Models for the Economics of Resilience

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Abstract: Estimating the economic burden of disasters requires appropriate models that account for key characteristics and decision-making needs. Natural disasters in 2011 resulted in $366 billion in direct damages and 29,782 fatalities worldwide. Average annual losses in the United States amount to about $55 billion. Enhancing community and system resilience could lead to significant savings through risk reduction and expeditious recovery. The management of such reduction and recovery is facilitated by an appropriate definition of resilience and associated metrics with models for examining the economics of resilience. This paper provides such microeconomic models, compares them, examines their sensitivities to key parameters, and illustrates their uses. Such models enable improving the resiliency of systems to meet target levels. **DOI: 10.1061/AJRUA6.0000867.**

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Background

Microeconomics plays a central role in informing decisions relating to enhancing system resilience at the structure, network, community, etc. levels. Having structured and consistent thinking across such diverse systems could lead to significant savings through risk reduction and expeditious recovery. The management of such reduction and recovery requires the development of models for the economics of resilience. Having models that are applicable across a broad spectrum of systems ensures consistency and enhances resource utilization and defensibility. Current models used in risk analysis and insurance practices have applicable attributes making them worthy of consideration (Ayyub 2014a). This paper provides a review of resilience definitions and metrics and proposes models for the economics of resilience. These metrics would provide a sound basis for the development of effective decision-making tools for multihazard environments.

It is essential to develop models using effective and economically sound assumptions. Resilience metrics and economic models should also meet a set of requirements necessary to link them to other metrics and enable aggregation at a system level. This section provides important background considerations.

Resilience Definitions

The concept of resilience was formally introduced in ecology, defined as the persistence of relationships within a system (Holling 1973), and measured by the system's ability to absorb change-state variables, driving variables, and parameters and still persist. It, however, appears in different domains ranging from ecology to psychology and psychiatry to infrastructure systems. Ayyub (2014b) and Gilbert (2010) provide summaries of definitions by several reputable entities in their high-impact documents that include the following as the most prominent ones:

- In the *Presidential policy directive on critical infrastructure security and resilience* (PPD-21), (PPD 2013), the "term resilience means the ability to prepare for and adapt to changing conditions and withstand and recover rapidly from disruptions. Resilience includes the ability to withstand and recover from deliberate attacks, accidents, or naturally occurring threats or incidents."
- Ayyub (2014b) suggested a resilience definition that builds on the PPD-21 (PPD 2013) and lends itself for measurement as "the resilience of a system is the persistence of its functions and performances under uncertainty in the face of disturbances." This definition is intended to have a broad use ranging from infrastructures to networks to communities, and enables the measurement of resilience through metrics by meeting the following requirements: (1) building on previous notional definitions; (2) considering initial and residual strength, i.e., capacity and robustness; (3) accounting for abilities to prepare and plan for, absorb, recover from or adapt to adverse events; (4) treating disturbances as events with occurrence rates of stochastic processes; (5) permitting the use of several performance attributes; (6) accounting for changes over time, e.g., aging or improvements; (7) considering full or partial recovery and times to recovery; (8) considering potential enhancements to system performance after recovery; (9) being compatible with other familiar notions such as reliability and risk; and (10) enabling the development of resilience metrics with meaningful units.

Resilience Metrics

Any model that purports to identify cost-effective strategies for increasing resilience should be based at some level on a metric for resilience. Resilience metrics are available as reviewed by Ayyub (2014b) towards suggesting a generalized metric, shown in Fig. 1. The figure shows a schematic representation of a system whose baseline performance is Q, which is a function of time, and is illustrated in Fig. 1 as having aging effects. At time of incident, t_i, it might lead to a failure event with a duration Δt_f. The failure event concludes at time t_f. The failure event is followed by a recovery event with a duration Δt_r. The recovery event concludes at time t_r.

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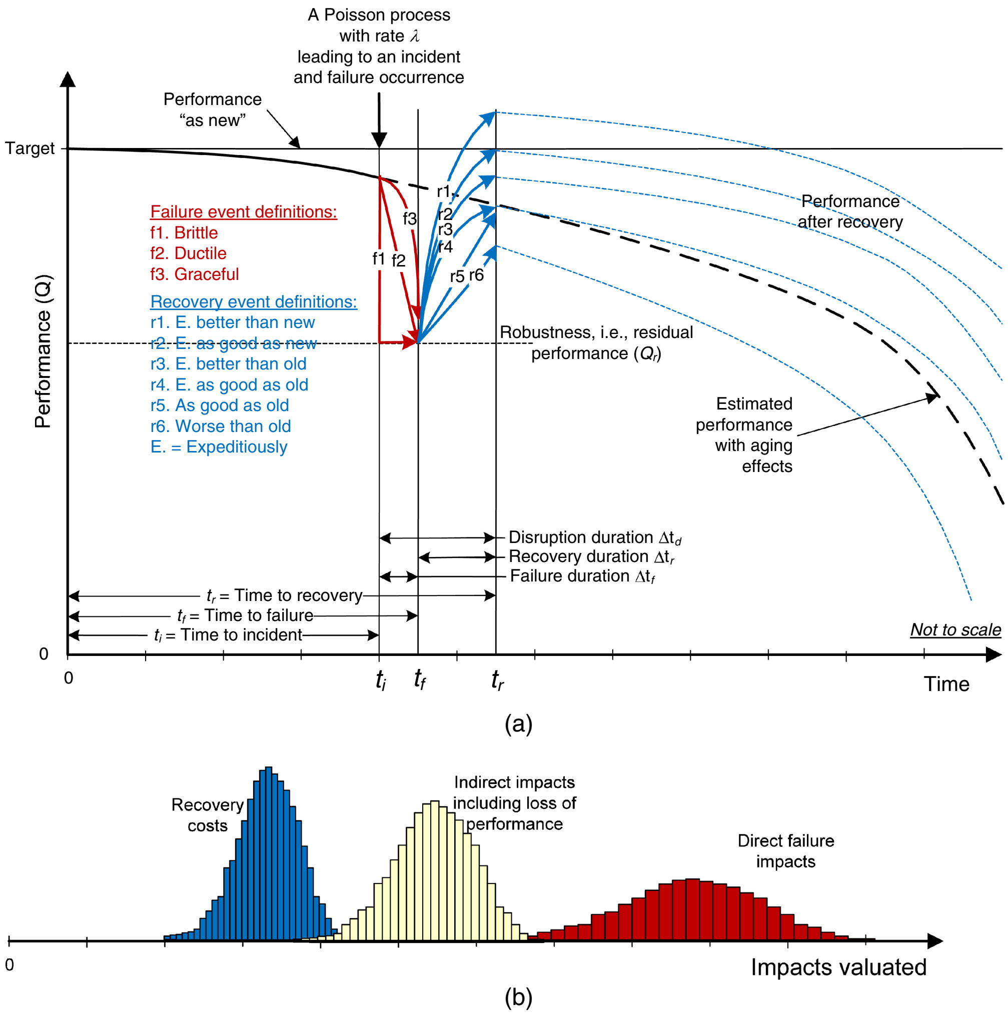

Fig. 1. Modeling resilience (reprinted from Ayyub 2015, © ASCE): (a) definition of resilience; (b) notional probability distributions over the magnitude of costs and direct and indirect losses from disasters

The total disruption has a duration of $\Delta t_d = \Delta t_f + \Delta t_r$. The failure event ($f$) is illustrated in Fig. 1, and is a function of time from t_i to t_f, and represents the loss in performance as a function of time during the failure event. Similarly, the recovery event (r) is illustrated in Fig. 1, and is a function of time from t_f to t_r, and represents the recovery in performance as a function of time during the recovery event.

