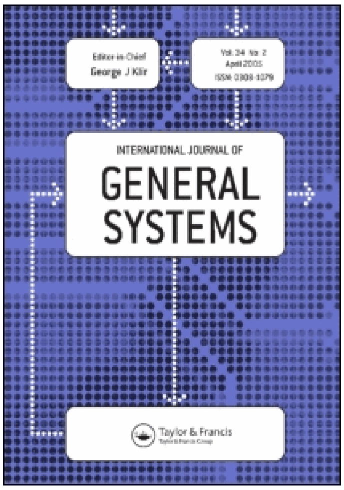

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On uncertainty in information and ignorance in knowledge

Bilal M. Ayyub [a]
[a] Department of Civil and Environmental Engineering, Center for Technology and Systems Management, University of Maryland, College Park, MD, USA

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On uncertainty in information and ignorance in knowledge

Bilal M. Ayyub*

Department of Civil and Environmental Engineering, Center for Technology and Systems Management, University of Maryland, College Park, MD 20742, USA

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This paper provides an overview of working definitions of knowledge, ignorance, information and uncertainty and summarises formalised philosophical and mathematical framework for their analyses. It provides a comparative examination of the generalised information theory and the generalised theory of uncertainty. It summarises foundational bases for assessing the reliability of knowledge constructed as a collective set of justified true beliefs. It discusses system complexity for ancestor simulation potentials. It offers value-driven communication means of knowledge and contrarian knowledge using memes and memetics.

Keywords: complexity; generalised information theory; generalised theory of uncertainty; ignorance; memes

1. Introduction

Philosophers have concerned themselves with the study of knowledge, truth and reality and knowledge acquisition since the early days of the Greek philosophers who first proposed a rational explanation of the natural world and its powers. Ayyub (2001) provides an introduction to knowledge, epistemology, their development and related terminology to form a basis for analysing and understanding knowledge, information, ignorance and uncertainty. Philosophers defined knowledge, its nature and methods of acquisitions that evolved over time producing various schools of thought.

The objective of this paper is to provide an overview of working definitions of knowledge, ignorance, information and uncertainty and to summarise formalised philosophical, analytical and mathematical frameworks that are suitable for systems in engineering and sciences. It summarises foundational bases for assessing the reliability of knowledge constructed as a collective set of justified true beliefs (JTBs), addresses system complexity, conceptually relates mass, energy, entropy and information towards ancestor simulation potentials and offers value-driven communication means of knowledge and contrarian knowledge using memes and memetics. This paper is not intended to offer a complete treatise on these subjects but to offer a cross-cutting thread from information and associated uncertainties to knowledge with its deficiencies as perceived and constructed by humans and propagated by cultures through memes and memetics.

*Email: ba@umd.edu

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2. Information and knowledge

2.1 *Systems framework: realism and constructivism*

The examination of uncertainty, information, ignorance and knowledge requires a systems framework in order to characterise the nature of knowledge. Two views have emerged in systems science for the purpose of system definition: *realism and constructivism*, as described by Ayyub and Klir (2006).

According to realism, a system that is obtained by applying correctly the principles and methods of science *represents* some aspects of the real world. This representation is only approximate, due to limited capability or resolution of our sensors and measuring instruments, and the representation of the system is viewed as a *homomorphic image* of its counterpart in the real world. Using more enhanced capability or refined instruments, the homomorphic mapping between entities of the system of concern and those of its real-world counterpart (the corresponding 'real system') becomes also more refined, and the system becomes a better representation of its real-world counterpart. Realism thus assumes the existence of systems in the real world, which are usually referred to as 'real systems'. It claims that any system obtained by sound scientific inquiry is an approximate (simplified) representation of a 'real system' via an appropriate mapping.

According to constructivism, all systems are artificial abstractions. They are not made by nature and presented to us to be discovered, but we construct them by our perceptual and mental capabilities within the domain of our experiences. The concept of a system that requires correspondence to real world is illusory because there is no way of checking such correspondence. We have no access to the real world except through experience. It seems that the constructivist view has become predominant, at least in systems science, particularly in the way formulated by von Glasersfeld (1995). According to this formulation, constructivism does not deal with ontological questions regarding the real world. It is intended as a theory of knowing, not a theory of being. It does not require analysts to deny ontological reality. Moreover, the constructed systems are not arbitrary: they must not collide with the constraints of the experiential domain. The aim of constructing systems is to organise our experiences in useful ways. A system is useful if it helps us to achieve some aims, for example, to predict, retrodict, control and make proper decisions.

We perceive reality as a continuum in its composition of objects, concepts and propositions. We construct knowledge in quanta to meet constraints related to our cognitive abilities and limitations, producing what can be termed as *quantum knowledge*. This quantum knowledge leads to and contains ignorance – manifested in two forms: (1) ignorance of some states of ignorance and (2) incompleteness and/or inconsistency, as discussed in detail in subsequent sections. The ignorance of a state of ignorance is called *blind ignorance*. The incompleteness form of ignorance stems from quantum knowledge that does not cover the entire domain of inquiry. The inconsistency form of ignorance rises from specialisation and focuses on a particular specialty discipline or science or phenomenon without, for example, accounting for interactions with or from other sciences or disciplines or phenomena.

2.2 *Data, information, knowledge and opinions defined*

Many disciplines of engineering and sciences rely on the development and use of predictive models that in turn require data, information, knowledge and sometimes subjective opinions of experts. Analysts commonly encounter the challenge of constructing knowledge from data of mixed types collected for diverse purposes.

An overview of working definitions of data, information, knowledge and opinions is necessary and is provided in this section.

Data are unconnected numbers or symbols (e.g. names, dates and positions) or sound, images etc, representing objects and entities with appropriate levels of reliability or belief.

Information as a concept has many meanings and is derived from the Latin accusative form (informationem), derived from the verb 'informare' (to inform) in the sense of 'to give form to the mind', 'to discipline', 'instruct' or 'teach'. Information subsumes data with the addition of context. It can be simply defined as a message received and understood and in terms of data, can be defined as a collection thereof from which conclusions may be drawn. Information can be viewed as a pre-processed input to our intellect system of cognition and knowledge acquisition and creation. Information can lead to knowledge through investigation, study and reflection. However, knowledge and information about a system might not constitute the eventual evolutionary knowledge state about the system as a result of not meeting the justification condition as discussed under the definition of knowledge or the ongoing evolutionary process or both.

The definition of knowledge can be based on *evolutionary epistemology* (Honderich 1995) using an *evolutionary model*. Knowledge can be viewed to consist of two types, *non-propositional* and *propositional* knowledge. The non-propositional knowledge can be further broken down into *know-how and concept knowledge* and *familiarity knowledge* (commonly called *object knowledge*). The concept and know-how knowledge types require someone to be familiar with particular words, phrases, images, doctrines and/or thoughts and know how to do a specific activity, function, procedure, etc., such as riding a bicycle. The concept knowledge can be empirical in nature. In evolutionary epistemology, the concept and know-how knowledge types are viewed as historical antecedents to propositional knowledge. The object knowledge is based on a direct acquaintance with a person, place or thing, for example, Mr Smith knows the President of the United States. The propositional knowledge is based on propositions that can be either true or false, for example, Mr Smith knows that the Rockies are in North America (Sober 1991, di Carlo 1998). This proposition can be expressed as

$$\text{Mr Smith knows that the Rockies are in North America} \tag{1a}$$

or

$$S \text{ knows } P, \tag{1b}$$

where S is the subject, that is, Mr Smith, and P is the claim 'the Rockies are in North America'. Epistemologists require the following three conditions for making an acceptable claim:

- S must believe P,
- P must be true and
- S must have a reason to believe P, that is, S must be justified in believing P.

