Structural Life Expectancy of Marine Vessels: Ultimate Strength, Corrosion, Fatigue, Fracture, and Systems

This paper provides a methodology for the structural reliability analysis of marine vessels based on failure modes of their hull girders, stiffened panels including buckling, fatigue, and fracture and corresponding life predictions at the component and system levels. Factors affecting structural integrity such as operational environment and structural response entail uncertainties requiring the use of probabilistic methods to estimate reliabilities associated with various alternatives being considered for design, maintenance, and repair. Variability of corrosion experienced on marine vessels is a specific example of factors affecting structural integrity requiring probabilistic methods. The Structural Life Assessment of Ship Hulls (SLASH) methodology developed in this paper produces time-dependent reliability functions for hull girders, stiffened panels, fatigue details, and fracture at the component and system levels. The methodology was implemented as a web-enabled, cloud-computing-based tool with a database for managing vessels analyzed with associated stations, components, details, and results, and users. Innovative numerical and simulation methods were developed for reliability predictions with the use of conditional expectation. Examples are provided to illustrate the computations.

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1 Background

Ship structural adequacy considerations require reliability assessments to support engineering, operational, and maintenance decisions based on risk and benefit trade-offs. The structural adequacy considerations include the ultimate hull girder strength based on stiffened panel structural configurations, fatigue, and fracture life predictions. Effects of homeport rotations on structural fatigue loading and corrosion are subject to variability that is describable only in probabilistic terms. Factors affecting structural integrity such as operational environments and structural responses entail uncertainties requiring the use of probabilistic methods to estimate reliabilities associated with various alternatives being considered for design criteria, maintenance, and repair. For example, the variability of corrosion experienced by vessels is one of the factors affecting structural integrity requiring a probabilistic treatment.

Reliability and life expectancy analysis are typically based on particular failure modes at the component and system levels. The following primary failure modes with corrosion effects are considered in this paper: (1) failure of a hull girder as a stiffened thin-walled structure by reaching its ultimate strength; (2) failure of a stiffened panel by material yielding or instability of one or a group of its components; (3) fatigue of structural details due to cyclic loading; and (4) fracture of a member. Numerical and simulation methods are typically used for assessing the time-dependent reliabilities for these failure modes.

The first and second failure modes are based on ultimate strength and extreme loading conditions with corrosion consideration that degrade strengths of the hulls and members. These two failure modes require the use of extreme value analysis and first-crossing time-dependent reliability analysis. These modes were investigated extensively for marine vessels, bridge girders, and other similar structures [e.g., 1,2]. The fatigue performance of marine vessels has been an area of interest and investigation using the stress cycles to failure (i.e., S-N) cumulative damage fatigue analysis [3]. The S-N formulation provides a prediction of fatigue life based on defining failure by crack initiation. Fracture mechanics methods assess life based on crack propagation from an initial crack size to full penetration in a structural member. A combination of fracture mechanics and stochastic modeling of loads provides the necessary tools for these reliability calculations. A limit state function can be formulated by applying linear elastic fracture mechanics. The uncertainties of key influencing parameters can be taken into account by treating them as basic random variables. Such methods should account for corrosion effects on member thicknesses and geometry. Additionally, a fracture mechanics approach produces crack size distribution associated with a vessel’s operational life and use [4].

Examining the hull girder of a vessel as a system entails the reliability analysis of hundreds of components in terms of ultimate strength, fatigue, and fracture with spatial and environmental exposure correlations. The reliability of the system can be approximated based on a discretized vessel in the form of a hull girder, stiffened panels, fatigue details, and fracture locations at critical regions. Weakest link modeling for system reliability forms a practical basis for assessing the time-dependent reliability of the system.
This system modeling formulation can account for interdependence among the components based on the assumption of either perfect dependence or independence using bounding methods for system reliability assessment.

