# Practical Resilience Metrics for Planning, Design, and Decision Making

Bilal M. Ayyub, Ph.D., P.E., F.ASCE[1]

**Abstract:** Natural disasters in 2011 alone resulted in $366 billion (2011 US$) in direct damages and 29,782 fatalities worldwide. Storms and floods accounted for up to 70% of the 302 natural disasters worldwide, with earthquakes producing the greatest number of fatalities. Managing these risks rationally requires an appropriate definition of resilience and associated metrics. This paper provides a resilience definition that meets a set of requirements with clear relationships to reliability and risk as key relevant metrics. The resilience definition proposed is of the intension type, which is of the highest order. Resilience metrics are reviewed, and simplified ones are proposed to meet logically consistent requirements drawn from measure theory. Such metrics provide a sound basis for the development of effective decision-making tools for multihazard environments. The paper also examines recovery, with its classifications based on level, spatial, and temporal considerations. Three case studies are developed and used to gain insights to help define recovery profiles. Two recovery profiles, linear and step functions, are introduced. Computational examples and parametric analysis illustrate the reasonableness of the metrics proposed. **DOI: 10.1061/AJRUA6.0000826.** *© 2015 American Society of Civil Engineers.*

**Author keywords:** Community; Definition; Failure; Measure; Metrics; Recovery; Resilience; Risk; Robustness.



## Background

Enhancing system resilience at the structure, network, community, etc. levels could lead to massive savings through risk reduction and expeditious recovery. The rational management of such reduction and recovery is facilitated by practical and fundamental resilience metrics. Current metrics do not always lend themselves easily and intuitively to practical application in effective and efficient manners. This paper reviews existing resilience definitions and metrics, and proposes fundamental metrics that are consistent with recently suggested rigorous metrics. These metrics would provide a sound basis for the development of effective decision-making tools for multihazard environments.

It is essential to develop metrics using effective and rational definitions. The metrics should also meet a set of requirements necessary to link them to other metrics and enable aggregation at a system level. This section addresses these considerations in detail.

### *Types of Definitions*

Definitions have a foundational nature for the development, placement, and effectiveness of guidance documents, codes, standards, laws, or legal regimes. In terms of their appropriateness and quality, definitions span a continuum from the most incisive to the loosest. This section offers perspectives on the types of definitions and their limitations to guide developers and users for their appropriate uses and increase awareness of limitations.

[1]Professor of Civil and Environmental Engineering, Univ. of Maryland, College Park, MD 20742; and Visitor, National Security Analysis Dept., Applied Physics Laboratory, Johns Hopkins Univ., Laurel, MD 20723. E-mail: ba@umd.edu

Note. This manuscript was submitted on December 14, 2014; approved on March 18, 2015; published online on May 15, 2015. Discussion period open until October 15, 2015; separate discussions must be submitted for individual papers. This paper is part of the ***ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering***, © ASCE, 04015008(11)/$25.00.

The common meaning of a definition as provided in dictionaries, such as Dictionary.com (2015), is "the act of making definite, distinct, or clear"; "the formal statement of the meaning or significance of a word, phrase, idiom, etc."; and "the condition of being definite, distinct, or clearly outlined." A definition should capture the essence of a thing or its essential attributes. Lock (2015) made a distinction between nominal and real essence. This leads to a corresponding distinction between nominal and real definitions. A nominal definition is the definition explaining what a word means, whereas a real definition is one expressing the real nature of the thing. This document concerns itself with nominal definitions. The phraseology used in defining a term should possess high levels of lexical accessibility.

Defining a word or phrase requires the use of other words or phrases that are lexically more accessible than the word or phrase being defined; otherwise the definition defeats its purpose. The development of a definition could benefit from listing requirements for its objective, scope, context, and uses. Additionally, its relationship to other established notions or definitions could help establish the necessary precision, or sometimes ambiguity, desired. Definition requirements may include (1) capturing essential attributes; (2) being void of circularity; (3) encompassing enough and nothing else; (4) being lexically nonobscure; and (5) avoiding the use of negation where possible. Additionally, it is inappropriate to have the definition of a term or phrase by another term or phrase requiring subsequent explanations of the defining terms or phrase. The definition of the term or phase is called Type I definition; whereas the subsequent explanation is Type II definition. If a Type II definition is necessary, then the primary Type I definition is inappropriate and should be restated.

At any given time, natural languages have finite numbers of words. Any comprehensive list of definitions must either be circular or rely upon primitive notions. For example, a dictionary, by its nature of providing a comprehensive list of lexical definitions, must resort to circularity. Two schools of philosophical thoughts are available in this regard as follows:

- Undefinable simples: In this school of thought, some simple terms are left undefined based on the claim that the highest genera, i.e., a group marked by common characteristics, cannot be defined without assigning them to a genus. The logic is that some simple concepts can go undefined, and a formal language based on lexical atoms is conceivable.

- Context-dependent simples: This case is based on the notion that what counts as "simple" in one circumstance might not be so in another. This case also claims that explanation of a term is only needed to avoid misunderstanding.

Some things or terms, such as individuals and names, cannot be defined, and are learned by connecting an idea with a sound, so that speakers and listeners have the same idea when the same words are used. Additionally, sometimes things are grouped together based on a family resemblance, making it impossible and unnecessary to state a definition as long as the use of the term is understood.

Definitions are classified at a high level as follows:

- An abstractive definition is for guiding a specific intercourse. Such definitions are used in cases demanding nonambiguity, such as mathematics. A definition of this type can only be disproved by showing a logical contradiction. Metrics fall in this class.
- An interpretive definition is intended for general usage, and has a varied degree of truthfulness, usefulness, or value. It has the characteristic of being descriptive. The use of qualifiers or criteria offers bases for reducing the ambiguity associated with this definition type.

Interpretive definitions are classified further from the lowest to highest level of precision degeneracy, i.e., increased ambiguity, as follows:

1. An intension definition specifies the necessary and sufficient conditions for a thing to belong to a particular set. Example intension definitions are "Earth is a planet" and "Asia is a continent of Earth". The definition in this case provides a one-to-one conceptual mapping (Russell 1993).
2. An extension definition specifies a list naming every object that is a member of a specific set. An example extension definition is "The continents of Earth are Asia, Africa, North America, South America, Antarctica, Europe, and Australia". The definition in this case provides a one-to-many mapping (Russell 1993).
3. An inclusion definition specifies the members of a class by meeting particular conditions or requirements. For example, "Living things are systems that reproduce for sustainment and survival."
4. An exclusion definition specifies the members of a class by pointing out things that do belong to the class by not meeting particular conditions or requirements. For example, "Living things are not rocks, water, oil, air, or fire."
5. Combinations. Combinations of the five classes lead to other definition types as long as the combined types have internal consistency. Generally a definition from a higher precision level can be combined with others of lower levels, but not vice versa.

Partitions and divisions provide bases for further defining subsets and sets of subsets based on the preceding classes and types. Definitions can be divided in a nested structure using genus and differentia. For example, a triangle (i.e., having three bounding sides) or a quadrilateral (i.e., having four bounding sides) as differentia belong to the same genera, i.e., a plane figure. Nesting offers partial ordering, which helps to maintain consistency. A definition or the thing being defined can have a recursive property where the thing could evolve or degenerate based on varying self-created criteria. For example, a political party can be defined using recursive rules. The thing being defined in this case could have evolving or degenerating or changing attributes.