The justification in the third condition can take various forms; however, simplistically, it can be taken as justification through rational reasoning or empirical evidence. Therefore, propositional knowledge is defined as a body of propositions that meet the conditions of JTB. This general definition does not satisfy a class of examples called the *Gettier problem*, initially revealed in 1963 by Edmund Gettier (Austin 1998). Gettier showed that we can have highly reliable evidence and still not have knowledge (Ayyub 2001). Also, someone can sceptically argue that as long as it is possible for S to be mistaken in believing

P (i.e. not meeting third condition), the proposition is false. This argument, sometimes called a Cartesian argument, undermines empirical knowledge. In evolutionary epistemology, this high level of scrutiny is not needed and need not be satisfied in the biological world. According to evolutionary epistemology, true beliefs can be justified causally from reliably attained law-governed procedures, where law refers to natural law. Sober (1991) noted that there are very few instances, if ever, where we have perfectly infallible evidence. Almost all of our common sense beliefs are based on evidence that is not infallible even though some may have overwhelming reliability. The presence of a small doubt in meeting the justification condition does not make our evidence infallible, but only reliable. Evidence reliability and infallibility arguments form the bases of the *reliability theory of knowledge*. Figure 1 shows a breakdown of knowledge by types, sources and objects that was based on a summary provided by Honderich (1995).

Knowledge is defined in the context of the humankind and associated evolution, language and communication methods, social and economic dialectic processes and cultures and cannot be removed from them. As a result, knowledge would always reflect the imperfect and evolutionary nature of humans that can be attributed to their reliance on their senses for information acquisition, their dialectic processes, persistence of cultures and their mind for extrapolation, creativity, reflection and imagination with associated biases as a result of preconceived notions due to time asymmetry, specialisation and other factors. An important dimension in defining the state of knowledge and truth about a system is non-knowledge or ignorance or the illusion of knowledge as discussed in subsequent sections.

Opinions rendered by experts which are based on information and existing knowledge can be defined as preliminary propositions with claims that are not fully justified or are justified with an inadequate reliability level but are not necessarily infallible. Expert

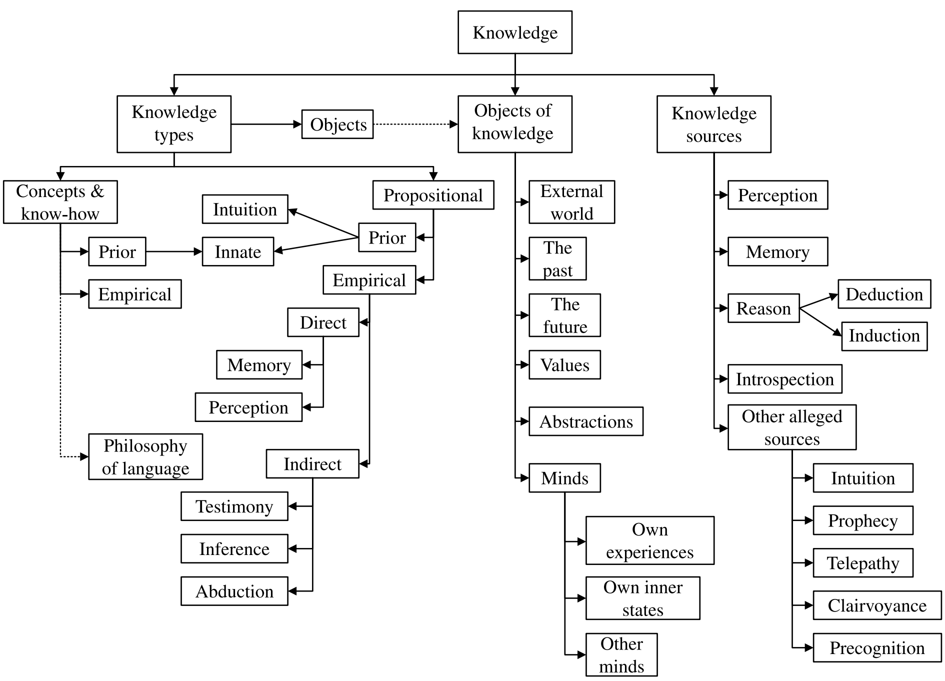

Figure 1. Knowledge sources (Ayyub 2001 after Honderich 1995).

opinions are seeds of propositional knowledge that do not meet one or more of the conditions required for the JTB and according to the reliability theory of knowledge. Opinions are valuable and necessary in many situations where analysts might not have any credible substitutes and offer the means for knowledge expansion; however, decisions made based on opinions can be risky because of their preliminary nature and the potential for someone to invalidate them in the future.

The relationships among knowledge, information, opinions and evolutionary epistemology are schematically shown in Figure 2. The outcomes of the dialectic processes are dependent on the effectiveness of the communication methods, such as languages, visual and audio formats, economic factors, schools of thought and political and social processes within peers, groups, societies and cultures.

2.3 *Potential propositional outcomes for knowledge construction*

A primary component of knowledge is propositions meeting the conditions of JTBs according to Equation (1(a) and (b)) as discussed in the previous section. The justification process according to JTB produces potential outcomes as provided in Table 1. According to the table, an analyst hypothesises a belief (as shown in column 1) to test a perceived state of reality (i.e. column 5). The belief is expressed in an affirmative or contrarian manner to the state of reality. The analyst obtains a body of evidence (either credible or not credible according to column 2); however, the evidence credibility is not known to the analyst with certainty. The analyst then utilises some logic founded in some mathematical framework to test the hypothesis as provided in column 3 of the table (using either suitable or not suitable logic; however, the suitability of logic for the case in hand is not known to

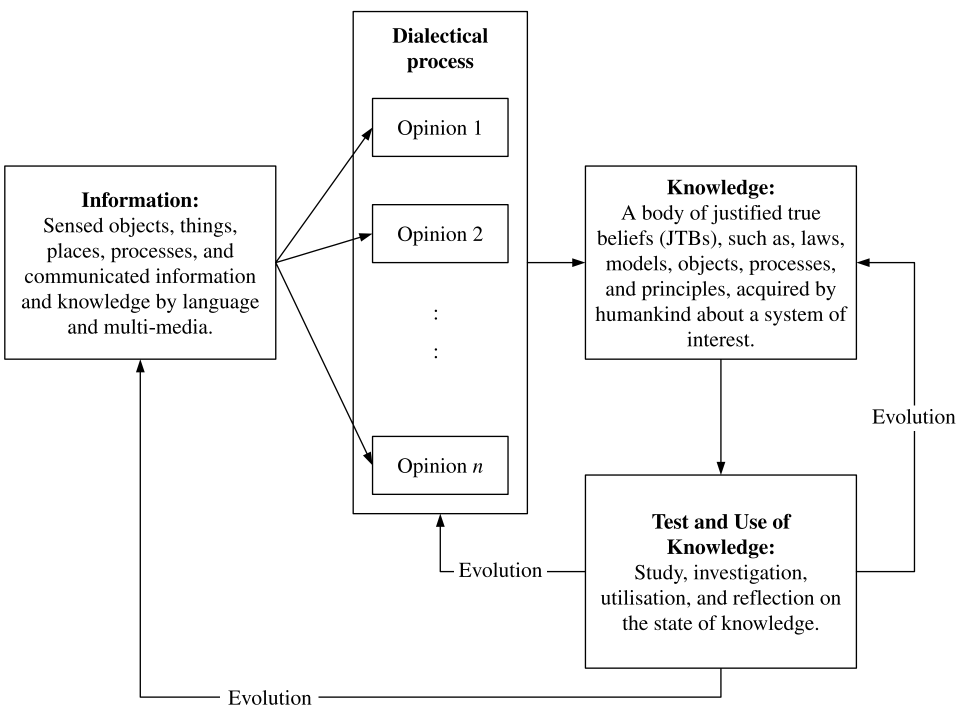

Figure 2. Information, opinion, knowledge and evolution (Ayyub 2001).