Reliability and life expectancy analysis of marine vessels requires the management of voluminous and potentially dispersed information. The management in this case can be facilitated by the use of web-enabled databases and analytical tools. With the support of the U.S. Coast Guard (USCG), BMA Engineering, Inc. (BMA) developed the Structural Life Assessment of Ship Hulls (SLASH) methodology to compute the time-dependent reliability functions for hulls, stiffened panels, fatigue details, and fracture at the component and system levels [5]. SLASH requires specification of the basic random variables for the corresponding performance functions representing a structural failure mode. BMA has developed a tool that implements the SLASH methodologies. This web-enabled, cloud-computing-based tool with a database can be used to manage ships analyzed and associated stations, components, details and results, and users. The methods are illustrated using examples with insightful observations in this tool.

In this paper, the reliability and life expectancy analysis methodology is fully developed and illustrated using a USCG cutter. This type of analysis is being used to support decision making for maintenance, operation, and repair as part of fleet support and hull structural life evaluations for the USCG's aging legacy cutters and recapitalization of new cutters. The assessments are being used as a basis for making statements relating to the structural life based on the probability of failure for a critical location, groupings of similar structural components, and critical regions of a vessel.

2 Component Reliability Analysis

2.1 Introduction. The methodology proposed in this paper is presented at two levels, the component level and the system level. The component level produces time-dependent reliability functions for hulls, stiffened panels fatigue, and fracture. The system level aggregates the results from the component-level analyses to produce time-dependent reliability functions for a particular group (or groups) of components treating the system as in series, i.e., with a weakest link, to compute the system reliability.

This section presents the methods for assessing the reliability at the component level. The components are defined by structural performance functions for potential failure modes including the ultimate collapse of a hull, the ultimate failure including buckling of a stiffened panel, the fatigue failure of a structural detail, and the fracture of a structural member. For each failure mode, the methodology produces a time-dependent reliability function. The consistency in the outputs for all the failure modes sets a working basis for system reliability analysis as discussed in a subsequent section.

The section starts with introducing a practical corrosion model since it is applicable to all the failure modes. The section then covers methods for the four failure modes.

2.2 Corrosion Model. Corrosion reduces the section modulus of the hull of a vessel by thinning the thickness of primary structural members. It reduces the ability of the structure to resist the externally induced bending moment. Several models of general corrosion growth have been suggested [1,6,7]. In the presence of corrosion, the ultimate strength (Su) of a structural member is given by

\[
S_u(t) = \begin{cases} 
S_{0u} & t \leq t_r, \\
c(t)S_{0u} & t > t_r,
\end{cases}
\]

where Su is the ultimate strength (i.e., resistance) of a structural component; tr is the life of coating (years) as a threshold time; t is the age of the vessel (years), S0u is the initial ultimate strength of a structural component at t = 0; c(t) is a strength reduction factor accounting for corrosion of dimensionless nature in the range [0,1], a model that may take the following form:

\[
c(t) = 1 - at(t - t_r)^b
\]

where a1 = annual thickness reduction factor for general corrosion, a2 = strength reduction factor per unit value of a1, and b = a model coefficient to account for trend nonlinearity, commonly taken as 1. In the case of fatigue and fracture, the effect of corrosion leads to an increase in the local stresses (S) that can be expressed as follows:

\[
S(t) = \begin{cases} 
S_0 & t \leq t_r, \\
S_0/c(t) & t > t_r.
\end{cases}
\]

2.3 Reliability of Stiffened Panels and Hulls

2.3.1 Ultimate Strength Including Buckling. Based on reevaluation of 215 tests by various researchers and using an empirical formulation, Herzog [8] developed models for the ultimate strength of stiffened panels that are subjected to uniaxial compression with or without lateral loads. In this paper, the case of uniaxial compression without lateral pressure is presented as an example since the hydrostatic lateral load is relatively small. The ultimate stress Fu of a longitudinally stiffened panel is given by the following empirical model from [8]:

\[
F_u = \begin{cases} 
\frac{mF_s}{0.5 + 0.5\left(1 - \frac{t_l}{t_r}\right)\sqrt{\frac{Fu}{F_y}}} & \text{for } \frac{t_l}{t_r} \leq 45, \\
\frac{mF_s}{0.5 + 0.5\left(1 - \frac{t_l}{t_r}\right)\sqrt{\frac{Fu}{F_y}}} \left[1 - 0.007\left(\frac{t_l}{t_r} - 45\right)\right] & \text{for } \frac{t_l}{t_r} > 45
\end{cases}
\]