The model to measure resilience is expressed as

$$\text{Resillence}(R_e) = \frac{t_i + F\Delta t_f + R\Delta t_r}{t_i + \Delta t_f + \Delta t_r} \quad (1)$$

The corresponding *failure profile* F is measured as follows:

$$\text{Failure}(F) = \frac{\int_{t_i}^{t_f} f dt}{\int_{t_i}^{t_f} Q dt} \quad (2)$$

This represents the average performance during the failure period as a percentage of baseline. The corresponding *recovery profile* R is measured as

$$\text{Recovery}(R) = \frac{\int_{t_f}^{t_r} r dt}{\int_{t_f}^{t_r} Q dt} \quad (3)$$

This represents the average performance during the recovery period as a percentage of baseline. Resilience (R_e), then, is approximately a time-weighted average of the performance of the system through a disaster. The failure-profile value (F) can be considered as a measure of robustness and redundancy, whereas the recovery-profile value (R) can be considered as a measure of resourcefulness and rapidity. The time to failure (t_f) can be characterized by its probability density function computed as follows:

$$-\frac{d}{dt}\int_{s=0}^{\infty} \exp\left(-\lambda t\left\{1 - \frac{1}{t}\int_{\tau=0}^{t} F_L[\alpha(t)s]d\tau\right\}\right) f_{S_0}(s)ds \quad (4)$$

where failure = when the load on the system (L) exceeds the system's strength (S); both L and S = random variables; F_L = cumulative probability distribution function of L; and f_S = probability density function of S. The aging effects are considered in this model by the term $\alpha(t)$ representing a degradation mechanism as a function of time t. The term $\alpha(t)$ can also represent improvement to the system. Eq. (4) is based on a Poisson process with an incident occurrence, such as loading, rate of λ, and is based on Ellingwood and Mori (1993). The probability density function of t_f as shown in Eq. (4) is the negative of the derivative of the reliability function.

Fig. 1 shows the losses and costs associated with a disruption consisting of consequences, recovery cost, and indirect cost. This paper provides models to address the question of how much should be invested at the present and throughout the system's life in order to reduce these consequences and losses in a cost-effective manner. It will be shown how the economic models relate to these definitions and metrics of resilience.

Economic Valuation and Discount Rates

Total Economic Valuation

Improving the resiliency of a system to meet target levels requires the examination of system enhancement alternatives in economic terms within a decision-making framework. Relevant decision-analysis methods would typically require the examination of resilience based on its valuation by society at large. Methods for the total economic valuation of resilience are needed and should satisfy the essential requirement of consistency with respect to the definition and metrics of resilience. Concepts from risk analysis and management can be used for this purpose (Ayyub 2014a). The use of a valuation approach with the following characteristics is recommended:

- Anthropocentric in nature based on utilitarian principles;
- Consideration of all instrumental values, including existence value;
- Possess a utilitarian basis to permit the potential for substitution among different sources of value that contribute to human welfare;
- Individual's preferences or marginal willingness to trade one good or service for another that can be influenced by culture, income level and information making it time- and context-specific; and
- Societal values as the aggregation of individual values.

This approach is consistent with National Research Council (2004) and does not capture nonanthropocentric values, e.g., biocentric values (e.g., the intrinsic value of ecosystems, or the value of some species to the ecosystem of which it is a part) and intrinsic values as it related to rights (e.g., the intrinsic value of freedom). In some decisions including environmental policy and law, biocentric intrinsic values should be included in agreement with previous practices, e.g., the Endangered Species Act (1973).

A total economic value (TEV) framework can be constructed based on the preceding characteristics and using individual preferences and values. The TEV framework is necessary to ensure that all components of value are recognized and included while avoiding double counting of values (Bishop et al. 1987; Randall 1991). Economic valuation, as commonly used in decision analysis, is defined as the worth of a good or service as determined by the market. Economists have dealt with this concept initially by estimating the value of a good to an individual alone, and then extend it broadly as it relates to markets for exchange between buyers and sellers for wealth maximization.

An economic measure of the value of a good or the benefit from a service can be defined as the maximum amount a person is willing to pay for this good or service. The concept of willingness to pay (WTP) is central to economic valuation. An alternate measure is the willingness to accept (WTA) of an amount by the person to forgo taking possession of the good or receiving the service. WTP and WTA produce amounts that are expected to be close; however, generally WTA generated amounts are greater than WTP generated amounts due primarily to income levels and affordability factors. The valuation of resilience can be based on the savings in potential direct and indirect losses as well as cost of recovery as illustrated in Fig. 1. Alternatives for enhancing resilience that can reduce these potential losses can be analyzed using models for benefit–cost analysis (Ayyub 2014a).

Choosing Planning Horizons and Discount Rates

Models for the economics of resilience require the consideration of time and the time-value of money. The bases of such a consideration are a planning horizon and a discount rate that are necessarily independent of each other.

The planning horizon impacts the results in a number of ways. Use of a fixed planning horizon either requires the assumption that mitigation measures last for the duration of the planning period and cease to be effective at the end of it, or they require careful estimation of their residual value [ASTM E917-13 (ASTM 2013) for a more-detailed discussion of the handling of residual values]. An infinite planning horizon usually requires assuming that mitigation measures are renewed indefinitely and typically requires the cost of replacement to be accounted for. If these issues are not properly accounted for, then the estimate of the present expected value net benefits will be biased. If, for example, the beneficial effects of a mitigation measure extend beyond the end of the planning period and the residual value is not properly accounted for, then the model will underestimate the benefits. If a mitigation measure loses its effectiveness or is replaced by a different mitigation measure before the end of the (potentially infinite) planning horizon and the modeled stream of benefits does not reflect that, then the model will overestimate its net benefits.

Ayyub (2014a) provides guidance on choosing an appropriate discount rate. It should be based on the situation under consideration. A discount rate associated with a highway system might be different than the discount rate associated with growing the global energy generation capacity with global climate change considerations. In both example cases, cost–benefit analysis can be used and two different rates can be justified. See Rambaud et al. (2005) for a more-complete discussion of the issues involved in discount rates for long-term projects.