Table 1. Potential propositional outcomes for constructing knowledge.

	Outcome justification				
Hypothesised belief relating to reality state (1)	Evidence credibility (2)	Logic suitability (3)	Outcome characterising reality state (4)	Reality state (unknown to an analyst) (5)	Comments relating to knowledge type (i.e. the state of mind of an analyst) (6)
True	Credible	Suitable	True	True	Lasting knowledge gained
			Not true	True	Illusion of knowledge
	Credible	Not suitable	True	True	Knowledge with *ignoratio elenchi*, that is, logical fallacy
			Not true	True	Knowledge illusion with *ignoratio elenchi*, that is, logical fallacy
	Not credible	Suitable	True	True	Gettier-like knowledge
			Not true	True	Illusion of knowledge
	Not credible	Not suitable	True	True	Gettier-like knowledge
			Not true	True	Knowledge illusion with *ignoratio elenchi*, that is, logical fallacy
True	Credible	Suitable	True	Not true	Illusion of knowledge
			Not true	Not true	Lasting knowledge gained
	Credible	Not suitable	True	Not true	Knowledge illusion with *ignoratio elenchi*, that is, logical fallacy
			Not true	Not true	Knowledge with *ignoratio elenchi*, that is, logical fallacy
	Not credible	Suitable	True	Not true	Illusion of knowledge
			Not true	Not true	Gettier-like knowledge
	Not credible	Not suitable	True	Not true	Knowledge illusion with *ignoratio elenchi*, that is, logical fallacy
			Not true	Not true	Gettier-like knowledge
Not true	Credible	Suitable	True	Not true	Illusion of knowledge
			Not true	Not true	Lasting knowledge gained
	Credible	Not suitable	True	Not true	Knowledge illusion with *ignoratio elenchi*, that is, logical fallacy
			Not true	Not true	Knowledge with *ignoratio elenchi*, that is, logical fallacy
	Not credible	Suitable	True	Not true	Illusion of knowledge
			Not true	Not true	Gettier-like knowledge
	Not credible	Not suitable	True	Not true	Knowledge illusion with *ignoratio elenchi*, that is, logical fallacy
			Not true	Not true	Gettier-like knowledge
Not true	Credible	Suitable	True	True	Lasting knowledge gained
			Not true	True	Illusion of knowledge
	Credible	Not suitable	True	True	Knowledge with *ignoratio elenchi*, that is, logical fallacy
			Not true	True	Knowledge illusion with *ignoratio elenchi*, that is, logical fallacy
	Not credible	Suitable	True	True	Gettier-like knowledge
			Not true	True	Illusion of knowledge
	Not credible	Not suitable	True	True	Gettier-like knowledge
			Not true	True	Knowledge illusion with *ignoratio elenchi*, that is, logical fallacy

the analyst with certainty). Two outcomes (i.e. column 4) are possible that characterise the state of reality of either matching or not matching the *unknown* state of reality (i.e. column 5). The last column of the table is a classification of the resulting knowledge as a state of mind of the analyst. These states range from lasting knowledge gained to a state of illusion in knowledge. According to Stephen Hawking, 'The greatest enemy of knowledge is not *ignorance*, it is the *illusion of knowledge*'. A value judgement on 'how desirable the outcomes are' depends on the objective and role of the analyst. For example, scientists working in a laboratory should desire achieving a state of *lasting knowledge gained*. On the other hand, an intelligence or propaganda analyst might desire to feed evidence to adversaries to lead the adversaries to a state of an *illusion of knowledge*. Similarly, a politician might desire to achieve a state of an *illusion of knowledge* by a constituency base to defeat an opponent in an election for a public office.

3. Ignorance and uncertainty

Generally, we tend to focus on and emphasise 'what is known' and not 'what is unknown'. Even the English language lends itself for this emphasis. For example, we can easily state that Expert A *informed* Expert B, whereas we cannot *directly* state the contrary. We can only state it by using the negation of the earlier statement as 'Expert A *did not inform* Expert B'. Statements such as 'Expert A *misinformed* Expert B' or 'Expert A *ignored* Expert B' do not convey the same (intended) meaning. Another example is 'John *knows* David', for which a meaningful *direct* contrary statement does not exist. The emphasis on knowledge and not on ignorance can also be noted in sociology by having a field of study called the *sociology of knowledge* and not having *sociology of ignorance*, although Weinstein and Weinstein (1978) introduced the *sociology of non-knowledge* and Smithson (1985) introduced the *theory of ignorance*.

The state of *ignorance* for a person (or a society) can be unintentional or deliberate due to an erroneous cognition state and not knowing relevant information or ignoring information and deliberate inattention to something for various reasons such as limited resources or cultural opposition, respectively. The latter type is a state of *conscious ignorance* which is not intentional, and once recognised, evolutionary species try to correct for that state for survival reasons with varying levels of success. The former ignorance type belongs to the *blind ignorance* category. Therefore, ignoring means that someone can either *unconsciously* or *deliberately* refuse to acknowledge or regard or leave out an account or consideration for relevant information (di Carlo 1998). These two states should be treated in developing a hierarchal breakdown of ignorance.

According to evolutionary epistemology, ignorance has factitious, that is, human-made, perspectives. Smithson (1988) provided a working definition of ignorance based on 'Expert A is ignorant from B's viewpoint if A fails to agree with or shows awareness of ideas which B defines as actually or potentially valid'. This definition allows for self-attributed ignorance, and either Expert A or Expert B can be the attributer or perpetrator of ignorance. Our ignorance and claimed knowledge depend on our current historical setting which is relative to various natural and cultural factors such as language, logical systems, technologies and standards which have developed and evolved over time. Therefore, humans evolved from blind ignorance through gambles to a state of incomplete knowledge with reflective ignorance recognised through factitious perspectives. In many scientific fields, the level of reflective ignorance becomes larger as the level of knowledge increases. Duncan and Weston-Smith (1997) stated in the *Encyclopedia of Ignorance* that compared to our pond of knowledge, our ignorance remains Atlantic. They invited scientists to state what they would like to know in their respective fields and noted that the more eminent they were, the more readily and generously they described their ignorance. Clearly, before solving a problem, it needs to be articulated. Thomas Sowell, a senior fellow on public policy at the Hoover Institution (http://www.hoover.stanford.edu/), said 'It takes considerable knowledge to realize the extent of your ignorance'.

Ignorance can be viewed to have a hierarchal classification based on its sources and nature as shown in Figure 3 with the brief definitions provided in Table 2. Ignorance can be classified into two types, blind ignorance (also called meta-ignorance) and conscious ignorance (also called reflective ignorance).