### *Resilience Definitions*

The concept of resilience appears in different domains ranging from ecology to psychology and psychiatry to infrastructure systems. It was formally introduced in ecology, defined as the persistence of relationships within a system (Holling 1973), and measured by the system's ability to absorb change-state variables, driving variables, and parameters yet still persist. Several reputable entities defined resilience in their high-impact documents, most notably the following (Ayyub 2014b):

- In the Presidential Policy Directive (PPD-21, 2013) on Critical Infrastructure Security and Resilience, the "term *resilience* means the ability to prepare for and adapt to changing conditions and withstand and recover rapidly from disruptions. Resilience includes the ability to withstand and recover from deliberate attacks, accidents, or naturally occurring threats or incidents."
- The National Research Council (2012) defined resilience as the ability to prepare and plan for, absorb, recover from, or more successfully adapt to actual or potential adverse events as a consistent definition with U.S. governmental agency definitions (SDR 2005; DHS 2008; PPD-8 2011; NRC 2012).
- The ASCE Committee on Critical Infrastructure (ASCE 2015) states that resilience refers to the capability to mitigate against significant all-hazards risks and incidents, and to expeditiously recover and reconstitute critical services with minimum damage to public safety and health, the economy, and national security.
- The National Infrastructure Advisory Council (2015) defines infrastructure resilience as the ability to reduce the magnitude and/or duration of disruptive events. The effectiveness of a resilient system depends upon its ability to anticipate, absorb, adapt to, and/or rapidly recover from a potentially disruptive event.
- The Multidisciplinary Center for Earthquake Engineering Research (MCEER 2015) of the State University of New York at Buffalo lists characteristics of resilience to include robustness, redundancy, resourcefulness, and rapidity.
- The United Nations Office for Disaster Risk Reduction (UNISDR 2015) characterized a resilient city by its capacity to withstand or absorb the impact of a hazard through resistance or adaptation, which enable it to maintain certain basic functions and structures during a crisis, and bounce back or recover from an event.
- The Civil Contingencies Secretariat of the Cabinet Office, London (2003) defined resilience as the ability of a system or organization to withstand and recover from adversity.

Ayyub (2014b) suggested a resilience definition that builds on the PPD-21 (2013) and lends itself for measurement as follows:

"Resilience notionally means the ability to prepare for and adapt to changing conditions and withstand and recover rapidly from disruptions. Resilience includes the ability to withstand and recover from disturbances of the deliberate attack types, accidents, or naturally occurring threats or incidents."

This combination definition has intension and inclusion class features. A proposed definition that belongs to the intension class is as follows:

The resilience of a system is the persistence of its functions and performances under uncertainty in the face of disturbances.

This proposed definition is intended to have a broad use ranging from infrastructures to networks to communities. It enables the measurement of resilience through metrics by meeting the following requirements as demonstrated by Ayyub (2014b): (1) building on previous notional definitions; (2) considering initial and residual strength, i.e., capacity and robustness; (3) accounting for abilities to prepare and plan for, absorb, recover from, or adapt to adverse events; (4) treating disturbances as events with occurrence rates of stochastic processes; (5) permitting the use of several performance attributes; (6) accounting for changes over time, e.g., aging or improvements; (7) considering full or partial recovery and times to recovery; (8) considering potential enhancements to system performance after recovery; (9) being compatible with other familiar notions such as reliability and risk; and (10) enabling the development of resilience metrics with meaningful units.

The proposed definition is consistent with the ISO (2009) risk definition of the "effect of uncertainty on objectives." The resilience measure includes three key words that offer a basis for quantification. These words are listed in a suggested order for their analysis as follows:

1. System's performance defined in terms of requirements or objectives, and examined in the form of output, throughput, structural integrity, lifecycle cost, etc.;
2. Uncertainty relating to events such as storms, disturbance, conditions, system states, etc.; and
3. Persistence examined in terms of enduring the events, recovery, continuance, and/or resumption of performance.

Most resilience definitions do not always lend themselves naturally and intuitively to the development of consistent resilience metrics with clear relationships to the most relevant metrics of the abstract notions of reliability and risk. The use of the operative word of *ability*, as provided by other definitions, sometimes has resulted in setting the measurement process on tracks that focus on the abilities rather than the outcomes of these abilities. The primary outcome of these abilities is the continuance of the performance of a system, including bouncing-back, a characteristic that could be appropriately termed as performance persistence for a particular function of the system. Performance persistence would naturally set measurement in terms of availability of the performance or continuance of system's states of normalcy. Subsequent sections of this paper provide metrics based on the proposed definition that meet logically consistent requirements drawn partly from measure theory as provided by Ayyub (2014b), and provide a sound basis for the development of effective decision-making tools for multihazard environments.

## Resilience Measurement and Metrics

### *Available Metrics and Their Limitations*

Bruneau and Reinhorn (2007) proposed metrics for measuring resiliency based on the size of expected degradation in the quality of an infrastructure by quantifying robustness, redundancy, resourcefulness, and rapidity to recovery. Garbin and Shortle (2007) outline an approach to quantitatively measure the resilience of a network as the percentage of links damaged versus the network performance, and the percentage of nodes damaged versus the network performance. Tierney and Bruneau (2007) suggested measuring resilience based on observing that resilient systems reduce the probabilities of failure and enhance recovery, and therefore resilience can be measured by the functionality of an infrastructure system after an external shock including the time it takes to return to the initial level of performance. They illustrated the concept as shown in Fig. 1, calling it the resilience triangle. Attoh-Okine et al. (2009) used several potential paths of infrastructure performance during normal operation and cases of unexpected events, for example, a path demonstrating sudden failure as shown in Fig. 1, as well as a path demonstrating decrease in service life, and a path

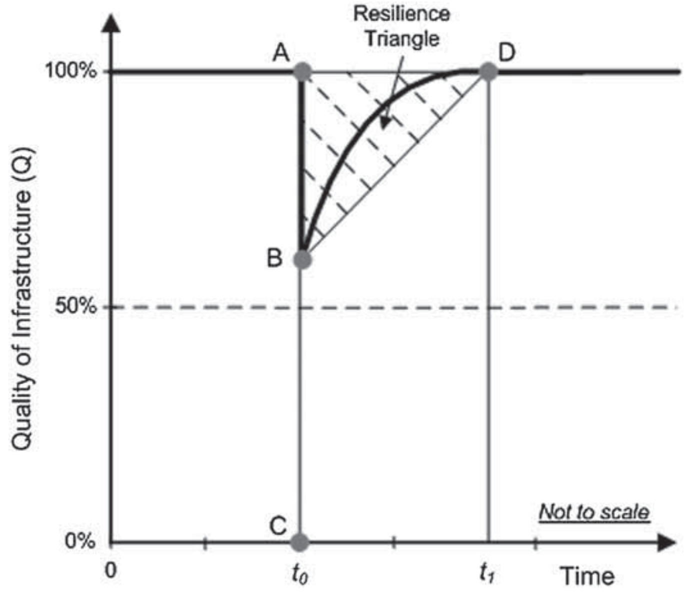


**Fig. 1.** Resilience properties and triangle

for the normal operation of the system. They used the concept of resilience as illustrated in Fig. 1 to define a resilience index as follows:

$$\text{Resilience} = \frac{\int_{t_0}^{t_1} Q(t)dt}{100(t_0 - t_1)} \tag{1}$$

where $Q$ = infrastructure quality, or the performance of a system; $t_0$ = time of incident or disturbance occurrence; and $t_1$ = time to full recovery. According to this model, the units of resilience are performance per unit time, where performance can be measured in percent according to Eq. (1). Eq. (1) was also used by the earthquake community (Tierney and Bruneau 2007) with a suggested framework of resilience, called the four Rs, as follows:

1. Robustness as the ability of the system and system elements to withstand external shocks without significant loss of performance;
2. Redundancy as the extent to which the system and other elements satisfy and sustain functional requirements in the event of disturbance;
3. Resourcefulness as the ability to diagnose and prioritize problems and to initiate solutions by identifying and monitoring all resources, including economic, technical, and social information; and
4. Rapidity as the ability to recover and contain losses and avoid future disruptions.

These properties are defined in Table 1 with reference to Fig. 1 based on models provided by Shinozuka et al. (2004). In this table, robustness and rapidity are defined as follows:

**Table 1.** Definition of Resilience Properties

| Property | Models (Points A, B, C, and D per Fig. 1) | Units |
|---|---|---|
| Robustness | Robustness $= B - C$ | Percentage |
| Redundancy | Not defined | — |
| Resourcefulness | Not defined | — |
| Rapidity | Rapidity $= (A - B)/(t_0 - t_1)$ | Average recovery rate in percentage per time |

$$\text{Robustness} = B - C \tag{2}$$

$$\text{Rapidity} = \frac{A - B}{t_0 - t_1} \tag{3}$$

Li and Lence (2007) refined the resilience index developed by Hashimoto et al. (1982) by using the performance ratio over two different time periods. Omer et al. (2009) measure resilience for Internet infrastructure systems as the ratio of the difference in information transmission before, i.e., initial, and after an event divided by the initial information transmission. Attoh-Okine et al. (2009) also provided formulation of a resilience index of urban infrastructure using belief functions. McGill and Ayyub (2009) related resilience concepts to regional capabilities performance assessment for human-caused hazards in homeland security.