The discount rate is a fundamental assumption for estimating the value, e.g., net present value, of developing and enhancing highway systems, power plants, schools, environmental protections, etc., and associated potential losses and costs. Decision-makers or policy-makers must quantify the social marginal cost and the social marginal benefit for each project and compare these projects in order to allocate limited resources. The discount rate appears in both sides of a cost–benefit analysis, i.e., future costs such as maintenance and future benefits such as reduced pollution emissions. Generally calculating the marginal cost is easier than measuring the marginal benefit. Also, the uncertainty in the former is smaller than in the latter. The examination of the effects including benefits require valuating time of people affected, human health and safety, ecological impacts, etc., that have differing time periods associated with the respective effects. A primary issue arises also in decisions spanning multiple generations, creating many situations of mismatch between generations bearing the costs and those generations reaping the benefits.

In risk studies that do not have significant social or society-wide impacts, economic efficiency dictates the use of a discount rate representing the opportunity cost of what else an entity, e.g., a decision-maker, could accomplish with those same funds used to cover the costs of an alternative selected. For example, if the funds could be instead used to invest in the private sector yielding 3% as the next best alternative for using the funds, then 3% would be the discount rate.

In the case of social-project funding, justifiably choosing discount rates requires making ethically subtle choices about the benefits to others. For example, nowadays consumption could impact future generations due to global change in temperature. In this case, choosing a discount rate for the costs and benefits of reducing CO_2 emissions and other harmful greenhouse gases is very important and could drive alternatives considered and decisions made. The discount rate for cost–benefit analysis ranges from 1.4 to about 3% based on various considerations. The small discount rate is from the Stern Review on the Economics of Climate Change (Stern 2006) The U.S. Office of Management and Budget (OMB) provides guidance on this matter and uses a pretax discount rate of 7% as an example in its Circular No. A-94 for benefit-cost analysis of federal programs (U.S. General Accounting Office 1992) and Rushing et al. (2013) provide additional guidance on discount rates for life cycle cost analysis.

Basic Economic Model

The basic economic model balances costs and benefits from the implementation of disaster-mitigation measures. The economic model starts by recognizing that disasters impose losses on a community when they occur. Disaster-mitigation measures are intended to reduce losses from disasters. The objective of an economic model of disaster resilience is to enable a community to identify cost-effective mitigation measures. This paper follows Vugrin et al. (2009) in distinguish between losses and response and recovery costs. Losses are damages caused directly or indirectly by the disaster. They would include, among other things, capital losses, deaths and injuries resulting from the disaster, and business interruption costs. Response and recovery costs are those costs incurred by a community in the time around the time of the disaster whose purpose is to mitigate the losses from the disaster and return the community to normalcy. They would include, among other things, costs from emergency response during the disaster, bottled water if the water supply is interrupted, and debris removal.

Costs are also associated with disaster planning, prevention, mitigation, response, and recovery. What mainly distinguishes mitigation costs from response and recovery costs is that mitigation costs occur on an ongoing basis, while response and recovery costs are specific to the time in and around a disaster. Mitigation costs include things like the incremental costs of disaster-resilient building codes, maintenance of Emergency Operations Centers, etc. Response and recovery costs include things like emergency response during disasters, cleanup, and temporary housing.

Benefits are the reduction in losses and response and recovery costs that result from the implementation of a mitigation plan.

Since the objective is to find cost-effective strategies for increasing resilience, this should be relatable back to the definition of the resilience metric. One way of doing so would define the recovery costs and losses as

$$D_i(P) + R_i(P) = \int_{t_i}^{t_r} Q dt - \int_{t_i}^{t_r} f dt - \int_{t_i}^{t_r} r dt \qquad (5)$$

For any mitigation plan, the net benefit is

$$J(P) = E\{L[\mathfrak{D}(P_0), k] - L[\mathfrak{D}(P), k] - \sum_t C(t, P)e^{-kt}\} \qquad (6)$$

where E = expected value operator. The difference in costs and losses represents the fact that it is the changes from the status quo that a particular mitigation plan represents that are of interest. In some cases, the reduction in disaster costs and losses, $L[\mathfrak{D}(P_0), k] - L[\mathfrak{D}(P), k]$, will be referred to as *savings*.

Since the choices are forward-looking—it is future costs and future losses that are of interest—the exact values of many of the terms are uncertain. Timings of future disasters and their associated losses are uncertain. Response and recovery costs associated with those disasters are also uncertain. In many cases, the future costs of selected mitigation measures are uncertain.

The typical approach for handling this uncertainty is to base decisions on the expected value of the present value of the future net benefits. The expected value is essentially the average of all possible ranges of future values, weighted for their probability. In order to compute the expected value the probabilities of the possible outcomes need to be identified.

It is sometimes useful to distinguish between five different levels of uncertainty. They form a continuum from the fully known to the completely unknown. Each level of uncertainty will now be discussed.

Something that is *known* has no uncertainty associated with it. In general, contractual payments and bond payments are known. Their timing and amount is fixed in advance.

Something with *well-characterized uncertainty* is something for which the value is not known in advance, but has a known and well-defined probability distribution. Hazards often fit this category. It is not known in advance when a hazard will strike or how much damage it will do when it does. But there are often well-defined probability distributions over hazards. For example with earthquakes the USGS has developed maps giving the peak ground acceleration with a 10% chance of exceedance in 50 years for most of the United States.

Something that is *ambiguous* is something for which the exact probability distribution is unknown, but for which a set of probability distributions could be identified. The discount rate any particular community prefers is not known, nor is there a well-defined probability distribution for the discount factor that is known to the literature. However plausible bounds can be put on the discount factor, which makes it possible to define a set of plausible probability distributions over the value of the discount factor.

Poorly-defined uncertainty applies to items where it is difficult to even construct a probability distribution. Model uncertainty is an example. With model uncertainty the range of possible (or even plausible) models is very large and difficult to define. Defining a probability distribution over those plausible models (let alone a range of distributions) is not a practical possibility.

Unknowns are things of whose existence the decision-maker is not even aware—i.e., the so-called unknown unknowns.

In general, a local community is interested in identifying the most cost effective mitigation plan. That is, a community is interested in solving the following problem:

$$\max_{P \in \mathcal{P}} J(P) \qquad (7)$$

Loss Models

This paper introduces and compares two different loss models, a Discounted Model and an Insurance Model (Ayyub 2014a). These two models were selected because they are commonly used in economics and in the insurance industry.

The loss models have a number of features in common. In particular, they both make the following assumptions about the occurrence and magnitude of disasters:

1. Disaster damages and times between disasters are independent of each other; and
2. The occurrence of disasters follows a Poisson process, with rate parameter λ.

λ is approximately equal to the inverse of the return interval for the disaster.

Both loss models assume that there is a planning period, $T_{\max}$, over which the analysis is considered, although the Discounted Model allows for the possibility that $T_{\max}$ is infinite.