in which E = material modulus of elasticity. The average yield strength FY is

\[
F_Y = \frac{F_{us}A_s + F_{ys}A_p}{A_s + A_p}
\]
The parameter \( A_p \), cross-sectional area of the plate and the stiffener and is given by

\[
A_p = bt \frac{t}{12} + \frac{d_w t_w}{12} + d_u t_u \frac{t}{12} - \frac{d_u^2}{12} + d_u t_u \frac{t}{12} - \frac{d_u^2}{12}
\]

where \( A_p \) = yield strength of plating; \( A_s \) = yield strength of stiffener; \( A_i \) = cross-sectional area of plating; \( A_s \) = cross-sectional area of stiffener and \( A = A_i + A_s \), cross-sectional area of plate-stiffener. The parameter \( k = 1.0, 0.8, \) or 0.65, corresponding to the following respective end conditions: (1) both ends are simply supported, (2) one end is simply supported and the other is clamped, and (3) both ends clamped. The parameter \( m = 1.2, 1.0, \) or 0.8 per Herzog [8], corresponds to the following respective cases: (1) no or average imperfection and no residual stress, (2) average imperfection and average residual stress, and (3) average or large imperfection and high value for the residual stress; \( a = \text{span (length)} \) of stiffener, \( b = \text{stiffener spacing}; \) \( t = \text{plate thickness}; \) and \( r = \text{radius of gyration of one stiffener with fully effective plating} \) and is given by

\[
r = \sqrt{\frac{I}{A}}
\]

where \( A = \text{sectional area of the plate and the stiffener} \) and is given by

\[
A = bt + d_w t_w + f_u t_f
\]

The moment of inertia of one stiffener with fully effective plating \( (I) \) is given by

\[
I = \frac{bt^3}{12} + bt \left( \frac{z_0 - \frac{t}{2}}{2} \right)^2 + d_w t_w \left( \frac{z_0 - \frac{d_u}{2}}{2} \right)^2 + f_u t_f \left( z_0 - t - d_u - \frac{t}{2} \right)^2
\]

where \( z_0 = \text{distance of neutral axis from the base line of the plate} \) and the thickness of stiffener web; \( t_f = \text{thickness of webbia HW} \) and the thickness of stiffener web; \( t_f = \text{thickness of webbia HW} \).
of stiffener flange; \( d_w \) = stiffener web height; and \( f_w \) = stiffener flange width. The parameter \( m \) of Eq. 4 was necessary since the 215 tests evaluated by Herzog belong to three distinct groups. Group I (75 tests) consisted of small values for imperfection and residual stress, Group II (64 tests) had average values for imperfection and residual stress, while the third group (Group III, 76 tests) consisted of higher values for imperfection and residual stress. Figure 2 defines the panel geometry [2].

The Herzog model was compared with experimental and numerical data [9,10,11] and proved to predict the strength value reasonably well. Assakkaf and Ayyub [11] concluded that the empirical model [8] for predicting the ultimate strength of stiffened panels has the least bias with a relatively low coefficient of variation among the other competing models. Table 3 provides the recommended stiffened panel strength model for the load and resistance factor design (LRFD) development [2,11]. The table also shows their probabilistic characteristics and biases.

In cases where the MAESTRO [12] finite-element models (see reference list for information on this computer program) are available for a vessel, the strength of the stiffened panels can be obtained from the MAESTRO [12] models and used as an empirical approximation of the ultimate strength of the panels. In cases where the panels are subjected to tension, the strength can be taken as the yield strength [13] as summarized in Table 3.

As for hulls, the probabilistic characteristics can be based on the MAESTRO [12] model of Table 3 with a bias of 1, a coefficient of variation (COV) of 0.18, and a lognormal probability distribution.