Blind ignorance includes *not knowing* relevant know-how, object-related information and relevant propositions that can be justified. The unknowable knowledge can be defined as knowledge that cannot be attained by humans based on current evolutionary progressions, or cannot be attained at all due to human limitations or can only be attained through quantum leaps by humans. Blind ignorance also includes irrelevant knowledge that can be of two types: (1) relevant knowledge that is dismissed as irrelevant or ignored and (2) irrelevant knowledge that is believed to be relevant through non-reliable or weak justification or as a result of *ignoratio elenchi* (Table 1). The irrelevance type can be due to untopicality, taboo and undecidability. Untopicality can be attributed to intuitions of experts that could not be negotiated with others in terms of cognitive relevance. Taboo is due to socially reinforced irrelevance. Issues that people must not know, deal with, enquire about or investigate define the domain of taboo. The undecidedness type deals with issues that cannot be designated true or false because they are considered insoluble, or solutions that are not verifiable or as a result of *ignoratio elenchi*. A third component of blind ignorance is fallacy that can be defined as erroneous beliefs due to misleading notions. The Newsweek Magazine (29 December 2003) selected a quote by the US Secretary of Defence Donald Rumsfeld in its 2003 quotes of the year used to clarify the US policy on the war on terror at a Pentagon briefing that includes elements related to Figure 3 as 'There are known knowns. These are things that we know. There are known unknowns. That is to say, there are things that we know we don't know. But there are also unknown unknowns. There are things we don't know we don't know'.

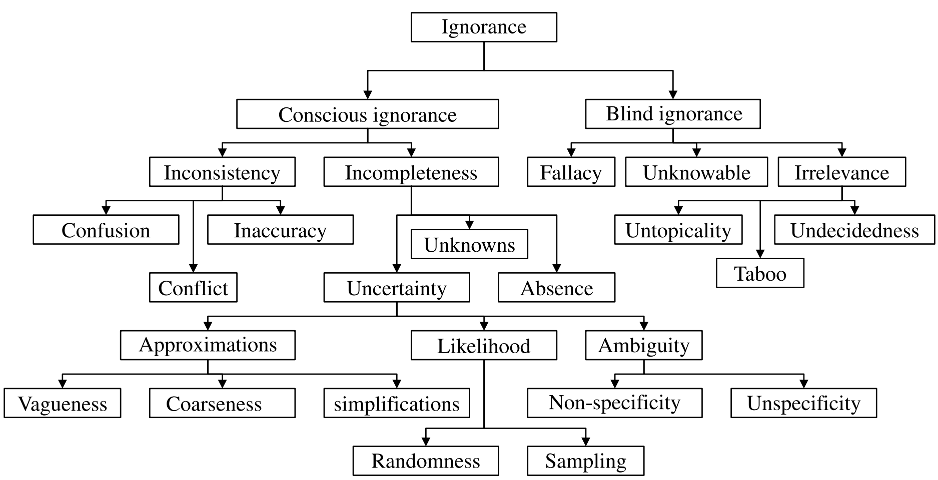

Figure 3. Ignorance hierarchy (Ayyub 2001).

Table 2. Taxonomy of Ignorance.

Term	Meaning
1. Blind ignorance	Ignorance of self-ignorance or called meta-ignorance.
1.1. Unknowable	Knowledge that cannot be attained by humans based on current evolutionary progressions, or cannot be attained at all due to human limitations, or can only be attained through quantum leaps by humans.
1.2. Irrelevance	Ignoring something.
1.2.1. Untopicality	Intuitions of experts that could not be negotiated with others in terms of cognitive relevance.
1.2.2. Taboo	Socially reinforced irrelevance. Issues that people must not know, deal with, enquire about or investigate.
1.2.3. Undecidedness	Issues that cannot be designated true or false because they are considered insoluble, or solutions that are not verifiable, or *ignoratio elenchi*.
1.3. Fallacy	Erroneous belief due to misleading notions.
2. Conscious ignorance	A recognised self-ignorance through reflection.
2.1. Inconsistency	Inconsistency in knowledge can be attributed to distorted information as a result of inaccuracy, conflict, contradiction and/or confusion.
2.1.1. Confusion	Wrongful substitutions.
2.1.2. Conflict	Conflicting or contradictory assignments or substitutions.
2.1.3. Inaccuracy	Bias and distortion in degree.
2.2. Incompleteness	Lacking or non-whole knowledge in its extent due to absence or uncertainty.
2.2.1. Absence	Incompleteness in kind.
2.2.2. Unknowns	The difference between the *becoming* knowledge state and *current* knowledge state
2.2.3. Uncertainty	Knowledge incompleteness due to inherent deficiencies with acquired knowledge.
2.2.3.1. Ambiguity	The possibility of having multi-outcomes for processes or systems.
(a) Unspecificity	Outcomes or assignments that are incompletely defined.
(b) Non-specificity	Outcomes or assignments that are improperly or incorrectly defined.
2.2.3.2. Approximations	A process that involves the use of vague semantics in language, approximate reasoning and dealing with complexity by emphasising relevance.
(a) Vagueness	Non-crispness of belonging and non-belonging of elements to a set or a notion of interest.
(b) Coarseness	Approximating a crisp set by subsets of an underlying partition of the set's universe that would bound the set of interest.
(c) Simplifications	Assumptions needed to make problems and solutions tractable.
2.2.3.3. Likelihood	Defined by its components of randomness, statistical and modelling.
(a) Randomness	Non-predictability of outcomes.
(b) Sampling	Samples versus populations.

Kurt Gödel (1906–1978) proved the incompleteness of axioms for arithmetic as well as the relative consistency of the axiom of choice and continuum hypothesis with the other axioms of set theory (Hofstadter 1999, Nagel and Newman 2001). According to Gödel, mathematicians hoped that their axioms could be proven consistent, that is, free from contradictions, and complete, that is, strong enough to provide proofs of all true statements. Gödel, however, showed these hopes were overly naive by proving that any consistent formal system strong enough to axiomatise arithmetic must be incomplete; that is, there are statements that are true but not provable. Also, one cannot hope to prove the consistency of such a system using the axioms themselves (Hofstadter 1999, Nagel and Newman 2001). Moreover, many systems defined in engineering and sciences are not based on a *closed universe* defined by sets and accompanying axioms, but on potentially an *open universe* defined by sets, including vague sets, and axioms, as constructed and deemed appropriate by humans.

Within the context of the collective propositional knowledge of humans (with the redundancy herein, since all knowledge is attributable to humans, is for emphasis), we could state that humans cannot be both consistent and complete and could not prove completeness without proving inconsistency and vice versa. This view can be used as a basis for classifying the conscious ignorance into *inconsistency* and *incompleteness*. This classification is also consistent with the concept of *quantum knowledge* previously discussed.

Inconsistency in knowledge can be attributed to distorted information as a result of inaccuracy, conflict, contradiction and/or confusion as shown in Figure 3. Inconsistency can result from assignments and substitutions that are wrong, conflicting or biased producing confusion, conflict or inaccuracy, respectively. The confusion and conflict result from in-kind inconsistent assignments and substitutions, whereas inaccuracy results from a level bias or an error in these assignments and substitutions.