Discounted Loss Model

For the first model, losses are the discounted sum of disaster losses over the planning period

$$L[\mathfrak{D}(P), k] = \sum_{i \in I(T_{\max})} [D_i(P) + K_i(P)] e^{-kT_i} = \sum_{i \in I(T_{\max})} S_i(P) e^{-kT_i} \quad (8)$$

The two assumptions provided above result in the following theorem:

Theorem 1: Given Assumptions 1 and 2 defined earlier, the expected value and variance of disaster losses when $k > 0$ and $T_{\max} < \infty$ for the Discounted Model are

$$\bar{V}_1(P, T_{\max}, \lambda, k) = \frac{\lambda}{k}(1 - e^{-kT_{\max}})\bar{S}(P) \quad (9)$$

and

$$\sigma_1^2(P, T_{\max}, \lambda, k) = \frac{\lambda}{2k}(1 - e^{-2kT_{\max}})[\sigma_S^2(P) + \bar{S}^2(P)] \quad (10)$$

For the special case where $T_{\max}$ is infinite, this becomes

$$\bar{V}_1(P, \infty, \lambda, k) = \frac{\lambda}{k}\bar{S}(P) \quad (11)$$

and

$$\sigma_1^2(P, \infty, \lambda, k) = \frac{\lambda}{2k}[\sigma_S^2(P) + \bar{S}^2(P)] \quad (12)$$

And for the special case where the discount factor k goes to zero, this becomes:

$$\bar{V}_1(P, T_{\max}, \lambda, 0) = \lambda T_{\max}\bar{S}(P) \quad (13)$$

and

$$\sigma_1^2(P, T_{\max}, \lambda, 0) = \lambda T_{\max}[\sigma_S^2(P) + \bar{S}^2(P)] \quad (14)$$

Proof is in Appendix I.

Insurance Loss Model

The insurance industry has developed its models and practices for building business cases for new insurance products and updating these models using Bayesian methods. The basic Insurance model is

$$L[\mathfrak{D}(P), k] = \sum_{i \in I(T_{\max})} D_i(P) + K_i(P) = \sum_{i \in I(T_{\max})} S_i(P) \quad (15)$$

The Insurance Model also adds the following two additional assumptions

3. The probability distribution over damages, F_S, is a normal distribution with mean $\mu(P)$, and standard deviation, $\sigma(P)$; and
4. The actual value of the Poisson Parameter, λ, is subject to well-characterized uncertainty, with a probability distribution f_λ.

Up through Assumption 3, this is just a special case of the Discounted Model above, with discount factor, k, set to zero and a distribution specified for damages. The inclusion of Assumption 4 serves as an extension to the Discounted Model.

That leads to Theorem 2, which is based on the development in Ayyub (2014a):

Theorem 2: Given Assumptions 1 through 3 given previously, the cumulative probability distribution of losses for the Insurance Model is

$$F[s; \lambda T_{\max}, \mu(P), \sigma(P)] = \sum_{n=0}^{\infty} e^{-\lambda T_{\max}} \frac{(\lambda T_{\max})^n}{n!} F_S[s; n\mu(P), \sqrt{n}\sigma(P)] \quad (16)$$

with mean and standard deviation given by Eqs. (8) and (9).

Given Assumptions 1 through 4, the cumulative probability distribution of losses for the Insurance Model (after taking expectation over λ) is

$$F[s; T_{\max}, \mu(P), \sigma(P)] = \int_0^\infty F[s; \lambda T_{\max}, \mu(P), \sigma(P)] f_\lambda(\lambda) d\lambda \quad (17)$$

Proof is in Appendix II.

Since differentiation is a linear operator, Eq. (15) could be rewritten by replacing the cumulative distribution functions with the probability distribution functions without making any further changes.

Comparison of the Loss Models

In interpreting the preceding results, it is worth noting that $\lambda\bar{S}(P)$ and $\lambda\mu(P)$ are the average annual losses in the two loss models. Similarly, $\lambda\bar{S}(P)T_{\max}$ and $\lambda\mu(P)T_{\max}$ are the average losses in the two loss models that would occur over a period of $T_{\max}$ years.

There are a few key differences between these loss models:

1. The Discounted Model explicitly accounts for discounting over time, while the Insurance Model does not discount time at all;
2. The Discounted Model makes no assumption about the damage distribution, while the Insurance Model explicitly assumes that damages are normally distributed;
3. The Discounted Model assumes a fixed λ, while the Insurance Model explicitly assumes a distribution for λ; and
4. The Discounted Model simply develops the expected value, while the Insurance Model explicitly develops a probability distribution over damages. The assumption of Normality for the damage distribution is what allows a closed-form solution to the probability distribution of losses to be identified.

Discussion of the Assumptions

Assumptions 1 and 2 listed earlier are certainly incorrect, but their impact on the overall results is likely to be minimal. Weather-related hazards are correlated across time, and fire and geologic events tend to have reduced probabilities of occurrence after major events. Earthquake magnitude is correlated with time since the previous event. These could be modeled, for example, by assuming that the time between events follows a Weibull distribution, or more generally by assuming that the time between events is governed by a known hazard function. Neither alternative approach will result in a simple close-form solution and, given the information that is likely to be available, Assumptions 1 and 2 will probably provide as good an approximation to the actual disaster sequence as any other that could be made.

The assumption of normality for damages, used in the Insurance Model is also certainly wrong when applied to disasters. Damage values are very highly skewed and nonnegative. On the other hand, the assumption allows for a major simplification of the model. In particular, it allows a closed-form expression to be used for the probability distribution over the net present value of losses. The normality assumption is not required to develop a probability distribution over net present value of losses using the model with discounting, but without the assumption of normality determination of the distribution of losses typically requires the use of Monte Carlo techniques.

The impact of the fourth assumption, that the rate at which disasters occur is itself unknown and subject to well-characterized uncertainty, will depend on the distribution used. One that would likely be applied would be a uniform distribution. As with the earlier assumption, any inaccuracy that brings will likely be minimal, at least when compared to any other assumption that could be made.

The Discounted Model assumes that future costs and benefits are discounted relative to the present, while the Insurance Model does not discount future cash flows. For very short time planning periods, the difference between the two loss models will be minimal, but for the longer planning periods that are likely to be used in evaluating disaster resilience projects, the differences will be significant.

Examples

These examples illustrate the economic model and demonstrate the similarities and differences of the loss models.

Suppose that a city is considering an investment in a disaster mitigation project. The project will cost $10 million, have $0.5 million in annual operations and maintenance costs, and have a lifetime of 30 years. For the sake of simplicity, it is assumed here that the costs are known, however that will typically not be the case. When costs have some degree of uncertainty associated with them, it will need to be accounted for in the analysis of the proposed project.

Here estimated savings from the mitigation measures are used. Given the assumptions defined in preceding sections, it can be shown that the equations in Theorem 1 apply directly to savings, reducing the computational effort required.