### Table 3: Recommended stiffened panels strength models for reliability analysis of stiffened panels and hulls

<table>
<thead>
<tr>
<th>Loading case</th>
<th>Mean (description)</th>
<th>Total bias</th>
<th>Coefficient of variation (COV)</th>
<th>Distribution type</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniaxial compression</td>
<td>Eq. (4)</td>
<td>1.0</td>
<td>0.18</td>
<td>Lognormal</td>
<td>Herzog [8]</td>
</tr>
<tr>
<td>Uniaxial compression</td>
<td>MAESTRO model</td>
<td>1.0</td>
<td>0.18</td>
<td>Lognormal</td>
<td>MAESTRO [12]</td>
</tr>
<tr>
<td>Uniaxial tension</td>
<td>( F_y ) (ordinary steel)</td>
<td>1.11</td>
<td>0.07</td>
<td>Lognormal</td>
<td>Hess et al. [13]</td>
</tr>
<tr>
<td>Uniaxial tension</td>
<td>( F_y ) (high-strength steel)</td>
<td>1.22</td>
<td>0.09</td>
<td>Lognormal</td>
<td>Hess et al. [13]</td>
</tr>
</tbody>
</table>

### Table 4: Recommended primary loads for reliability analysis of stiffened panels and hulls

<table>
<thead>
<tr>
<th>Loading type</th>
<th>Mean (description)</th>
<th>Total bias</th>
<th>Coefficient of variation</th>
<th>Distribution type</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stillwater vertical bending moment</td>
<td>Using fundamental naval architecture (naval vessels)</td>
<td>0.70</td>
<td>0.15</td>
<td>Normal</td>
<td></td>
</tr>
<tr>
<td>Combined wave and dynamic vertical bending moment</td>
<td>A annual load occurrence using SPECTRA</td>
<td>1.0</td>
<td>0.25</td>
<td>Weibull</td>
<td>Michaelson [14]</td>
</tr>
<tr>
<td>Combined wave and dynamic vertical bending moment</td>
<td>Annual extreme value using SPECTRA</td>
<td>1.0</td>
<td>0.25</td>
<td>Weibull</td>
<td>Ayyub et al. [3]</td>
</tr>
<tr>
<td>Load encounter rate</td>
<td>Annual load rate (( \lambda ))</td>
<td>NA</td>
<td>NA</td>
<td>Deterministic</td>
<td></td>
</tr>
</tbody>
</table>

Note: NA = not applicable.
performing these computations, called SPECTRA, that produces annual loading that can be represented by a Weibull probability distribution as provided in Table 4 [15]. The combined wave and dynamic vertical bending moment produced by SPECTRA is for a single encounter, whereas the combined wave and dynamic vertical bending moment [3] is the annual extreme value.

2.3.3 Performance Function and Reliability Assessment. The reliability of a ship structural component can be defined as the likelihood of it maintaining its ability to fulfill its design purpose for some time period under specified environmental and operational conditions. In this paper, calculating time-dependent reliabilities based on its ultimate strength is based on hulls or stiffened panels in a particular region of interest of a vessel. A strength performance function can be expressed in consistent units as follows:

$$g(t) = S_n(t) - L_{sw}(t) - L_w(t)$$  \(8\)

where \(S_n\) is the strength of a stiffened panel random variable accounting for all relevant uncertainties, e.g., \(F_u\) in Eq. 4; \(L_{sw}\) is the stillwater loading random variable accounting for modeling uncertainty in still water; and \(L_w\) is the wave loading random variable accounting for modeling uncertainty, nonlinearities, and dynamic effects. Typical values are provided in Tables 3 and 4.

The instantaneous reliability may be obtained based on the limit state defined in Eq. 8, where the failure domain is defined by \(\Omega = \{g(t) < 0\}\) and its complement \(\{g(t) > 0\}\) defines the safe domain. The instantaneous failure probability at time \(t\) is defined by

$$P_f(t) = \int_{\Omega} f(x(t))dx$$  \(9\)

where \(f(x(t))\) is the joint probability density function of the basic random variables defining strength and loading random variables at time \(t\). In general, the joint probability density function is unknown, and evaluating the convolution integral is a formidable task. Several practical approaches including the first-order reliability method (FORM), second-order reliability method (SORM), advanced second moment (ASM) method, or Monte Carlo simulation are usually used. The theory of FORM, SORM, and Monte Carlo simulation are well established and can be found in Refs. [16,17]. The initial probability of failure \((P_f)\) at design and construction, i.e., at \(t = 0\), is given by Eq. (9) using \(t = 0\). The survival probability \((P_s)\) is

$$P_f = 1 - P_s$$  \(10\)