Gilbert (2010) provides extensive coverage of and mathematical models for recovery after a storm in the context of a disaster cycle consisting of response, recovery, mitigation, and preparedness. His discussion includes partial recovery and full recovery including instant urban renewal of population recovery, physical infrastructure, economy, social networks, government services, and environments. He also develops simulation models of recovery and provides validation examples [Miles and Chang (2003) for the Kobe Earthquake]. Generally the recovery trends shown have decreasing slopes as shown in Fig. 1.

Bonstrom and Corotis (2014) provide a first-order reliability approach to quantify and improve building portfolio resilience. The approach builds on the concept of Fig. 1 affecting the relative quality at point B, the shape of the resilience triangle as represented by the nonlinear curve connecting B and D, and the position of point D at $t_1$. It also includes a fundamental uncertainty treatment with a reliability framework.

Ayyub (2014b) suggested resilience metrics that are consistent with the resilience definition provided earlier as persistence of the system's functions and performances under uncertainty in the face of disturbances, and meet the requirements imposed on such a definition. The definition captures the details of resilience concept of Fig. 1 at both the quality and time axes. Ayyub (2014b) used Fig. 2 to provide a schematic representation of a system performance ($Q$) with aging effects and an incident occurrence with a rate ($\lambda$) according to a Poisson process. At time $t_i$, it might lead to a failure event with a duration $\Delta T_f$. The failure event concludes at time $t_f$.

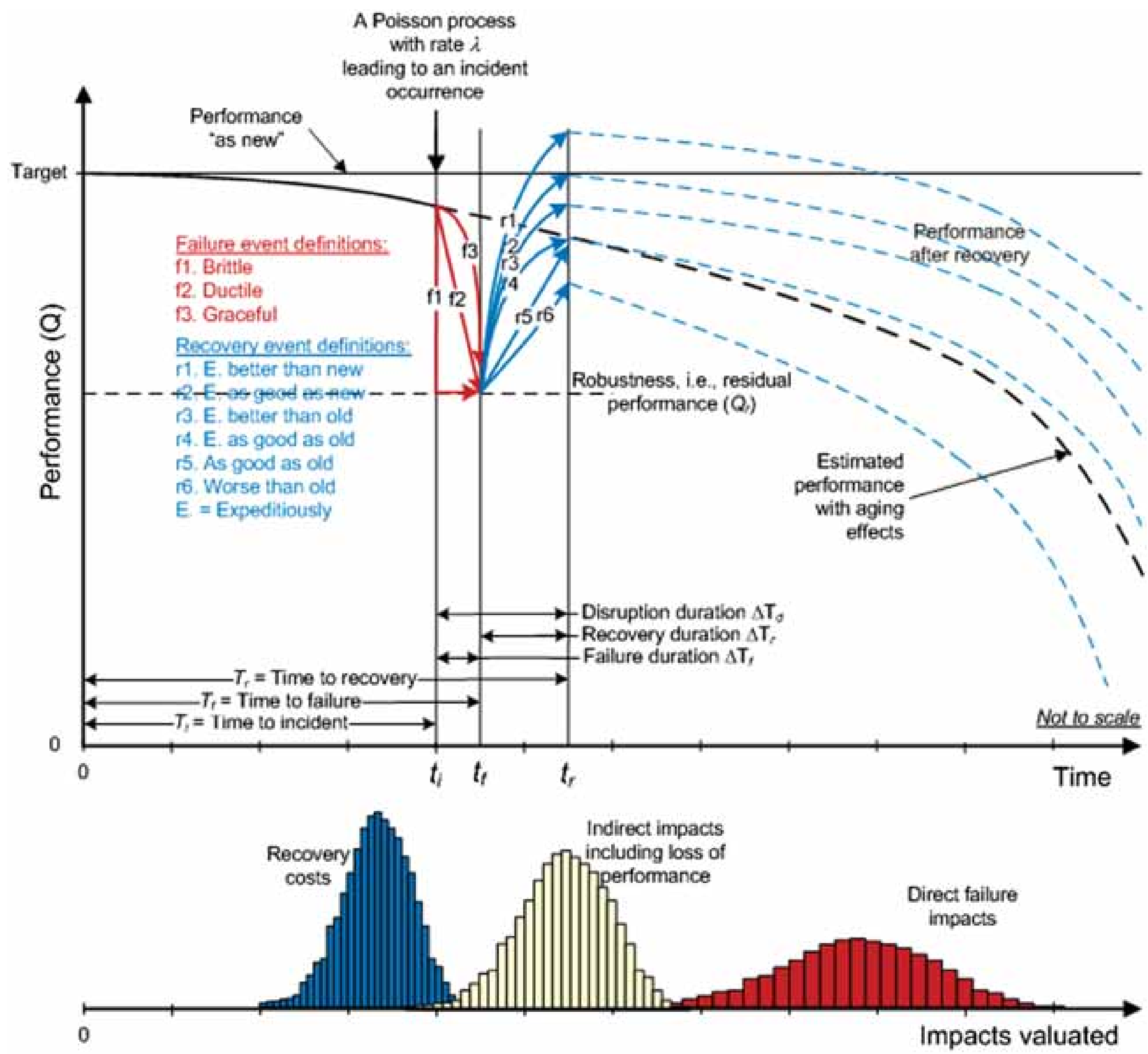


**Fig. 2.** Proposed definitions of resilience metrics

The failure event is followed by a recovery event with a duration $\Delta T_r$. The recovery event concludes at time $t_r$. The total disruption ($D$) has a duration of $\Delta T_d = \Delta T_f + \Delta T_r$. The figure shows for illustration purposes three failure events: brittle (f1), ductile (f2), and graceful (f3), and six recovery events: expeditious recovery to better than new (r1); expeditious recovery to as good as new (r2); expeditious recovery to better than old (r3); expeditious recovery to as good as new (r4); recovery to as good as old (r5); and recovery to worse than old (r6). These events define various rates of change of performance of the system. The figure also shows the aging performance trajectory and the estimated trajectory after recovery. The proposed model to measure resilience is

$$\text{Resilience}\,(R_e) = \frac{T_i + F\Delta T_f + R\Delta T_r}{T_i + \Delta T_f + \Delta T_r} \tag{4}$$

where for any failure event ($f$) as illustrated in Fig. 2, the corresponding *failure profile* $F$ is measured as follows:

$$\text{Failure}\,(F) = \frac{\int_{t_i}^{t_f} f\,dt}{\int_{t_i}^{t_f} Q\,dt} \tag{5}$$

Similarly for any recover event ($r$) as illustrated in Fig. 2, the corresponding *recovery profile* $R$ is measured as follows:

$$\text{Recovery}\,(R) = \frac{\int_{t_f}^{t_r} r\,dt}{\int_{t_f}^{t_r} Q\,dt} \tag{6}$$

The failure-profile value ($F$) can be considered as a measure of robustness and redundancy, and is proposed to address the notion offered by Eq. (2); whereas the recovery-profile value ($R$) can be considered as a measure of resourcefulness and rapidity, and is proposed to address the notion offered by Eq. (3). The time to failure ($T_f$) can be characterized by its probability density function computed as follows:

$$-\frac{d}{dt}\int_{s=0}^{\infty} \exp\left(-\lambda t\left\{1 - \frac{1}{t}\int_{\tau=0}^{t} F_L[\alpha(\tau)s]d\tau\right\}\right) f_{S_0}(s)ds \tag{7}$$

where $Q$ = system's performance in terms of its strength ($S$) minus the corresponding load effect ($L$) in consistent units, i.e., $Q = S - L$. Both $L$ and $S$ are treated as random variables, with $F_L$ = the cumulative probability distribution function of $L$; and $f_S$ = the probability density function of $S$. The aging effects are considered in this model by the term $\alpha\,(t)$ representing a degradation mechanism as a function of time $t$. The term $\alpha\,(t)$ can also represent improvement to the system. Eq. (7) is based on a Poisson process with an incident occurrence, such as loading, rate of $\lambda$, and is based on Ellingwood and Mori (1993). The probability density function of $T_f$ as shown in Eq. (7) is the negative of the derivative of the reliability function. The times $T_i$, $T_f$, and $T_r$ are random variables as shown in Fig. 2, and are related to durations as follows:

$$\Delta T_f = T_f - T_i \tag{8}$$

$$\Delta T_r = T_r - T_f \tag{9}$$

The disruption duration is given by

$$\Delta T_d = \Delta T_f + \Delta T_r \tag{10}$$

The proposed model of Eq. (4) for measuring resilience meets the set of requirements previously described (Ayyub 2014b).