In this example, two cases are considered. In Case 1, disasters are rare but relatively large, while in Case 2 disasters are relatively common but small. Details of the disaster occurrence and value of savings are listed in Table 1.

Table 1. Expected Savings from Mitigation Measure

Case	λ	Reduction in losses ($ millions)	σ
Case 1	0.02	50	8
Case 2	0.5	2.5	1

Table 2. Expected Present Value of Costs and Savings for the Two Loss Cases for Various Discount Rates

Discount rate (%)	Costs	Savings ($) Case 1	Savings ($) Case 2
0.0	25.00	30.00	37.50
1.4	22.15	24.46	30.58
3.0	19.70	19.70	24.63
6.39	16.31	13.05	16.31
7.0	15.89	12.22	15.27
10.0	14.31	9.09	11.36

Table 2 lists the present expected value for costs and the present expected value for savings using the Discounted Model for a variety of discount rates. A plot of costs and savings versus discount rate is also shown in Fig. 2. In Case 1, the expected savings for the proposed mitigation measure exceed the present value of costs for any discount rate less than about 3%, while in Case 2, the savings exceeds costs for discount rates below about 6.4%. Costs are less responsive to discount rates because a large portion of the costs of the mitigation plan are incurred up front, while disasters (and hence losses and savings) are entirely future.

Remembering that the Insurance Model is a special case of the Discounted Model, the present expected values of savings for the Insurance Model are listed in line 1 of Table 2. It is important when using the Insurance Model to use the same discount rate of zero for costs as for savings. So the appropriate value of costs for comparison is also that on Line 1 of Table 2.

The Insurance Model also allows probability distributions over savings to be computed. Probability distributions for savings for the proposed mitigation measure are included as Figs. 3 and 4. Fig. 3 shows the probability density of savings (note that in both cases, there is a nonzero probability that zero damages will occur, which is not displayed on the figure). Fig. 4 shows the cumulative probability density function for savings. On both figures, the cost of the proposed mitigation measure is included for reference. For Case 1, savings will exceed costs 45% of the time, while for Case 2, savings will exceed costs 89% of the time.

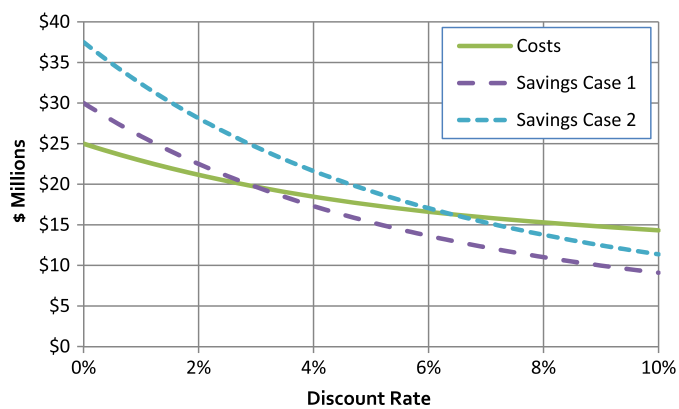

Fig. 2. Costs and savings versus discount rate

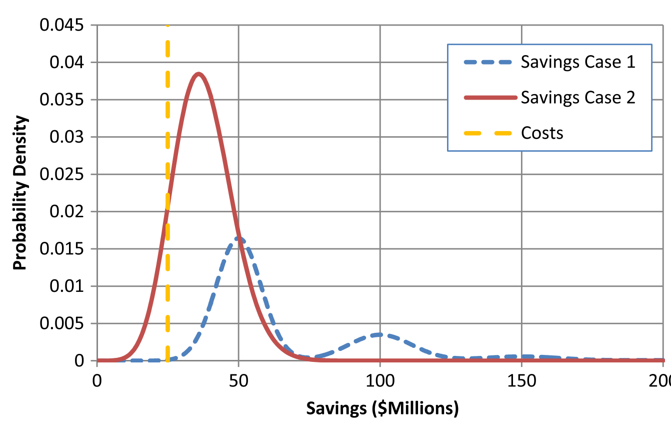

Fig. 3. Probability distribution of costs and savings using the insurance model

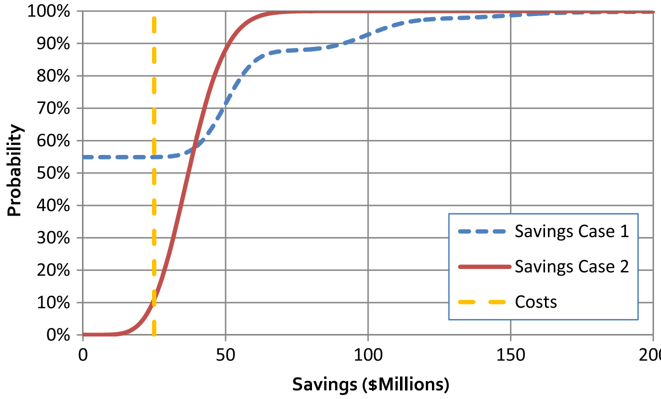

Fig. 4. Cumulative distribution function of costs and savings using the insurance model

Table 3. Sensitivity Analysis for Case 1

Discount rate (%)	$\lambda = 0.01$ ($)	$\lambda = 0.05$ ($)	$T = 25$ ($)	$T = 35$ ($)	$\mu = 45$ ($)	$\mu = 55$ ($)
0.0	15.00	75.00	25.00	35.00	27.00	33.00
1.4	12.25	61.24	21.09	27.67	22.05	26.95
3.0	9.91	49.54	17.62	21.71	17.84	21.80
7.0	6.27	31.34	11.80	13.05	11.28	13.79
10.0	4.75	23.76	9.18	9.70	8.55	10.45

Table 4. Sensitivity Analysis for Case 2

Discount rate (%)	$\lambda = 0.4$ ($)	$\lambda = 0.6$ ($)	$T = 25$ ($)	$T = 35$ ($)	$\mu = 2$ ($)	$\mu = 3$ ($)
0.0	30.00	45.00	31.25	43.75	30.00	45.00
1.4	24.50	36.74	26.37	34.59	24.50	36.74
3.0	19.82	29.73	22.02	27.14	19.82	29.73
7.0	12.54	18.80	14.75	16.32	12.54	18.80
10.0	9.50	14.25	11.47	12.12	9.50	14.25

A simple sensitivity analysis of the expected value of savings for the Discounted Model is included in Tables 3 and 4. For each case, notional upper and lower bounds were used for each of the input variables (holding all the other variables constant). The sensitivity analysis provides a sense of which variables have the greatest impact on the results (taking into account the degree of certainty regarding each of the variables), and provides a sense of how much variation is associated with the level of uncertainty in the problem. Since the Insurance Model is a special case of the Discounted Model, the first line in Tables 3 and 4 represent the sensitivity of the expected value of savings for the Insurance Model.