In the presence of degradation mechanisms such as corrosion, the ultimate strength \(S_n(t)\) is a decreasing function of time according to Eq. 1; therefore, the probability of failure is also a function of time. By varying the time period \(t\) from zero to an expected service life, the decreasing values of ultimate strength \(S_n(t)\) can be estimated. Furthermore, the instantaneous failure probability at any time \(t\), defined by \(\Omega = \{g(t) < 0\}\) without regard to survival of a vessel in the previous years, can be obtained using Eq. 9.

Several methods for analytical time-dependent reliability assessment are available. In these methods, significant loads as a sequence of pulses can be described by a Poisson process with mean occurrence rate \(\lambda\), random intensity \(L\), and duration \(\tau\). The treatment is based on reliability theory [e.g., 17] and its subsequent adaptation for strength-degraded structures [18]. The performance function \((Z)\) of a component or system at any instant of time \((t)\) can be defined as

$$Z(t) = S(t) - L(t)$$  \(11\)

where \(S(t)\) is the strength at time \(t\) and \(L(t)\) is the load at time \(t\) as shown in Fig. 3. The instantaneous probability of failure at time \(t\) can then be defined as the probability of \(S(t)\) less than \(L(t)\); however, this instantaneous probability treatment does not recognize what has previously happened to the component or system from the start of its life to the present, represented by time \(t\). One is usually interested in the first occurrence of \(L\) exceeding \(S\), not the instantaneous occurrence, requiring the imposition of a condition on the probability of \(L\) exceeding \(S\) of being the first time in its life. Such a conditional probability concept is the basis for computing what is termed time-dependent reliability, and estimated using the reliability function \(R(t)\).

The reliability function, \(R(t)\), is defined as the probability that a component or a system survives during interval of time \((0, t)\) based on a performance function \(Z\). Assuming the load to follow a Poisson process with a rate \(\lambda\) means that the time to a load occurrence is exponentially distributed. Characterizing the time to failure requires not only the time to load occurrence but also the consideration that only some of the load occurrences may lead to failure; therefore, the following expression can be made based on the exponential distribution:

$$R(t) = \exp(-\lambda P_f)$$  \(12\)

where \(\lambda P_f\) is the product of load rate \((\lambda)\) and the average failure probability \((P_f)\) over the period \((0, t)\) that should account for any degradation of the strength \((S)\). The strength degradation, for example due to the corrosion of a structural member, can be modeled by a function \(0 < c(t) < 1\) and used as a multiplier to an initial strength \((S_0\) at \(t = 0\).) This probability, \(P_f\), is taken as the average value over the period \((0, t)\) as follows:

$$P_f = 1 - \frac{1}{t} \int_{t=0}^{t} P(Z > 0)dt = 1 - \frac{1}{t} \int_{t=0}^{t} P(cS > L)dt$$  \(13\)

where \(Z = S - L\) is an example performance function. Substituting Eq. 13 into Eq. 12 and accounting for the uncertainty in the initial strength produces the following expression:

$$R(t) = \int_{t=0}^{\infty} \exp\left[-\lambda in(R(t))\right] f_{S_0}(s)ds$$  \(14\)

where \(f_{S_0}(s)\) is the probability density function of the initial strength \((S_0)\). Noting that the expression \(P(c(\tau)s > L)\) in Eq. 14 is the cumulative distribution function of \(L\) evaluated at \(c(\tau)s\), the reliability function can be written as

$$R(t) = \int_{t=0}^{\infty} \exp\left[-\lambda in(R(t))\right] f_{S_0}(s)ds$$  \(15\)

The reliability can be expressed in terms of the failure rate or hazard function, \(h(t)\), as

$$h(t) = -\frac{d}{dt} \ln(R(t))$$  \(16a\)
which is also related as follows:

\[ R(t) = \exp \left( - \int_0^t h(\tau) d\tau \right) \]  

(16b)

and

\[ h(t) = \frac{f(t)}{1 - F(t)} \]  

(16c)

The reliability \( R(t) \) is based on complete survival during the service life interval \((0, t)\). It means the probability of successful performance during the service life interval \((0, t)\). Therefore, the probability of failure, \( P_f(t) \) or \( F(t) \), can be computed as the probability of the complementary event, \( P_f(t) = 1 - R(t) \), not being equivalent to \( P[S(t) < L(t)] \), the latter being just an instantaneous failure at time \( t \) without regard to previous performance.