Incompleteness is defined as lacking or non-whole knowledge in its extent. Knowledge incompleteness consists of (1) absence and unknowns as incompleteness in kind and (2) uncertainty. The unknowns or unknown knowledge can be viewed in evolutionary epistemology as the difference between the *becoming* knowledge state and *current* knowledge state. The knowledge absence component can lead to one of the scenarios: (1) no action and working without the knowledge, (2) unintentionally acquiring irrelevant knowledge leading to blind ignorance and (3) acquiring relevant knowledge that can be with various uncertainties and levels. The fourth possible scenario of deliberately acquiring irrelevant knowledge is not listed since it is not realistic.

Uncertainty can be defined as knowledge incompleteness due to inherent deficiencies with acquired knowledge. Klir (2006) formally defines uncertainty as information deficiency including deficiency types of incompleteness, imprecision, fragmentation, unreliability, vagueness or contradiction. Uncertainty can be classified based on its sources into three types: ambiguity, approximations and likelihood. The ambiguity comes from the possibility of having multi-outcomes for processes or systems. Recognising only some of the possible outcomes creates uncertainty. The recognised outcomes might constitute only a partial list of all possible outcomes leading to *unspecificity*. In this context, unspecificity results from outcomes or assignments that are *incompletely* defined. The *improper* or *incorrect* definition of outcomes, that is, error in defining outcomes, can be called *non-specificity*. In this context, non-specificity results from outcomes or assignments that are improperly defined. The unspecificity is a form of knowledge absence and can be treated similarly to the absence category under incompleteness. The non-specificity can be viewed as a state of blind ignorance.

The human mind has the ability to perform approximations through reduction and generalisations, that is, induction and deduction, respectively, in developing knowledge. The process of approximation can involve the use of vague semantics in language, approximate reasoning and dealing with complexity by emphasising relevance. Approximations can be viewed to include vagueness, coarseness and simplification. Vagueness results from the *imprecise* nature of belonging and non-belonging of elements to a set or a notion of interest, whereas coarseness results from approximating a set by subsets of an underlying partition of the set's universe that would bound the crisp set of interest. Simplifications are assumptions introduced to make problems and solutions tractable.

Likelihood can be defined in the context of chance, odds and gambling. Likelihood has primary components of randomness and sampling. Randomness stems from the non-predictability of outcomes. Engineers and scientists commonly use samples to characterise populations, hence the last type.

4. Generalised theories of information and uncertainty

Defining uncertainty as information deficiency means that information contains uncertainty, and reducing uncertainty means enhancing information. Therefore, information and uncertainty can be viewed as a duality defining two mutually antagonistic principles as bases for knowledge construction and associated ignorance. It is an eventuality that we have two generalised theories to address them, as synthesised and created by Klir (1996) and Zadeh (2005, 2006), respectively. The purpose of this section is to briefly introduce the two theories and the appropriate sources for necessary comparative examinations in future studies.

4.1 *Generalised information theory*

The recognition that *scientific knowledge* can be organised, by and large, in terms of *systems* of various types is an important outcome of systems science (Klir 1972, 1985). Systems are constructed for various *purposes*, such as predicting, retrodicting, prescribing, diagnosing and controlling. Uncertainty was recognised as a primary driver in systems analysis for several centuries; however, it was liberated from its probabilistic confines only in the second half of the twentieth century (Klir 2004, 2005, 2006). It is closely connected with two important generalisations in mathematics. One of them is the generalisation of classical measure theory (Halmos 1950) to the theory of generalised measures, which was first suggested by Gustave Choquet (1953–1954) in his theory of capacities. The second one is the generalisation of classical set theory to fuzzy set theory, introduced by Zadeh (1965) and Klir and Yuan (1995). Generalised measures are obtained by abandoning the requirement of additivity of classical measures. Fuzzy sets are obtained by abandoning the requirement of sharp boundaries of classical sets. These generalisations enlarged substantially the framework for formalising uncertainty. As a consequence, they made it possible to conceive new theories of uncertainty as provided in Figure 4 (Klir and Wierman 1999, Ayyub and Klir 2006) according to formalised languages as classified in Figure 5 (Ayyub and Klir 2006).

In general, uncertainty is an expression of some information deficiency. This suggests that information could be measured in terms of uncertainty reduction. To reduce relevant uncertainty in a situation formalised within a mathematical theory it requires that some relevant action be taken by a cognitive agent, such as performing a relevant experiment, searching for a relevant fact or accepting and interpreting a relevant message. If results of the action taken (an experimental outcome, a discovered fact, etc.) reduce uncertainty involved in the situation, then the amount of information obtained by the action is measured by the amount of uncertainty reduced – the difference between *a priori* and *a posteriori* uncertainty. Measuring information in this way is clearly contingent upon our capability to measure uncertainty within the various mathematical frameworks. Information measured solely by uncertainty reduction is an important, even though restricted, notion of information. To distinguish it from the various other conceptions of information, it is common to refer to it as *uncertainty-based information* (Klir 2006).

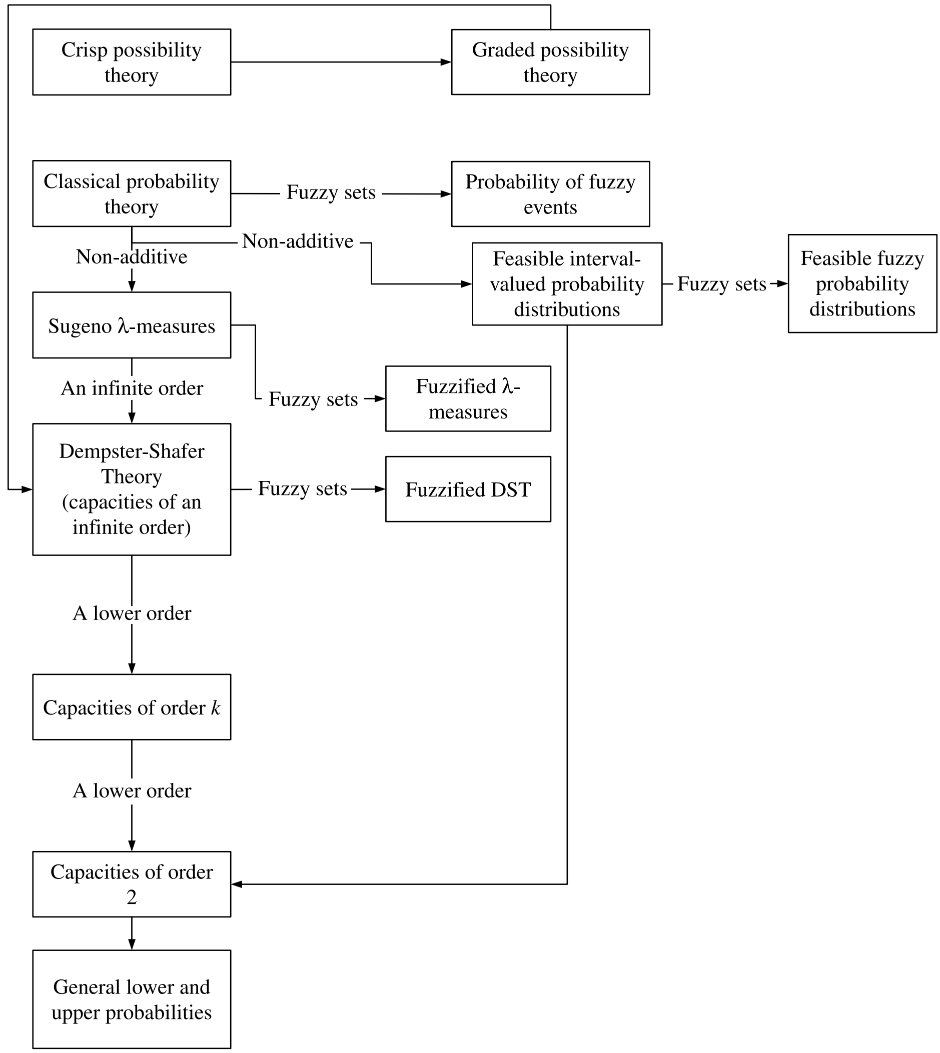

Figure 4. Ordering uncertainty theories by levels of their generality (Ayyub and Klir 2006).