## Performance Measurement for Resilience Metrics

The resilience model of Eq. (4) is applicable to varied systems, such as buildings, other structures, facilities, infrastructure, networks, and communities. The primary basis for evaluating Eq. (4) is the definition of performance ($Q$) at the system level with meaningful and appropriate units, followed by the development of an appropriate breakdown for this performance, using what is termed herein as performance segregation. The performance segregation should be based on some system-level logic that relates the components of the performance breakdown to the overall performance at the system level as the basis for a system model. This model can be used to aggregate the performance of components to the assess system-level performance. Such performance segregation and aggregation analysis is essential for examining the resilience of systems for buildings, other structures, facilities, infrastructure, networks, and communities. The uncertainties associated with the performance components can be modeled as random variables with any necessary performance events in order to use Boolean algebra and the mathematics of probability to characterize the performance $Q$ in Eq. (4). The units of performance at the system level vary depending on the system type and the objectives of the analysis. Table 2 shows examples of performance types and units of measurement for selected systems for demonstration purposes.

## Potential Losses and Costs

Fig. 2 also shows the associated costs including losses, recovery costs, and indirect costs. These losses and costs should be based on total economic valuations using anthropocentric considerations. Ayyub (2014b) provides background information on these concepts. Resilience metrics should lend themselves for total economic valuation and the development of frameworks for economic analysis, which are beyond the scope of this paper.

## Limitations

Primary limitations of available methods can be classified as (1) inappropriately structured, and (2) complex.

The model of Eq. (1) according to Table 1 and Fig. 1 measures resilience in the units of performance per unit of time. It focuses on the recovery slope, and does not account for the rates of stressor, the probability of failure as a result of a stressor, the failure profile, and recovering to states other than "as-good-as-new." Such models do not account for failure rates and probabilities. Therefore, they do not meet the resilience definition by not accounting for the ability of a system to withstand a stressor.

Although the model of Eqs. (4)–(10), Table 2 and Fig. 2 offer a comprehensive capture of the resilience attributes according to the

**Table 2.** Systems and Performance Measurements

| Systems | Performance | Units |
|---|---|---|
| Buildings | Space availability | Area per day |
| Other structures: Highway bridges | Throughput traffic | Count per day |
| Facilities: Water treatment plants | Water production capacity | Volume per day |
| Infrastructure: Water delivery | Water available for consumption | Volume |
| Network: electric power distribution | Power delivered | Power per day |
| Communities | Economic output | Dollars |
| | Quality of life (consumption) | Dollars |

resilience definition; it is complex and might be impractical. This paper addresses the limitations of this model through the development of simplified, practical metrics. The intended users of such simplified metrics are practitioners including engineers, city planners, decision makers, analysts, and policy makers.

## Practical Resilience Metrics

This paper offers proposed practical metrics that are consistent with recently suggested rigorous ones, e.g., Eqs. (4)–(10). Such metrics would provide a sound basis for the development of effective decision-making tools for multihazard environments. Achieving this objective requires simplifying the concepts presented in Fig. 2 and Eq. 4 while maintaining intent, comprehensiveness and accuracy. The accuracy level should be commensurate with the decisions to be informed by setting appropriate resolution levels in constructing a system's model.

### *Proposed Metrics*

Consider a fundamental case having a performance level that would be maintained and sustained over time, i.e., no aging effects, with a brittle failure profile, i.e., f1, in Fig. 2. Also, assume as-good-as-old recovery, i.e., r5 in Fig. 2. This fundamental case is shown in Fig. 3. Additionally, the following assumptions are made: (1) a planning horizon ($t$); (2) Poisson process of stressors with an annual rate ($\lambda$) as shown in Fig. 4 (Ayyub 2014a); (3) the planning horizon related to the stressor rate as $t = 1/\lambda$; (4) annual failure probability ($p$) due to a stressor; and (5) independent failures. The stressors have varied intensities (Fig. 4), and not all stressors fail the system and disrupt the system's performance. The failure probability is denoted as $p$. Two fundamental cases are presented in this section: the case of linear recovery and the case of step recovery.

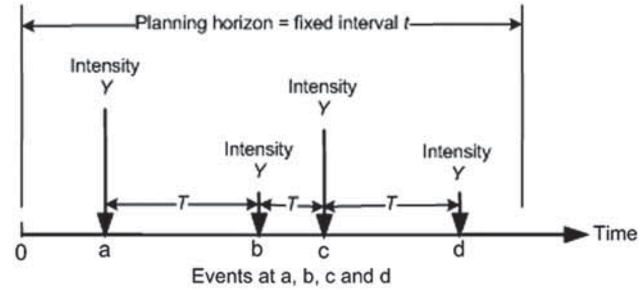


**Fig. 4.** Poisson process of a stressor with varied intensity

For the fundamental case of a linear recovery as shown in Fig. 4, the resilience metric of Eq. (4) for one failure-causing event is basically the ratio of two areas according to this figure; i.e., the rectangular area $tQ_{100}$, divided by the $tQ_{100}$ without the triangle representing the loss of functionality of the system. For a linear recovery path ($r$), it can be expressed as follows for one failure-inducing event:

$$\text{Linear recovery:Resilience per failure } (R_f) = 1 - \frac{(t_r - t_i)(Q_{100} - Q_r)}{2Q_{100}t} \tag{11}$$

For analytical and computational convenience, the concept of nonresilience can be introduced and defined as follows:

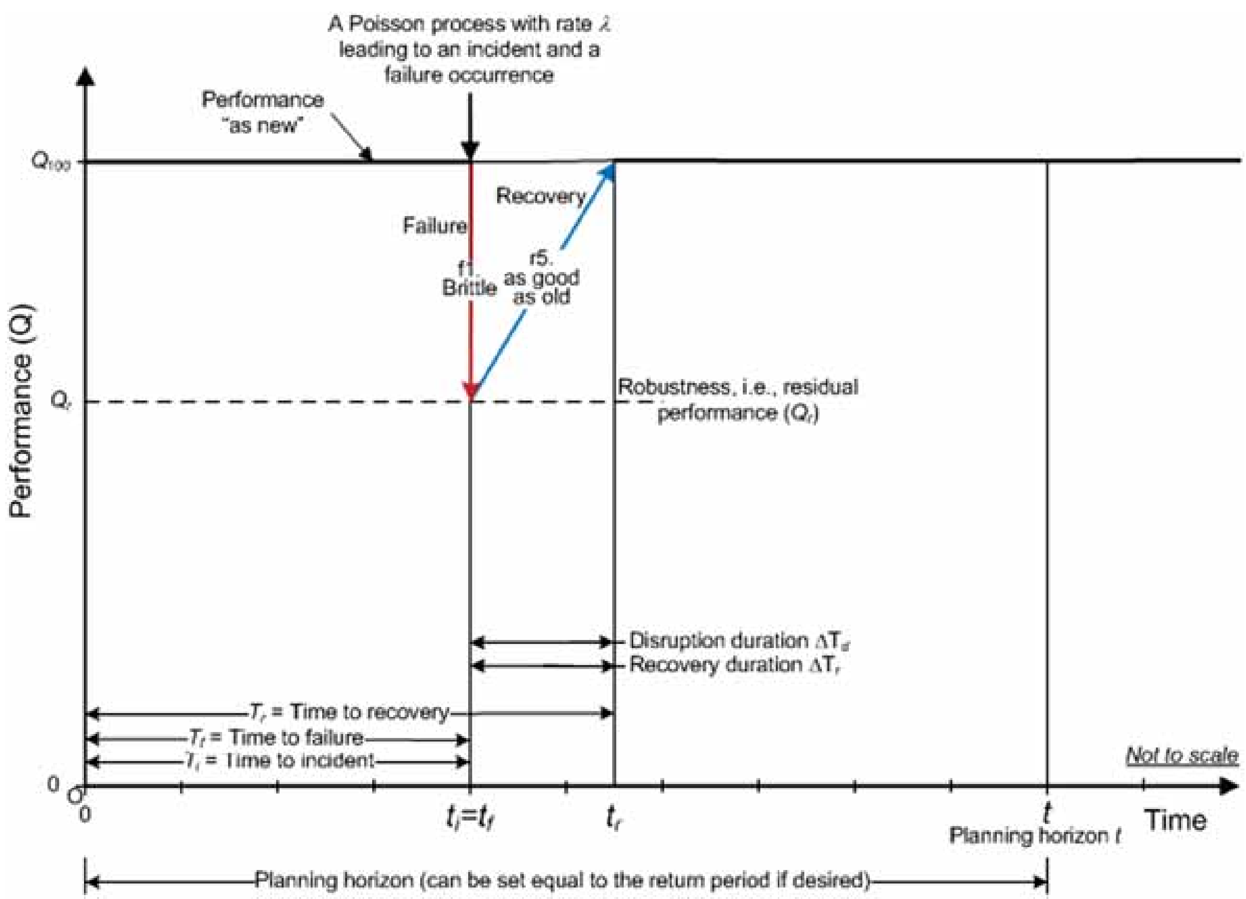


**Fig. 3.** Fundamental resilience Case 1 of linear recovery

Linear recovery:Nonresilience per failure $(\bar{R}_f)$

$$= \frac{(t_r - t_i)(Q_{100} - Q_r)}{2Q_{100}t} \tag{12}$$

The relationship between $R_f$ and $\bar{R}_f$ is

$$\bar{R}_f = 1 - R_f \tag{13}$$

Eqs. (11) and (12) can be generalized to account for the potential of multiple occurrences of failure-inducing events and their associated probabilities assuming that the system is fully recovered after each failure as follows:

$$\text{Resilience } (R_e) = 1 - \sum_{x=0}^{\infty}\left[\exp(-\lambda t)\frac{(\lambda t)^x}{x!}p^x \bar{R}_f^x\right] \tag{14}$$

The following infinite series can be used to reduce Eq. (14):

$$\exp(\lambda t) = 1 + (\lambda t) + \frac{(\lambda t)^2}{2!} + \frac{(\lambda t)^3}{3!} + \cdots \tag{15}$$

Eq. (14) is reduced to

$$\text{Resilience } (R_e) = 1 - \exp(-\lambda t)[\exp(\lambda t p \bar{R}_f) - 1] \tag{16}$$

Eq. (16) can be reduced further to the following practical form:

$$\text{Resilience } (R_e) = 1 - \exp[-\lambda t(1 - p\bar{R}_f)] + \exp(-\lambda t) \tag{17}$$

For the fundamental case of step recovery as shown in Fig. 5(a), the resilience metric of Eq. (4) for one failure-causing event is again the ratio of two areas according to this figure; i.e., the rectangular area $tQ_{100}$, divided by the $tQ_{100}$ without the rectangle representing the loss of functionality of the system. It can be expressed as follows for one failure-inducing event:

Step recovery:Resilience per failure $(R_f)$

$$= 1 - \frac{(t_r - t_i)(Q_{100} - Q_r)}{Q_{100}t} \tag{18}$$

For analytical and computational convenience, the concept of nonresilience can be introduced and defined as follows:

Step recovery:Nonresilience per failure $(\bar{R}_f)$

$$= \frac{(t_r - t_i)(Q_{100} - Q_r)}{Q_{100}t} \tag{19}$$

The relationship between $R_f$ and $\bar{R}_f$ is provided in Eq. (13). Eq. (17) can be used to compute the resilience. This fundamental case can be extended easily to a generalized case of multiple steps where recovery is achieved by bringing into operation portions of a system while remaining portions being restored. A two-step case is shown in Fig. 5(b).

Eq. (17) offers the simplicity and practicality desired for systems with time invariant performance and accounts for

1. Rate of stressor s, i.e., the rate $\lambda$ of a Poisson process;
2. Probability of failure ($p$) given a stressor, i.e., inherent strength of the system;
3. Capacity of the system ($Q_{100}$);
4. Robustness of the system ($Q_r$);
5. Brittle failure and linear or step recovery to as-good-as-old profiles;
6. Nonresilience associated with the occurrence of a failure-inducing event;
7. Planning horizon $t$; and
8. Stressor time as a result of the failure-inducing event.

For cases where the planning horizon ($t$) is equal to the return period ($1/\lambda$), Eq. (17) can be reduced to the following special case:

$$\text{Resilience } (R_e) = 1 - \exp[-(1 - p\bar{R}_f)] + \exp(-1) \tag{20}$$

### Examples

Consider the case of a planning horizon equal to the return period of stressors and linear recovery. Eq. (20) can be used in this case to evaluate resilience under varied failure probabilities ($p$) of 0.1, 0.25, and 0.5, recovery periods from 0 to 10 years, and a robustness of zero. The results are shown in Fig. 6. Fig. 7 shows the effect of robustness on resilience by varying robustness using values of 0.25, 0.5, 0.75, and 0.99. Fig. 8 shows the results differently by placing the failure probability on the abscissa, the recovery time for selected values of 1, 5, and 10, and robustness of 0.25.

## Recovery Profiles and Models

### Recovery Classification

Fig. 2 shows several recovery profiles for the purpose of illustration (Ayyub 2014b). Several recovery types are listed as follows:

1. Expeditious recovery to a performance level that is better than new (r1);
2. Expeditious recovery to a performance level that is as good as new (r2);
3. Expeditious recovery to a performance level that is better than old (r3);
4. Expeditious recovery to a performance level that is as good as old (r4);
5. Recovery to a performance level that is as good as old (r5); and
6. Recovery to a performance level that is worse than old (r6).

Fig. 2 also shows failure profiles that are in common use.

The type of systems may offer a useful basis for classification, such as: society or community recovery, economic recovery, regional recovery, country recovery, corporate recovery, market recovery, supply-chain recovery, infrastructure system recovery, such as electric power recovery, environmental recovery, Earth recovery, individual health or public health recovery, and relationship recovery.

The recovery level, spatial scale, and time might have utility as a basis for classification, such as

1. By level, such as no recovery, diminished recovery, and superior recovery.
2. By spatial scale, such as uniform versus nonuniform, spatially random, and spatially trending.
3. By timesuch as linear versus nonlinear, time lagging, gradual versus abrupt, step function, and asymptotic.
4. Combinations of level, space, and temporal cases

### Recovery Models

Fig. 2 shows several recovery profiles for the purpose of illustration (Ayyub 2014b). Cimellaro et al. (2010) Chang and Shinozuka (2004), and Kafali and Grigoriu (2005) suggested the use of linear, trigonometric, or exponential models for recovery profiles. These models offer the flexibility needed to accommodate the various cases shown in Fig. 2. This section focuses on the uncertainties associated with such recovery profiles. The two cases presented in Figs. 4 and 5; i.e., a triangular disruption or a linear model and a rectangular disruption or a step function, are recommended