The conditional expectation method [17] was implemented based on the following performance function:

\[ Z = c(t)S_a - L_{sw} - L_w \]  

(17)

The computational procedure for the conditional expectation method is as follows:

- In the \( i \)th simulation cycle, randomly generate \( S_a \) and \( L_{sw} \) as \( s_a \) and \( l_{sw} \).
- Evaluate \( R_i(t) \) using Eq. 15 for all the \( t \) values of interest, \( t = 1, 2, 3, \ldots, 50 \) years for each simulation cycle as follows:
  - For \( t = 1 \), evaluate
    \[ \frac{1}{t} \int_0^{t-1} F_{L_a}(c(\tau)s_{a_i} - l_{sw_i}) d\tau \]
  - using the trapezoidal rule based on say 100\( \tau \) increments, and then compute
    \[ R_i(t = 1) = \exp \left[ -\lambda t \left( \frac{1}{t} \int_0^{t-1} F_{L_a}(c(\tau)s_{a_i} - l_{sw_i}) d\tau \right) \right] \]
  - For \( t = 2 \), evaluate
    \[ \frac{1}{t} \int_1^{t-1} F_{L_a}(c(\tau)s_{a_i} - l_{sw_i}) d\tau \]
  - using the trapezoidal rule based on 100 additional increments and add the result to the previous one of
    \[ \frac{1}{t} \int_0^{t-1} F_{L_a}(c(\tau)s_{a_i} - l_{sw_i}) d\tau \]
  - then compute
    \[ R_i(t = 2) = \exp \left[ -\lambda t \left( \frac{1}{t} \int_0^{t-1} F_{L_a}(c(\tau)s_{a_i} - l_{sw_i}) d\tau \right) \right] \]
  - and repeat the process until \( t = 50 \).

Repeat the previous step for the next simulation cycle \( i + 1 \) to obtain \( R_{i+1}(t) \) for \( t = 1, 2, \ldots, 50 \) and until \( I = N \) cycles.

For each \( t \), compute the statistics of \( R(t) \) and check for convergence as follows:

\[ \bar{R}(t) = \frac{\sum_{i=1}^N R_i(t)}{N} \]  

(18)

where \( N \) is the number of simulation cycles. The accuracy of this estimate can be evaluated through the variance (Var) and COV as given by

\[ \text{Var}(\bar{R}(t)) = \frac{\sum_{i=1}^N (R_i(t) - \bar{R}(t))^2}{N(N - 1)} \]  

(19)

and

\[ \text{COV}(\bar{R}(t)) = \frac{\sqrt{\text{Var}(\bar{R}(t))}}{\bar{R}(t)} \]  

(20)

It should be noted that the composite trapezoidal rule with a uniform grid can be used to calculate the integrals

\[ \int_a^b R(x) dx \approx \frac{b - a}{N} \left( R(x_0) + 2 \sum_{i=1}^{N/2} R(x_{2i-1}) + R(x_N) \right) \]  

(21a)

where \( R(x) \) is an arbitrary function, \( N \) is the number of intervals for numerical integration, and \( x_i \) is \( i \)th value of \( x \) as follows:

\[ x_i = a + \frac{b - a}{N}, \quad \text{for } i = 0, 1, 2, \ldots, N \]  

(21b)

2.4 Fatigue Reliability. Fatigue design criteria are typically expressed in years of service, such as 30, 40, or 50 years, without crack initiation with some associated probability. A general design procedure for fatigue can be based on reliability methods [3,4,7,19,20,21,22].