A more-detailed analysis of the sensitivity to the input variables (results not shown) indicates that increasing k decreases the sensitivity to all the input variables. Increasing λ, and μ (or $\bar{S}$) increases sensitivity to all input variables. Increasing $T_{\max}$ increases sensitivity to all terms except $T_{\max}$ itself, where increasing $T_{\max}$ actually decreases the sensitivity.

Conclusions

Significant savings could be realized by enhancing the resilience of a system, including buildings, infrastructure, networks, and communities, through risk reduction and expeditious recovery. However care must be taken to ensure that measures taken to enhance resilience are cost-effective. This paper describes an economic model to estimate the costs versus the savings from proposed measures for enhancing disaster resilience, introduces two loss models for valuing the performance of a system, provides simple expressions for their computation, and provides an illustrative example of the workings of the economic model.

The Discounted Model assumes that costs and losses from disasters are independent and identically distributed, and occur according to a Poisson process with a fixed, known rate. Otherwise, it makes no assumption about the distribution of damages. The Insurance Model adds the assumption that costs and losses follow a normal distribution, ignores discounting over time, and assumes that the disaster rate is unknown, but with a known probability distribution. Based on those assumptions, this paper develops simple expression for the expected value and variance of the present expected value of damages. In addition, for the Insurance Model, it develops an expression for the probability distribution over damages.

The assumptions of independence and Poisson are certainly incorrect, but their impact on the overall results is likely to be minimal. Given the information that is likely to be available, these assumptions will probably provide as good an approximation to the actual disaster sequence as any other that could be made, and they allow the expected net present value of costs and losses to be expressed with a set of simple closed-form expressions.

The assumption of normality for damages, used in the Insurance Model, is also certainly wrong when applied to disasters. However, it allows for a major simplification of the model by allowing a closed-form expression to be used for the probability distribution over the net present value of losses. The use of nonnormal probability distributions would require solving convolution integrals as provided by Ayyub (2014a).

In summary, the expressions developed in this paper will simplify evaluating the cost-effectiveness of resilience-improving measures and thus help communities improve their resilience.

Appendix I. Theorem 1

Given Assumptions 1 and 2 stated earlier, the expected value and variance of disaster losses when $k > 0$ and $T_{\max} < \infty$ for the Discounted Model are

$$\bar{V}_1(P, T_{\max}, \lambda, k) = \frac{\lambda}{k}(1 - e^{-kT_{\max}})\bar{S}(P) \tag{18}$$

and

$$\sigma_1^2(P, T_{\max}, \lambda, k) = \frac{\lambda}{2k}(1 - e^{-2kT_{\max}})[\sigma_S^2(P) + \bar{S}^2(P)] \tag{19}$$

For the special case where $T_{\max}$ is infinite, this becomes

$$\bar{V}_1(P, \infty, \lambda, k) = \frac{\lambda}{k}\bar{S}(P) \tag{20}$$

and

$$\sigma_1^2(P, \infty, \lambda, k) = \frac{\lambda}{2k}[\sigma_S^2(P) + \bar{S}^2(P)] \tag{21}$$

And for the special case where the discount factor k goes to zero, this becomes

$$\bar{V}_1(P, T_{\max}, \lambda, 0) = \lambda T_{\max}\bar{S}(P) \tag{22}$$

and

$$\sigma_1^2(P, T_{\max}, \lambda, 0) = \lambda T_{\max}[\sigma_S^2(P) + \bar{S}^2(P)] \tag{23}$$

Proof: To simplify notation, the reference to plan (P) will be dropped throughout.

First the expected value of the case where $T_{\max}$ is infinite is computed. That is, the value of the following equation is determined:

$$\bar{V} = E\left(\sum_{i=1}^{\infty} S_i e^{-kT_i}\right) \tag{24}$$

where, for now, it is assume that k is strictly positive.

For convenience, define the following random variables:

$$V_H(t) = \sum_{i=1}^{\infty} I\{T_i > t\} S_i e^{-k(T_i - t)} \tag{25}$$

$$V_L(t) = \sum_{i=1}^{\infty} I\{T_i \le t\} S_i e^{-k(T_i - t)} \tag{26}$$

$$V_n = \sum_{i=n}^{\infty} S_i e^{-k(T_i - T_{n-1})} \tag{27}$$

where $I\{\cdot\}$ = indicator function; $V_H(t)$ = value of all disaster occurring after time t; $V_L(t)$ = value of all disaster occurring up to and including time t; and V_n = value of all disasters from the nth disaster on. In all cases value is determined from the perspective of time t, or the $n-1$th disaster. Now Eq. (27) is used to rewrite Eq. (24) as

$$\bar{V} = E\{(S_1 + V_2)e^{-kT_1}\} \tag{28}$$

Since, by Assumption 1, all values for damage and time-between are independent of each other, then S_1, T_1 and V_2 are independent of each other. So

$$\bar{V} = [E(S_1) + E(V_2)]E\{e^{-kT_1}\} \tag{29}$$

Or

$$\bar{V} = [\bar{S} + E(V_2)]E\{e^{-kT_1}\} \tag{30}$$

Since, by Assumption 1, the times-between and damages are all independent of each other

$$E(V_1) = E(V_2) = E(V_i) = E[V_H(t)] = \bar{V} \tag{31}$$

That gives

$$\bar{V} = (\bar{S} + \bar{V})E\{e^{-kT_1}\} \tag{32}$$

which can also be expressed as

$$\bar{V} = (\bar{V} + \bar{S})\int_0^{\infty} e^{-kT} f(T)dT \tag{33}$$

By Assumption 2, the disaster sequence is a Poisson process, so the time between events is an exponential distribution. Then this expression becomes

$$\bar{V} = (\bar{V} + \bar{S})\int_0^{\infty} \lambda e^{-kT} e^{-\lambda T} dT \tag{34}$$

Working out the math, this becomes

$$\bar{V} = \frac{\lambda}{k+\lambda}(\bar{V} + \bar{S})\int_0^{\infty}(k+\lambda)e^{-(k+\lambda)T}dT \tag{35}$$

$$\bar{V} = -\frac{\lambda}{k+\lambda}(\bar{V} + \bar{S})e^{-(k+\lambda)T}\Big]_0^{\infty} \tag{36}$$

$$\bar{V} = \frac{\lambda}{k+\lambda}(\bar{V} + \bar{S}) \tag{37}$$

$$\bar{V}\left(1 - \frac{\lambda}{k+\lambda}\right) = \frac{\lambda}{k+\lambda}\bar{S} \tag{38}$$

This then becomes

$$\bar{V} = \frac{\lambda}{k}\bar{S} \tag{39}$$

which is Eq. (20)

To determine the present expected value of any disasters occurring before a specified point of time, $T_{\max}$, this can be easily obtained by subtracting out the discounted value at time $T_{\max}$. That is

$$\bar{V}(t) = E[V_1 - V_H(t)e^{-kT_{\max}}] = \bar{V} - \bar{V}e^{-kt} = \frac{\lambda}{k}\bar{S}(1 - e^{-kT_{\max}}) \tag{40}$$

which is Eq. (18).