A research programme whose objective is to develop a broader treatment of uncertainty-based information, not restricted to probabilistic formalisation of uncertainty, was introduced in the early 1990s under the name ‘generalised information theory’ (GIT; Klir 1991). The ultimate goal of GIT is to develop the capability to deal with any type of uncertainty-based information that is recognised on intuitive grounds. To be able to deal with each recognised type of uncertainty (and the associated type of information) in an operational way, relevant issues must be addressed at each of the following four levels:

- *Level* 1 – find an appropriate mathematical representation of the conceived type of uncertainty;
- *Level* 2 – develop a calculus by which this type of uncertainty attributes can be properly quantified (i.e. measures) and manipulated;

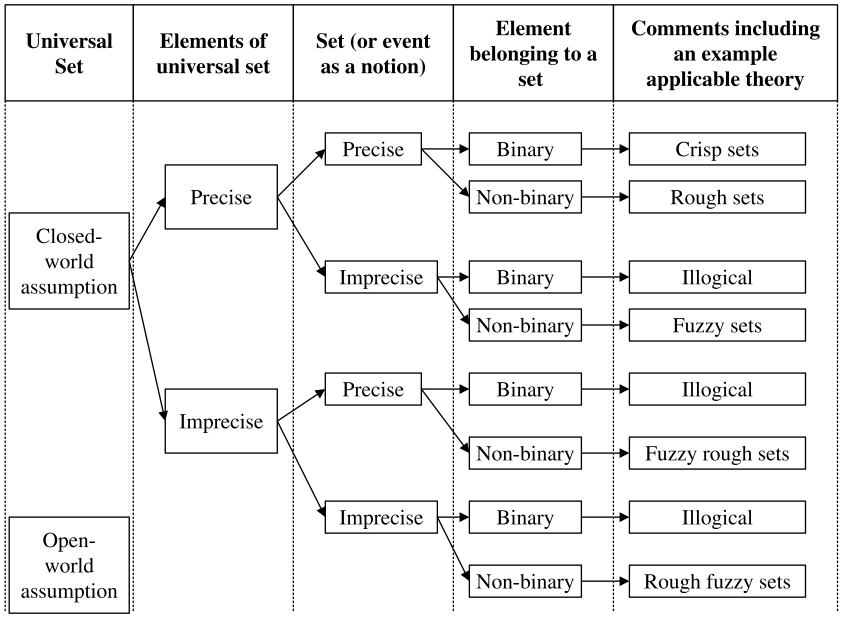

Figure 5. Formalised languages (Ayyub and Klir 2006).

- *Level* 3 – find a meaningful way of measuring relevant uncertainty in any situation formalised in the theory and
- *Level* 4 – develop methodological aspects of the theory, including procedures for making the various uncertainty principles operational within the theory.

4.2 *Generalised theory of uncertainty*

Zadeh (2005, 2006) introduced the generalised theory of uncertainty (GTU), where uncertainty is considered as an attribute of information. Information is conveyed by constraining the values of a variable, and a proposition is considered as a carrier of information. A proposition is therefore treated as a generalised constraint. For example, the statement 'Monika is young' includes a *fuzzy* constraint on Monika's age. Another example, the statement 'hotel checkout time is 1:00 pm' includes a *crisp* constraint on the checkout time. For example, the statement that X is 'approximately five' constrains the spectrum of possible values for X, where the constraint is represented as a fuzzy set 'approximately five'. Within the GTU, each uncertainty modality would be explicitly treated as a generalised constraint without the need to transform among theories. The general syntax for making statements under the GTU is $Z = X$ is$r$ Y, where the value of r specifies the modality of the constraint ('$r =$ blank' is possibilistic, '$r =$ p' is probabilistic, '$r =$ v' is veristic, '$r =$ u' is usuality, '$r =$ rs' is a random set, '$r =$ fg' is a fuzzy graph, '$r =$ bm' is bimodal and '$r =$ g' is a group). Operations such as conjunction, disjunction, projection and propagation can be performed on multiple constraints. According to GTU, each variable is constrained according to the modality of the uncertainty type present. Treating variables as information with possibilistic constraints (i.e. fuzzy sets) and using precisiated natural language, a deduction rule should be selected to match the protoforms of the data and query.

GTU and GIT offer two complementary tracks for handling available information in its native form. Where GTU differs is in the processing of information. GTU does not transform information of different modalities to facilitate aggregation under a common uncertainty framework but rather preserves the natural representation of the information and selects the appropriate deduction rules according to the types of constraints present in the information. In this sense, all information is processed in its native form without the need for transformation. The challenge, however, is to build and maintain a library of deduction rules for processing all possible combinations of uncertainty types, which Zadeh (2005) suggests to be an area of ongoing research.

5. System complexity and ancestor simulation

Several facets of complexity necessitate the use of several reasonable classifications of complexity investigations. Within the context of *system definition*, complexity can be classified into two broad categories: (1) complexity with structure and (2) complexity without structure. The complexity with structure was termed *organised complexity* by Weaver (1948). Organised complexity can be observed in a system that involves nonlinear differential equations with a lot of interactions among a large number of components and variables that define the system, such as in life, behavioural, social and environmental sciences. Such systems are usually non-deterministic in their nature. Advancements in computer technology and numerical methods have enhanced our ability to obtain solutions of these problems effectively and inexpensively. These computer and numerical advancements are not limitless, as the increasing computational demands lead to what is termed *transcomputational problems* capped by the *Bremermann's limit* (Bremermann 1962). This Bremermann's limit, which was derived on the basis of quantum theory, is expressed by the following proposition (Bremermann 1962): '*No data processing systems, whether artificial or living, can process more than* 2×10^{47} *bits per second per gram of its mass*', where data processing is defined as transmitting bits over one or several of a system's channels. Klir and Folger (1988) provide additional information on the theoretical basis for this proposition. Considering a hypothetical computer that has the entire mass of the Earth (6×10^{27} g) operating for a time period equal to an estimated age of the Earth (3.14×10^{17} s), this imaginary computer would be able to process 2.56×10^{92} bits, or rounded to the nearest power of 10, 10^{93} bits, defining the Bremermann's limit. Many scientific and engineering problems defined to include extensive details could easily exceed this limit. Klir and Folger (1988) provided the examples of pattern recognition and human vision that can easily reach transcomputational levels. In pattern recognition, consider a square $q \times q$ spatial array defining $n = q^2$ cells that partition the recognition space. Pattern recognition often involves colour. Using k colours, as an example, the number of possible colour patterns within the space is k^n. In order to stay within the Bremermann's limit, the inequality $k^n \leq 10^{93}$ must be met. According to Klir and Folger (1988), if we consider a retina of about one million cells with each cell having only two states of *active* and *inactive* in recognising an object, modelling the retina in its entirety would require the processing of $2^{1,000,000} = 10^{300}$ bits of information, far beyond the Bremermann's limit.