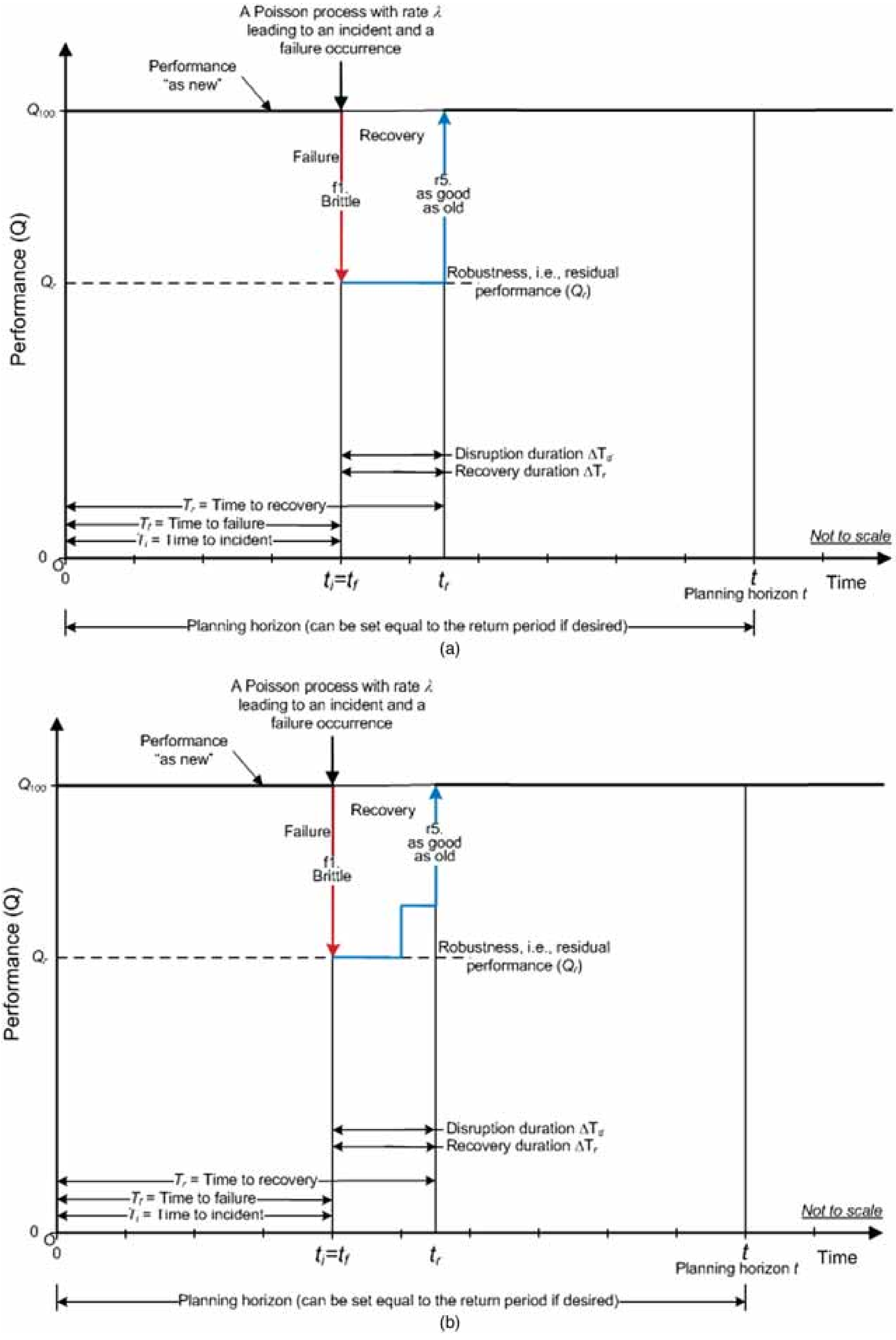


**Fig. 5.** Fundamental resilience Case 2 of step recovery: (a) one-step recovery; (b) two-step recovery

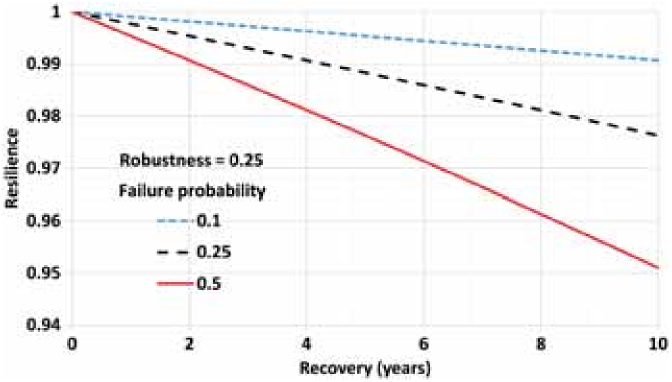


**Fig. 6.** Resilience computations for the case of robustness of zero

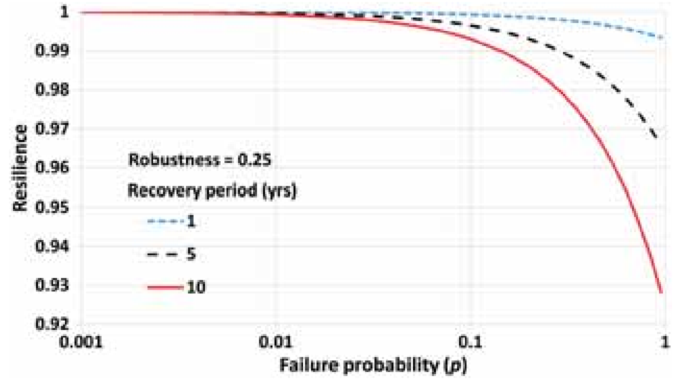


**Fig. 8.** Example resilience computations as a function of failure probability for robustness of 0.25

for now until data are collected and classifications made to justify other forms. The case studies presented at the end of this section offer some recovery features that might help to select appropriate profiles.

### *Recovery of Societies*

The Nobel Laureate Becker (2005) believes in the accuracy of the optimistic outlook of the great nineteenth century English economist and philosopher, John Stuart Mill (2015), who marveled at the "great rapidity with which countries recover from a state of devastation, the disappearance in a short time, of all traces of mischief done by earthquakes, floods, hurricanes, and the ravages of war." Both natural and human-caused disasters during the subsequent century-and-a-half are generally on the side of Mill's claim, such as (Becker 2005):

- The September 11, 2001, coordinated attacks in New York City and Washington, DC, resulted in 2,996 deaths and at least \$10 billion in property and infrastructure damage. These attacks had a slight overall impact on the course of Gross Domestic Product (GDP) and employment in the United States, although some industries and New York City were affected for several years;
- The Kobe earthquake of 1995 resulted in 6,000 deaths and the destruction of more than 100,000 homes. The economic recovery not only of Japan but also of Kobe was rapid;
- The flu pandemic of 1918–1919 resulted in about 30 million deaths worldwide without having a major impact on the world's economy; and
- The 2004 Indian Ocean earthquake and tsunami resulted in 227,898 deaths and about 1.5 million people displaced