The fatigue life of a structural detail, subjected to the action of cyclic stress, is defined as the total number of stress cycles required to initiate a dominant fatigue crack added to the number of stress cycles required to propagate this crack until the final failure. This total life, in a simplified view, is a function of the geometry of the structure (local and global) applied stress range, the mean stress and the environment where the structure is located.

The stress-based fatigue analysis methodologies, represented by the classical S-N diagram, embody the damage evolution, crack nucleation, and crack growth stages of fatigue into a single, experimentally characterized continuum formulation. These S-N curves, however, are developed experimentally based on relatively small structures and their failure does not necessarily correspond to the structure failure, which is based on the behavior of very large highly redundant structure. Another factor that could be taken into account in fatigue analysis is the corrosion effect. In simple terms, the corrosion process can cause reduction in plate thickness, making the stress range acting on the structural detail to become time-dependent. This paper presents a method to take into account the effect of corrosion on fatigue reliability assessment using this simplified approach; however, the methodology does not account for corrosion-fatigue interaction. Such interaction should be considered in future studies.

This section provides a method to assess the fatigue life of ship structural details subjected to stress ranges induced by sea loading. That method is based on S-N curve approach for fatigue analysis. The damage associated with fatigue can be calculated as follows:

\[ D = \frac{1}{k_s A} \sum_{i}^{k} \frac{n_i}{S_i^f} \]  

(22)

where \( k_s \) expresses the uncertainty in stress range calculation, \( A \) is the stress life parameter of the S-N curve; \( B \) is the slope parameter of the S-N curve, \( n_i \) is the number of actual load cycles at the \( i \)th stress range level \( (S_i) \), and \( k \) is the number of stress range levels. The damage calculated according to Eq. 22 considers that the stress range acting on the structural detail, for each sea state condition, is constant during the ship structural life. Nevertheless, the ship structure can be corroded during its life.

The corrosion effect, based on the hypothesis of uniform corrosion of ship structural detail, is represented by a reduction of structure thickness, which affects the stress range magnitude and consequently the accumulated fatigue damage. The increase in the
local stresses ($S$) can be expressed by Eq. 3 using the corrosion model of Eq. 2.

Based on Eq. 22, the fatigue damage becomes dependent on the time variation of the stress range due to corrosion effects. The fatigue damage is calculated as follows:

$$D_j = \frac{1}{k} \sum_{j=1}^{n} \sum_{i=1}^{k} \frac{n_{ij}}{S_{ij}}$$

where $T$ is the number of operational years considered in the analysis, $n_{ij}$ is the number of actual load cycles at the $i$th stress range level ($S_i$) during one operational year, $k$ is the number of stress range levels, and $S_{ij}$ is the $i$th stress range level during one operational year calculated as follows:

$$S_{ij} = S_i \left(1 - a_i a_2 ((j - 1) - t_j)^b\right), \text{ for } j > (t + 1)$$

$$S_{ij} = S_i, \text{ for } j \leq (t + 1)$$

where $\Delta g$ is the fatigue damage ratio limit that has a mean value of 1 [23]. The method takes into account the corrosion damage effect according to Eq. 2.

### 2.5 Fracture Distribution and Reliability

The reliability method developed herein is based on the crack growth prediction executed according to linear elastic fracture mechanics principles, where the stress intensity factor ($K$) is used to define the stress field in the vicinity of a crack. The value of the stress intensity factor depends on the loading, body configuration, crack shape, and mode of crack displacement. According to Fuchs and Stephens [24], the basic equation that governs crack growth, named the Paris law, is given by

$$\frac{da}{dN} = C \Delta K^m$$

where $a$ is the crack size, $N$ is the number of fatigue cycles, $\Delta K$ is the range of stress intensity factor, and $C$ and $m$ are crack propagation parameters that come from fracture mechanics.

The range of the stress intensity factor is given by [24]

$$\Delta K = S f(a) \sqrt{\pi a}$$

in which $f(a)$ is a function of crack geometry and structure geometry and $S$ is the stress range induced by the cyclic loading. When the crack size $a$ reaches some critical crack size $a_{cr}$, failure is assumed to have occurred.