Organised complexity in nature offers another interesting aspect of complexity in that it can be decomposed into an underlying repeated unit (Flake 1998). For example, economic markets that defy prediction, or the pattern recognition capabilities of any of the vertebrates, or the human immune system's response to viral and bacterial attacks or the evolution of life on our planet are emergent in that they contain simple units that, when combined, form a more complex whole. They are examples of the whole of the system being greater than the sum of the parts, which is a fair definition of holism – the very opposite of reductionism. They are similar to an ant colony. Although a single ant exhibits a simple behaviour that includes a very small number of tasks depending on its caste, such as foraging for food, or caring for the queen's brood, or tending to the upkeep of the nest, defending against enemies or in the case of the queen laying eggs; the behaviour of the ant colony as a whole is very complex. The ant colony includes millions of workers that can sweep whole regions clean of animal life and the fungus-growing ants that collect vegetable matter as food for symbiotic fungi and then harvest a portion of the fungi as food for the colony. The physical structure of the colony that ants build often contains thousands of passageways and appears mazelike to human eyes but are easily navigated by the inhabitants. The point herein is that an ant colony is more than just a bunch of ants. An organised complexity exists that is challenging to scientists. By knowing how each caste in an ant species behaves would not enable a scientist to magically infer that ant colonies would possess so many sophisticated patterns of behaviour.

By increasing the complexity of the system model, our ability to make *relevant* assessments of the system's attributes diminishes. Therefore, there is always a tradeoff between relevance and precision in system modelling in this case. Our goal should be to model a system with a sufficient level of detail that can result in sufficient precision and can lean to relevant decisions in order to meet the objective of the system assessment. Living systems show signs of these tradeoffs between precision and relevance in order to deal with complexity. The survival instincts of living systems have evolved and manifest themselves as processes to cope with complexity and information overload. The ability of a living system to make relevant assessments diminishes with the increase in information input as discussed by Miller (1978). Living systems commonly need to process information in a continuous manner in order to survive. For example, a fish needs to process visual information constantly in order to avoid being eaten by another fish. When a school of larger fish rushes towards the fish, presenting it with images of threats and dangers, the fish might not be able to process all the information and images and becomes confused. Considering the information processing capabilities of living systems as input–output black boxes, the input and output to such systems can be measured and plotted in order to examine such relationships and any nonlinear characteristics that they might exhibit. Miller (1978) described these relationships for living systems using the following hypothesis that was analytically modelled and experimentally validated:

> As the information input to a single channel of a living system – measured in bits per second – increases, the information output – measured similarly – increases almost identically at first but gradually falls behind as it approaches a certain output rate, the channel capacity, which cannot be exceeded. The output then levels off at that rate, and finally, as the information input rate continues to go up, the output decreases gradually towards zero as breakdown or the confusion state occurs under overload.

The above hypothesis was used to construct families of curves to represent the effects of information input overload. Once the input overload is removed, most living systems recover instantly from the overload and the process is completely reversible; however, if the energy level of the input is much larger than the channel capacity, a living system might not fully recover from this input overload. Living systems also adjust the way they process information in order to deal with an information input overload using one or more of the following processes by varying degrees depending on the level of a living system in terms of complexity: (1) omission by failing to transmit information, (2) error by transmitting information incorrectly, (3) queuing by delaying transmission, (4) filtering by giving priority in processing, (5) abstracting by processing messages with less than complete details, (6) multiple channel processing by simultaneously transmitting messages over several parallel channels, (7) escape by acting to cut off information input and (8) chunking by transformation of information in meaningful chunks. These actions can also be viewed as simplification means to cope with complexity and/or an information input overload.

Complexity can be viewed to be a product of human perception that is rooted in our attempt to understand the relationships among matter in terms of mass, energy, entropy and information. The relationship is schematically depicted in Figure 6. The schematic, triangular representation shows the laws of mass conservation and energy conservation and relates mass to energy through Einstein's law. The energy–entropy relationship might be controversial; however, thermodynamics may be viewed as an application of Shannon's information theory (Jaynes 1957). Thermodynamic entropy is interpreted as being an estimate of the amount of further Shannon information needed to define the detailed microscopic state of the system corresponding to a specified macrostate. For example, adding heat to a system increases its thermodynamic entropy because it increases the number of possible microscopic states that it could be in, thus making any complete state description longer. Mass and entropy cannot be credibly related at this stage due to several challenges including

- the number of microstates which is uncountably infinite,
- continuity and approximations (coarse graining and quantum mechanics),
- state definition versus behaviour function definition and
- the role of Bremermann's limit for a self-contained system of approximately 2×10^{47} bits per second per gram.

Understanding complexity and system modelling could enable detailed simulation including ancestor simulation. Bostrom (2003) posed the question 'Do we live in an ancestor-simulation?' He argues that at least one of the following propositions is true:

- the fraction of human-level civilisations that reach a post-human stage (i.e. technologically mature) is very close to zero,
- the fraction of post-human civilisations that are interested in running ancestor simulations is very close to zero or
- the fraction of all the people with our present level of technological maturity who are living in a simulation is high.

The belief, therefore, that there is a significant chance that we will one day become post-humans who run ancestor simulations is false, unless we are currently living in a simulation. Achieving the capability of ancestor simulation requires post-human computation requirements defined as follows:

- potential capacity – 10^{42} operations per second for a computer with a mass of the order of a relatively small planet, smaller than earth,
- human brains – $\sim[10^{14}, 10^{17}]$ operations per second for the entire human brain times several billion people,
- memory – it is not a stringent constraint like processing power,
- human sensors – maximum human sensory bandwidth is $\sim 10^{8}$ bits per second (negligible),
- environments – they are filled with appropriate scope, granularity and other features on *ad hoc* bases and

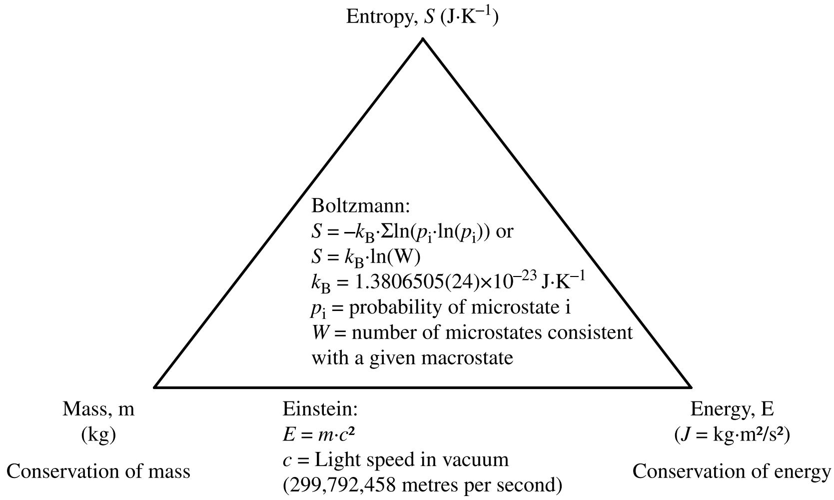

Figure 6. Mass, energy, entropy and information.

- potential demand – 100 billion humans $\times$ 50 years/human $\times$ 30 million s/year $\times$ $[10^{14}, 10^{17}]$ operations in each human brain per second $\approx [10^{33}, 10^{36}]$ operations.