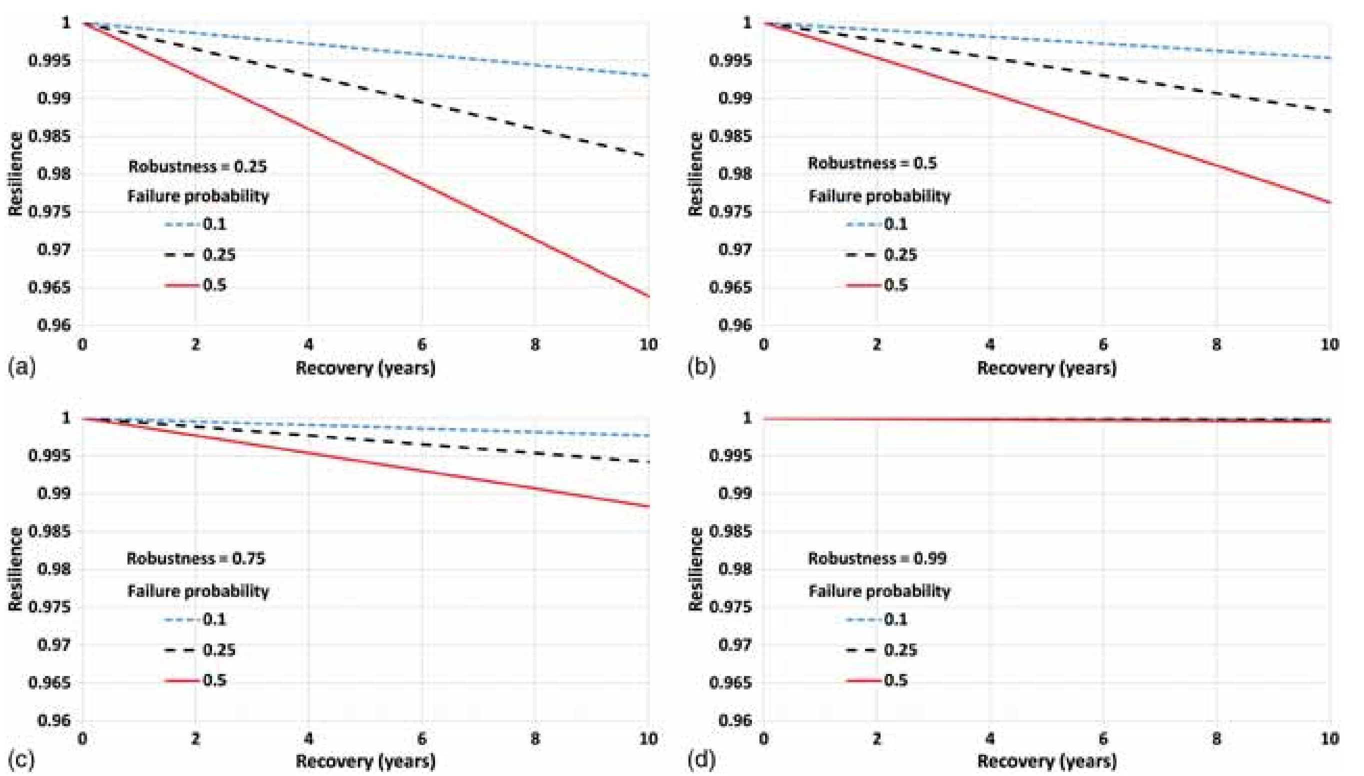


**Fig. 7.** Example resilience computations as a function of recovery period: (a) robustness = 0.25; (b) robustness = 0.5; (c) robustness = 0.75; (d) robustness = 0.99

in 15 countries; however, the Asian stock markets did not change much.

General conclusions cannot be made based on these examples due to several factors including (1) the wealth of a nation; (2) the extent and sophistication of insurance coverage; (3) education and earning levels of populations; (4) access to healthcare and emergency response national and international channels; (5) governance; and (6) social and cultural considerations. Interconnectedness and moral risk effects add other levels of complexity that are difficult to address.

The ability of a society to recover after a disaster mainly depends on whether its population size has been significantly reduced by death or displacement. With the same skill and knowledge that they possessed prior to the disaster, with their land and primary improvements undestroyed, a society has nearly all the requisites for their former amount of production. This ability can be associated as an attribute of living systems, and the society being alive, i.e., vis medicatrix naturae. The recovery trajectory is greatly affected by the post-disaster land, infrastructure availability, and population size.

## *Case Studies*

### Case Study I: Hurricane Katrina and New Orleans (2005)

Hurricane Katrina (August 23–31, 2005) was an extraordinary act of nature and the most destructive natural disaster in American history, creating a human tragedy and laying waste to 90,000 mi$^2$ (233,000 km$^2$) of land, an area the size of the United Kingdom. In Louisiana and Mississippi, the storm surge obliterated coastal communities and left thousands destitute. One of the primary contributors to the flooding of New Orleans was the failures of levees and floodwalls that make up the hurricane protection system (HPS). The utilization of engineered systems leads to risks that result from humans using technology in an attempt to gain benefits, such as control of naturally occurring conditions.

According to Federal Emergency Management Agency (FEMA), the total damage from Katrina is \$108 billion (in 2005 US\$). The direct and indirect fatalities stand at 1,833 (FEMA 2015). It is estimated that 40% of the deaths in Louisiana were caused by drowning. 25% were caused by injury and trauma and 11% were caused by heart conditions. Also, it is estimated that about 50% of the fatalities in Louisiana were people over the age of 74. Payments to fulfill insurance claims add up to an estimated \$41.1 billion from private insurers, and \$16.1 billion from the National Flood Insurance Program. More than one million people in the Gulf region were displaced by the storm. Additionally, the federal government has spent post-Katrina \$120.5 billion on the Gulf Region. The hurricane destroyed levees that lead to flooding 80% of the city and reducing its population from 484,674 in April 2000 to 230,172 in July 2006, a decrease of over 50%. By 2012, the population had increased to 369,250. About 70% of its housing was damaged (CNN 2015).

The U.S. Bureau of Economic Analysis (2015) publishes economic statistics and used to construct Fig. 9. The figure shows the GDP for all industries in the US and the GDP of all industries in New Orleans indexed to the year 2001. Estimating recovery time for the city from Fig. 9 is not easy because of the multidimensionality of recovery. The recovery period can be estimated approximately to be eight years based on GDP although the population growth has not kept up with the GDP growth. This disparity perhaps is attributable to changes in the composition of the industries, population skill levels, and incomes. The recovery profile is almost linear and similar to Fig. 4.

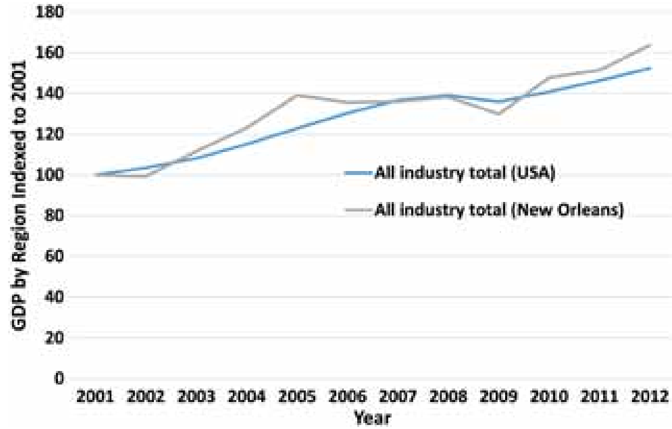


**Fig. 9.** Economic recovery of New Orleans

### Case Study II: Highway I-35W Mississippi River Bridge Collapse (2007)

An eight-lane, steel truss arch bridge carrying Interstate 35 W across the Saint Anthony Falls of the Mississippi River in Minneapolis, Minnesota, collapsed during rush hour on August 1, 2007, resulting in the death of 13 and the injury of 145 people. The bridge was carrying average daily traffic of about 140,000 vehicles. The National Transportation Safety Board (NTSB 2015) investigated and identified a design flaw as the likely cause of collapse. A replacement bridge was fast-tracked through planning, design, and construction, and opened to traffic on September 18, 2008. The recovery time in this case is about one year. The bridge robustness in this case is 0% and the recovery profile is as provided in Fig. 5.

### Case Study III: The World Trade Center Collapse on (September 11, 2001)

Reconstruction of the World Trade Center after the September 11, 2001, destructive attack in New York City was deferred until 2006 due to disputes between the Port Authority and the developer. One World Trade Center opened 13 years later to tenants on November 3, 2014. Additional towers are underway. A recovery has not been achieved yet.

## Decision Analysis for Enhancing Resilience

The valuation of resilience can be based on the savings in potential direct and indirect losses, and cost of recovery as illustrated in Fig. 2. Alternatives for enhancing resilience that can reduce these potential losses can be analyzed using models for benefit–cost analysis, where the benefit ($B$) is the potential savings in losses and recovery costs due to the implementation of an alternative and the cost ($C$) is the cost of the alternative. The benefit and costs are treated as random variables (Ayyub 2014a). Assuming $B$ and $C$ to be normally distributed, a benefit–cost index ($\beta_{B/C}$) can be defined as follows:

$$\beta_{B/C} = \frac{\mu_B - \mu_C}{\sqrt{\sigma_B^2 + \sigma_C^2}} \tag{21a}$$

where $\mu$ and $\sigma$ = mean and standard deviation. In the case of lognormally distributed $B$ and $C$, the benefit–cost index ($\beta_{B/C}$) can be computed as

$$\beta_{B/C} = \frac{\ln\left(\frac{\mu_B}{\mu_C}\sqrt{\frac{\delta_C^2+1}{\delta_B^2+1}}\right)}{\sqrt{\ln[(\delta_B^2+1)(\delta_C^2+1)]}} \tag{21b}$$

where $\delta$ = coefficient of variation. In the case of mixed distributions or cases involving basic random variables of $B$ and $C$, other reliability methods can be used as described by Ayyub (2014a). The probability of cost exceeding benefit can be computed as

$$P_{f,B/C} = P(C > B) = 1 - \Phi(\beta) \tag{22}$$

where $\Phi$ = standard normal cumulative distribution function.