Variance can be determined similarly.

Let

$$\sigma_V^2 = \mathrm{Var}(V) = \mathrm{Var}\left(\sum_{i=n}^{\infty} S_i e^{-kT_1}\right) \tag{41}$$

Once again, inserting Eq. (27) into Eq. (41) gives

$$\sigma_V^2 = \mathrm{Var}(V) = \mathrm{Var}\{(S_1 + V_2)e^{-kT_1}\} \tag{42}$$

Or

$$\begin{aligned}\sigma_V^2 &= E\{(S_1 + V_2)^2 e^{-2kT_1}\} - \bar{V}^2\\ &= E\{(S_1^2 + 2S_1V_2 + V_2^2)e^{-2kT_1}\} - \bar{V}^2\\ &= E\{[(S_1^2 - \bar{S}^2) + 2S_1V_2 + (V_2^2 - \bar{V}^2) + \bar{S}^2 + \bar{V}^2]e^{-2kT_1}\} - \bar{V}^2\end{aligned} \tag{43}$$

Again, independence (Assumption 1) makes it possible to evaluate the expectation of each term separately

$$\begin{aligned}&= [E(S_1^2 - \bar{S}^2) + 2E(S_1)E(V_2) + E(V_2^2 - \bar{V}^2) + \bar{S}^2\\ &\quad + \bar{V}^2]E\{e^{-2\mathrm{kT_1}}\} - \bar{V}^2\end{aligned} \tag{44}$$

$$= [\sigma_S^2 + 2\bar{S}\,\bar{V} + E(V_2^2 - \bar{V}^2) + \bar{S}^2 + \bar{V}^2]E\{e^{-2\mathrm{kT_1}}\} - \bar{V}^2 \tag{45}$$

And again, since this is an infinite sequence where damages and times-between are independent

$$E(V^2 - \bar{V}^2) = E(V_1^2 - \bar{V}^2) = E(V_2^2 - \bar{V}^2) = E(V_i^2 - \bar{V}^2) = \sigma_V^2 \tag{46}$$

So this becomes

$$\sigma_V^2 = (\sigma_S^2 + \sigma_V^2 + \bar{S}^2 + 2\overline{SV} + \bar{V}^2)E\{e^{-2kT_1}\} - \bar{V}^2 \tag{47}$$

Again, since times between follow an exponential distribution (Assumption 2), this becomes

$$\sigma_V^2 = (\sigma_S^2 + \sigma_V^2 + \bar{S}^2 + 2\overline{SV} + \bar{V}^2) \int_0^\infty \lambda e^{-2kT} e^{-\lambda T} dT - \bar{V}^2 \quad (48)$$

$$\sigma_V^2 = \frac{\lambda}{2k+\lambda} (\sigma_S^2 + \sigma_V^2 + \bar{S}^2 + 2\bar{S}\,\bar{V} + \bar{V}^2) \int_0^\infty (2k + \lambda) e^{-(2k+\lambda)T} dT - \bar{V}^2 \quad (49)$$

$$\sigma_V^2 = \frac{\lambda}{2k+\lambda} (\sigma_S^2 + \sigma_V^2 + \bar{S}^2 + 2\bar{S}\,\bar{V} + \bar{V}^2) - \bar{V}^2 \quad (50)$$

Or

$$\sigma_V^2 \left(1 - \frac{\lambda}{2k+\lambda}\right) = \frac{\lambda}{2k+\lambda} (\sigma_S^2 + \bar{S}^2 + 2\bar{S}\,\bar{V}) - \bar{V}^2 \left(1 - \frac{\lambda}{2k+\lambda}\right) \quad (51)$$

$$\sigma_V^2 \frac{2k}{2k+\lambda} = \frac{\lambda}{2k+\lambda} (\sigma_S^2 + \bar{S}^2 + 2\bar{S}\,\bar{V}) - \bar{V}^2 \frac{2k}{2k+\lambda} \quad (52)$$

$$\sigma_V^2 = \frac{\lambda}{2k} (\sigma_S^2 + \bar{S}^2 + 2\bar{S}\,\bar{V}) - \bar{V}^2 \quad (53)$$

Substitute the expression for $\bar{V}$ from Eq. (39)

$$\sigma_V^2 = \frac{\lambda}{2k} \left(\sigma_S^2 + \bar{S}^2 + 2\bar{S}\frac{\lambda}{k}\bar{S}\right) - \left(\frac{\lambda}{k}\bar{S}\right)^2 \quad (54)$$

$$\sigma_V^2 = \frac{\lambda}{2k}\sigma_S^2 + \frac{\lambda}{2k}\left(\frac{k+2\lambda}{k}\right)\bar{S}^2 - \frac{\lambda^2}{\bar{k}^2} S^2 \quad (55)$$

$$\sigma_V^2 = \frac{\lambda}{2k}\sigma_S^2 + \left(\frac{\lambda}{2k} + \frac{\lambda^2}{\bar{k}^2} - \frac{\lambda^2}{\bar{k}^2}\right)\bar{S}^2 \quad (56)$$

Or

$$\sigma_V^2 = \frac{\lambda}{2k}\sigma_S^2 + \frac{\lambda}{2k}\bar{S}^2 = \frac{\lambda}{2k}(\sigma_S^2 + \bar{S}^2) \quad (57)$$

which is Eq. (21).

Again, to get the variance for a fixed time interval, partition V and take the variance

$$\sigma_V^2 = \text{Var}(V) = \text{Var}[V_L(T_{\max}) + V_H(T_{\max})e^{-kT_{\max}}] \quad (58)$$

Since, by Assumption 1, $V_L(T_{\max})$ and $V_H(T_{\max})$ are independent for fixed $T_{\max}$, and for any two independent random variables, X and Y, $\text{Var}(X+Y) = \text{Var}(X) + \text{Var}(Y)$, this becomes

$$\sigma_V^2 = \text{Var}[V_L(T_{\max})] + \text{Var}[V_H(T_{\max})e^{-kT_{\max}}] \quad (59)$$

Furthermore, $\sigma_V^2(T_{\max}) = \text{Var}[V_L(T_{\max})]$ is the expression whose value is to be determined

$$\sigma_V^2 = \sigma_V^2(T_{\max}) + \text{Var}[V_H(T_{\max})e^{-kT_{\max}}] \quad (60)$$

Expanding the remaining term

$$\sigma_V^2 = \sigma_V^2(T_{\max}) + E\{[V_H(T_{\max})e^{-kT_{\max}}]^2\} - \{E[V_H(T_{\max})e^{-kT_{\max}}]\}^2 \quad (61)$$