The potential demand of $[10^{33}, 10^{36}]$ operations per second is well within the stated capacity of 10^{42} operations per second.

6. Memes and memetics

Table 1 shows potential propositional outcomes for constructing knowledge. A value judgement on 'how desirable the outcomes are' depends on the objective and role of the analyst. As was stated earlier, scientists working in a laboratory should desire achieving a state of *lasting knowledge gained*. On the other hand, an intelligence or propaganda analyst might desire to feed evidence to adversaries to achieve for them a state of *knowledge illusion*. Memes and memetics could offer means to support both objectives.

The word 'meme' is a neologism coined by Dawkins (1976) in The Selfish Gene (although it may have had earlier roots) and is defined as a self-reproducing and propagating information structure analogous to a gene in biology. Dawkins focused on the meme as a replicator, analogous to the gene, able to affect human evolution through the evolutionary algorithm of variation, replication and differential fitness. But in an application, the relevant characteristics of the meme are that it consists of information which persists, propagates and influences human behaviour.

Several definitions are available for a meme such as a self-reproducing and propagating information structure analogous to a gene in biology, or a unit of cultural transmission (or a unit of imitation) that is a replicator that propagates in the meme pool leaping from brain to brain via (in a broad sense) imitation, such as tunes, ideas, catch-phrases, clothes fashions and ways of making pots or of building arches, or cultural information units that are the smallest elements that replicate themselves with reliability and fecundity. Finkelstein and Ayyub (2010) provided a pragmatic, functionally useful description of a meme as information transmitted by any number of sources to at least an order of magnitude more recipients than sources and propagated during at least 12 h.

A meme is transmitted after either being created in the mind of an individual or re-transmitted after being received by an individual from elsewhere. Arriving at a new potential host, the meme is received and decoded. The potential host becomes an actual host if the meme satisfies certain selection and fitness criteria. The new host replicates and transmits the meme (perhaps with a different vector, such as a text message instead of speech). Because the number of memes at any given time exceeds the number of recipients able to absorb them, fitness criteria determine which meme will survive, propagate, persist and have impact. The selection and fitness criteria include such human motivators as fear (e.g. of going to hell or failing in business) and reward (e.g. of going to heaven or succeeding in business), or the meme might be beneficial in a practical way (such as instruction on how to make a hard-boiled egg or an improvised explosive device), or the meme might be entertaining to the recipient, such as a joke (why did the terrorist cross the road?) or a song (Bomb bomb bomb, bomb bomb Iran', as sung by presidential candidate John McCain, as featured on YouTube.com), or it might consist of appreciative direct feedback to the recipient (such as providing emotional satisfaction, e.g. reinforcement and pride in membership in a nation, tribe, religion, ethnic group or ideology; Finkelstein and Ayyub 2010).

To be readily acceptable to the host, the meme should fit existing constructs or belief systems of the host or be a paradigm to which the host is receptive. Memes also aggregate and reinforce in complexes (memeplexes) so that a suitable existing framework in the mind of the host is especially susceptible to a new meme which fits the framework (such as a new precept by a religious leader being absorbed by a follower of that religion, whereas it would be ignored or escape notice by a non-follower). Suitable storage capacity, in memory or media, is necessary for the meme to persist, along with enduring vectors (e.g. the meme is literally chiselled in stone or reproduced in many, widely distributed copies of books or electronic media).

New research projects could provide a scientific and quantitative basis for memetics and an exploration of its prospective applicability and value, possibly discovering whether brief memes such as ‘axis of evil’, ‘war on terror’ or ‘Winston tastes good like a cigarette should’ are, in fact, cognitively and functionally different from non-memes such as ‘I like your hair’ or ‘please pass the salt’.

Further research is needed to enhance our understanding of the nature of memes and their attributes and to develop simulation methods and simulation environments, that is, sandboxes, to enable analysts to explore their development and examine their performance effectiveness in quantifiable metrics and submetrics.

Since Dawkins’ revelation about memes, the concept has attracted a coterie of proponents, sceptics and opponents. In 30 years, there has been no significant research on the concept to establish a scientific basis for it – but neither has there been a definitive refutation. To progress as a discipline with useful applications, memetics needs a general theory – a theoretical foundation for development of a scientific discipline of memetics. It needs a narrowly focused, pragmatically useful definition and, ultimately, the ability to make testable predictions and falsifiable hypotheses. The discrete meme must be defined, identified and distinguished in the near-continuum of information, just as the discrete gene can be identified (more or less) in a long string of DNA nucleotides (albeit, with current technology a gene may not be clearly identifiable). A quantitative basis for memes must be established, using, for example, such tools as information theory and entropy; genetic, memetic and evolutionary algorithms; neuroeconomics tools such as functional magnetic resonance imaging and biochemical analyses and modelling and simulation of social networks and information propagation and impact (Finkelstein and Ayyub 2010).

As an example, a transmission probability can be quantified using the following procedure provided herein for illustration purposes using uncertainty measures (Ayyub 2004, Ayyub and Klir 2006):

- assess the information content of memes (or memeplex) using uncertainty measures,
- assess the internal inconsistency,
- assess the inconsistency among memes within a host and other memes at potential hosts,
- assess utilities based on a value structure,
- assess shaping factors based on meme source, timing, complexity, impact, etc.,
- aggregate into an overall successful transmission likelihood.

The steps of this procedure can be impeded in an ancestor simulation as discussed in an earlier section to explore cultural progression and examine meme-based stimuli in anthropology.

7. Tribute and dedication

The author dedicates this article to Professor George J. Klir for his distinguished career. Professor Klir's seminal work and contributions in the areas of systems and information sciences and uncertainty analysis are not only unmatched but also unparalleled and have significantly impacted many fields of engineering and the sciences. This paper was presented on 21 September 2007 at the mini-symposium held in honour of Professor Klir at the University of Binghamton, State University of New York, http://emrs.binghamton.edu/Symposium/index.html.The author would like to acknowledge the valuable comments by the reviewers and Professor Radim Belohlavek.

Notes on contributor

Bilal M. Ayyub has more than 25 years of experience after his PhD degree from the Georgia Institute of Technology. He is a leading authority on uncertainty modelling and analysis, risk analysis and decision science. He completed several research projects that were funded by the National Science Foundation, Air Force, Coast Guard, Army Corps of Engineers, Department of Homeland Security, the Maryland State Highway Administration, the American Society of Mechanical Engineers and several engineering companies. Dr Ayyub is a fellow of ASCE, ASME and SNAME and has served the engineering community in various capacities through societies that include ASNE, ASCE, ASME, SNAME, IEEE-CS and NAFIPS. He is a multiple recipient of the ASNE *Jimmie* Hamilton Award for the best papers in the Naval Engineers Journal in 1985, 1992, 2000 and 2003. Also, he received the ASCE *Outstanding Research Oriented Paper* in the Journal of Water Resources Planning and Management for 1987, the ASCE Edmund Friedman Award in 1989, the ASCE Walter Huber Research Prize in 1997 and the K.S. Fu Award of NAFIPS in 1995. He received the Department of the Army Public Service Award in 2007 for leading the development of the risk model for the hurricane protection system of New Orleans. Dr Ayyub is the author and co-author of more than 550 publications in journals, conference proceedings and reports. Among his publications are more than 20 books, including textbooks on probability, statistics, reliability, risk and uncertainty, used by more than 50 universities worldwide.

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