## Conclusions

Massive savings could be realized by enhancing the resilience of a system, including buildings, infrastructure, networks, and communities through risk reduction and expeditious recovery. This paper provides a resilience definition that meets a set of requirements with clear relationships to metrics of the relevant abstract notions of reliability and risk. The paper also shows that this definition is the intension type, considered of the highest order. The paper also provides proposed metrics that are practical and simplified while capturing all the attribute set in the resilience definition. The paper provides recovery models with case studies, and illustrative examples.

## Acknowledgments

The author would like to acknowledge the accommodation of the National Security Analysis Department of the Applied Physics Laboratory at The Johns Hopkins University (APL-JHU) during his sabbatical leave in 2014-15. The opinions articulated in the paper are solely the personal opinions of the author.

## References

ASCE. (2015). "ASCE committee on critical infrastructure." ASCE Policy Statement 518, ⟨http://www.asce.org/issues-and-advocacy/public-policy/policy-statement-518—unified-definitions-for-critical-infrastructure-resilience/⟩ (Apr. 8, 2015).

Attoh-Okine, N., Cooper, A. T., and Mensah, S. A. (2009). "Formulation of resilience index of urban infrastructure using belief functions." *IEEE Syst. J.*, 3(2), 147–153.

Ayyub, B. M. (2014a). *Risk analysis in engineering and economics*, 2nd Ed., Chapman & Hall/CRC, Boca Raton, FL.

Ayyub, B. M. (2014b). "Systems resilience for multihazard environments: Definition, metrics, and valuation for decision making." *J. Risk Anal.*, 34(2), 340–355.

Becker, G. S. (2005). "...And the economics of disaster management." *Wall Street Journal*, Eastern Ed., New York.

Bonstrom, H., and Corotis, R. B. (2014). "First-order reliability approach to quantify and improve building portfolio resilience." *J. Struct. Eng.*, 10.1061/(ASCE)ST.1943-541X.0001213, C4014001.

Bruneau, M., and Reinhorn, A. (2007). "Exploring the concept of seismic resilience for acute care facilities." *Earthquake Spectra*, 23(1), 41–62.

Chang, S. E., and Shinozuka, M. (2004). "Measuring improvements in the disaster resilience of communities." *Earthquake Spectra*, 20(3), 739–755.

Cimellaro, G. P., Reinhorn, A. M., and Bruneau, M. (2010). "Seismic resilience of a hospital system." *Struct. Infrastruct. Eng.*, 6(1–2), 127–144.

Civil Contingencies Secretariat of the Cabinet Office, London. (2003). ⟨www.cabinetoffice.gov.uk⟩ (Apr. 8, 2015).

CNN (Cable News Network). (2015). ⟨www.cnn.com⟩ (Apr. 8, 2015).

DHS Risk Steering Committee. (2008). "DHS risk lexicon." Dept. of Homeland Security, ⟨www.dhs.gov/xlibrary/assets/dhs_risk_lexicon.pdf⟩ (Apr. 8, 2015).

Dicionary.com. (2015). ⟨http://dictionary.reference.com/⟩ (Apr. 8, 2015).

Ellingwood, B. R., and Mori, Y. (1993). "Probabilistic methods for condition assessment and life prediction of concrete structures in nuclear plants." *Nucl. Eng. Des.*, 142(2–3), 155–166.

FEMA (Federal Emergency management Agency). (2015). ⟨www.fema.gov⟩ (Apr. 8, 2015).

Garbin, D. A., and Shortle, J. F. (2007). "Measuring resilience in network-based infrastructures." *Critical thinking: Moving from infrastructure protection to infrastructure resiliency*, J. A. McCarthy, ed., George Mason Univ., Fairfax, VA.

Gilbert, S. W. (2010). "Disaster resilience: A guide to the literature." *NIST Special Publication 1117*, Office of Applied Economics, Engineering Laboratory, National Institute of Standards and Technology, Gaithersburg, MD.

Hashimoto, T., Stedinger, J. R., and Loucks, D. P. (1982). "Reliability, resiliency and vulnerability for water resources system performance evaluation." *Water Resour. Res.*, 18(1), 14–20.

Holling, C. S. (1973). "Resilience and stability of ecological systems." *Ann. Rev. Ecol. Syst.*, 4(1), 1–23.

ISO (International Organization of Standardizations). (2009). "Risk management—Risk assessment techniques." *ISO Standard IEC/FDIS 31010*, Geneva.

Kafali, C., and Grigoriu, M. (2005). "Rehabilitation decision analysis." *ICOSSAR'05 Proc., 9th Int. Conf. on Structural Safety and Reliability*, Millpress, Rotterdam, Denmark.

Li, Y., and Lence, B. (2007). "Estimating of water resources systems." *Water Resour.*, 43, W07422.

Lock, J. (2015). "Real essence." Stanford Encyclopedia of Philosophy, ⟨http://plato.stanford.edu/entries/real-essence/⟩ (Apr. 8, 2015).

MCEER (Multidisciplinary Center for Earthquake Engineering Research). (2015). ⟨http://mceer.buffalo.edu/⟩ (Apr. 8, 2015).

McGill, W. L., and Ayyub, B. M. (2009). "Regional capabilities performance assessment for homeland security." *IEEE Syst. J.*, 3(2), 192–198.

Miles, S., and Chang, S. (2003). "Urban disaster recovery: A framework and simulation model." Multidisciplinary Center for Earthquake Engineering Research (MCEER) of the State Univ. of New York at Buffalo, Buffalo, NY.

Mill, J. S. (2015). ⟨http://en.wikipedia.org/wiki/John_Stuart_Mill⟩ (Apr. 8, 2015).

National Infrastructure Advisory Council. (2015). ⟨http://www.dhs.gov/national-infrastructure-advisory-council⟩ (Apr. 8, 2015).

NRC (National Research Council). (2012). *Disaster resilience: A national imperative*, National Academies Press, Washington, DC.

NTSB (National Transportation Safety Board). (2015). ⟨www.ntsb.gov⟩ (Apr. 8, 2015).

Omer, M., Nilchiani, R., and Mostashari, A. (2009). "Measuring the resilience of the global internet infrastructure system." *3rd Annual Systems Conf.*, IEEE, Piscataway, NJ, 156–162.

PPD (Presidential Policy Directive). (2011). "National preparedness." *PPD-8*, ⟨http://www.dhs.gov/xabout/laws/gc_1215444247124.shtm⟩ (Apr. 8, 2015).

PPD (Presidential Policy Directive). (2013). "Critical infrastructure security and resilience." *PPD-21*, ⟨http://www.whitehouse.gov/the-press-office/2013/02/12/presidential-policy-directive-critical-infrastructure-security-and-resil⟩ (Apr. 8, 2015).

Russell, B. (1993). *Introduction to mathematical philosophy*, Dover, New York.

SDR (Subcommittee on Disaster Reduction). (2005). *Grand challenges for disaster reduction*, National Science and Technology Council, Washington, DC.

Shinozuka, M., et al. (2004). "Resilience of integrated power and water systems." *MCEER Research Progress and Accomplishments: 2003–2004, MCEER-04-SP01*, NY, 65–86.

Tierney, K., and Bruneau, M. (2007). "Conceptualized and measuring resilience." *TR News 250*, 14–17, ⟨http://onlinepubs.trb.org/onlinepubs/trnews/trnews250_p14-17.pdf⟩ (Apr. 8, 2015).

UNISDR (United Nations Office for Disaster Risk Reduction). (2015). ⟨http://www.unisdr.org/⟩ (Apr. 8, 2015).

U.S. Bureau of Economic Analysis. (2015). ⟨www.bea.gov⟩ (Apr. 8, 2015).