Since $T_{\max}$ is fixed and nonstochastic, it can be pulled out of the expression

$$\sigma_V^2 = \sigma_V^2(T_{\max}) + (E\{[V_H(T_{\max})]^2\} - \{E[V_H(T_{\max})]\}^2)e^{-2kT_{\max}} \quad (62)$$

$$\sigma_V^2 = \sigma_V^2(T_{\max}) + \text{Var}[V_H(T_{\max})]e^{-2kT_{\max}} \quad (63)$$

And as discussed earlier, $\sigma_V^2 = \text{Var}[V_H(T_{\max})]$, so this becomes

$$\sigma_V^2 = \sigma_V^2(T_{\max}) + \sigma_V^2 e^{-2kT_{\max}} \quad \text{Or}$$
$$\sigma_V^2(T_{\max}) = \sigma_V^2(1 - e^{-2kT_{\max}}) = \frac{\lambda}{2k}(\sigma_S^2 + \bar{S}^2)(1 - e^{-2kT_{\max}}) \quad (64)$$

which is Eq. (19).

As far as the case where $k = 0$, the following limit argument gives an expression for $\bar{V}(T_{\max})$ and $\sigma_V^2(T_{\max})$ when $k = 0$.

Starting with Eq. (40) from above

$$\bar{V}(T_{\max}) = \frac{\lambda}{k}\bar{S}(1 - e^{-kT_{\max}}) \quad (65)$$

Using L'Hôpital's rule (A. Pantelous, personal communication, 2015):

$$\lim_{k\to 0} \bar{V}(T_{\max}) = \lambda\bar{S} \lim_{k\to 0} \frac{1 - e^{-kT_{\max}}}{k} = \lambda\bar{S} \lim_{k\to 0} \frac{T_{\max}e^{-kT_{\max}}}{1} = \lambda\bar{S}T_{\max} \lim_{k\to 0} e^{-kT_{\max}} \quad (66)$$

Take the limit as $k \to 0$, this becomes

$$\lim_{k\to 0} \bar{V}(T_{\max}) = \bar{S}\lambda T_{\max} \quad (67)$$

Which is Eq. (22).

Similarly for $\sigma_V^2(T_{\max})$

$$\lim_{k\to 0} \sigma_V^2(T_{\max}) = \lambda(\sigma_S^2 + \bar{S}^2) \lim_{k\to 0} \frac{1 - e^{-2kT_{\max}}}{2k} = \lambda(\sigma_S^2 + \bar{S}^2) \lim_{k\to 0} \frac{2T_{\max}e^{-2kT_{\max}}}{2} = \lambda(\sigma_S^2 + \bar{S}^2)T_{\max} \lim_{k\to 0} e^{-2kT_{\max}} \quad (68)$$

So

$$\lim_{k\to 0} \sigma_V^2(T_{\max}) = (\sigma_S^2 + \bar{S}^2)\lambda T_{\max} \quad (69)$$

Which is Eq. (23).

Appendix II. Theorem 2

Given Assumptions 1 through 3 as stated earlier, the cumulative probability distribution of losses for the Insurance Model is

$$F[s; \lambda T_{\max}, \mu(P), \sigma(P)] = \sum_{n=0}^{\infty} e^{-\lambda T_{\max}} \frac{(\lambda T_{\max})^n}{n!} F_S[s; n\mu(P), \sqrt{n}\sigma(P)] \quad (70)$$

with mean and standard deviation given by Appendix I.

Given Assumptions 1 through 4, the probability distribution of losses for the Insurance Model (after taking expectation over λ) is

$$F[s;T_{\max},\mu(P),\sigma(P)] = \int_0^\infty F[s;\lambda T_{\max},\mu(P),\sigma(P)]f_\lambda(\lambda)d\lambda \tag{71}$$

Proof: The sum of n independent random variables distributed as $N(\mu,\sigma)$ has the cumulative probability distribution $N(n\mu,\sqrt{n}\sigma)$. Again, to simplify notation, the reference to plan (P) is dropped throughout.

Since disasters are a Poisson process with rate parameter, λ, then the probability that n disasters will occur is

$$e^{-\lambda t}\frac{(\lambda t)^n}{n!} \tag{72}$$

And the cumulative probability distribution is

$$F(s;\lambda t,\mu,\sigma) = P\{n=0\} + P\{n=1\}F_S(s;\mu,\sigma) + \cdots + P\{n=m\}F_S(s;n\mu,\sqrt{n}\sigma) + \cdots \tag{73}$$

or

$$F(s;\lambda T_{\max},\mu,\sigma) = \sum_{n=0}^{\infty} e^{-\lambda T_{\max}}\frac{(\lambda T_{\max})^n}{n!}F_S(s;n\mu,\sqrt{n}\sigma) \tag{74}$$

which is Eq. (70). Since all the conditions of Theorem 1 apply, mean and standard deviation will be as determined in Appendix I.

Taking the expectation over λ, then, is simply

$$F[s;T_{\max},\mu(P),\sigma(P)] = \int_0^\infty F[s;\lambda T_{\max},\mu(P),\sigma(P)]f_\lambda(\lambda)d\lambda \tag{75}$$

which is Eq. (71).

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Notation

The following symbols are used in this paper:

$C(t,P)$ = costs as a function of time. Costs are specific to the mitigation plan and are defined relative to the status quo. That is, by definition the costs associated with the status quo plan are zero;

$D_i(P)$ = loss from the ith disaster, losses depend on the mitigation plan. Since the disasters are in the future, this is a random variable;

$\mathfrak{D}(P)$ = sequence of disasters, where $\mathfrak{D}(P) = \{T_i, D_i(P), K_i(P)\}_{i=1}^{\infty}$. This is a random sequence;

$I(T_{\max}) = \{i|T_i \leq T_{\max}\}$ = set of all disasters that occur before time $T_{\max}$;

$K_i(P)$ = response and recovery costs from the ith disaster. Response and recovery costs depend on the mitigation plan. Since the disasters are in the future, this is a random variable;

k = discount rate;

$L[\mathfrak{D}(P),k]$ = present expected value of costs and losses as a function of the (stochastic) damage sequence, the mitigation plan, and the discount factor;

$P \in \mathcal{P}$ = some specific mitigation plan;

$P_0 \in \mathcal{P}$ = status quo mitigation plan;

$S_i(P) = D_i(P) + K_i(P)$, which is the total cost (including both losses and response and recovery costs) from the ith disaster given mitigation plan P;

$\bar{S}(P) = E[S_i(P)]$ is the expected total cost of a disaster given plan P;

T_i = time of the ith disaster. Since the disasters are in the future, this is a random variable;

$T_{\max}$ = planning period over which the analysis is considered; and

$\mathcal{P}$ = set of all possible mitigation plans. This includes any set of choices that could affect losses from disasters, including building codes, training of emergency response personnel, building and operation of an Emergency Operations Center, etc.

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