Although most laboratory testing is typically performed with constant amplitude stress ranges, Eq. 26 is always applied to variable stress range models that ignore sequence effects. Rearranging the variables in Eq. 26 and substituting Eq. 27 into Eq. 26, the number of cycles for the crack to grow from the initial size ($a_i$) to a particular crack size ($a_j$) can be computed by integration as follows:

$$N = \frac{1}{C(S)^m} \int_{a_i}^{a_j} \frac{da}{f(a)^m(\sqrt{\pi a})^m}$$

Modeling $f(a)$ as a constant for a particular structural detail with an assumed geometry, the crack growth prediction can be expressed by integrating Eq. 28 ([25]) up to $a = a_j$:

$$a_j^{(1-\frac{2}{m})} = a_i^{(1-\frac{2}{m})} + \left(1 - \frac{m}{2}\right) C S_{ij}^{m} N_i^{\frac{2}{m}} C R_{ij}^{m} N_j$$

The random variables are $a_i$, $k_i$, and $C$. The probabilistic characteristics of those random variables are provided by Ref. [25]. The stress range is considered deterministic. The value $N_i$ represents the number of loading cycles during an operational life. The final crack size distribution ($a_j$) is also a random variable with a distribution that can be simulated with the application of Monte Carlo simulation. The statistical properties of $a_j$, such as mean, median, and COV, can be calculated from the simulation results.

The stress fluctuation during a ship’s operational life due to sea state conditions requires modifying Eq. 29 to account for the planning time horizon of interest that can be different than the built-in number of operational days in typical sea-state databases. The crack growth analysis is modified as follows:

$$a_j^{(1-\frac{2}{m})} = a_i^{(1-\frac{2}{m})} + \left(1 - \frac{m}{2}\right) C S_{ij}^{m} N_i^{\frac{2}{m}} C R_{ij}^{m} N_i^{\frac{2}{m}}$$

where $T_{\text{operational}}$ represents the operational life for crack growth analysis (in days) according to the planning time horizon, $T_{\text{database}}$ represents the number of days used to define the stress range database for a particular sea state, $N_i$ is the frequency of occurrence of the $i$th stress range ($S_i$) recorded in the database, and $k$ is the number of stress ranges defining the underlying histogram.

The crack growth analysis produces the final crack size ($a_j$) distribution for a particular operational life based on the input of the probability distribution functions of $a_i$, $k_i$, and $C$, and the parameter $\alpha$ related to the crack geometry and structural detail. Prediction of the probability distribution of $a_j$ enables engineers to consider appropriately consider repair and maintenance decisions, and assigning vessels to new missions.

The following computation procedure based on Monte Carlo simulation is proposed:

- In the $i$th simulation cycle, generate the random variables $a_j$, $C$, and $k_i$ according to their respective probability distributions.
- Use the generated values of $a_j$, $C$, and $k_i$, and deterministic values of $m$, $\alpha$, $S_i$, $N_i$, the summation over $k$, $T_{\text{operational}}$, and $T_{\text{database}}$ to compute $a_j$ for the times $t = 1, 2, 3, \ldots$, the number of years in the planning horizon, e.g., 30 years.
- Terminate the $i$th simulation cycle either by reaching the number of the years in the planning horizon or once the time-dependent $a_j(t)$ exceeds the permissible crack size, e.g., 0.25 in.
- Repeat the simulation process $N$ times.
- Store the generated values of $a_j$, $C$, and $k_i$, and crack size $a_j$ as a function of time $t$ and for each simulation cycle.

The postprocessing of the stored results has the objectives of characterizing the crack size histogram as a function of time $t$, and estimating the reliability function $R(t)$. The following procedure is proposed:

- For each time $t$, using an increment of 1 year and up to the number of the years in the planning horizon, compute the count of values (denoted $n_t$) out of $N$ simulation cycles, average, median, standard deviation, and coefficient of variation of $a_j$, $C$, and $k_i$, and crack size $a_j$.
- For each time $t$, increasing the increment of one year and up to the number of the years in the planning horizon, estimate the reliability function $R(t)$ as the count of values out of $N$ simulation cycles divided by $N$, i.e., $n_t / N$, and compute the cumulative distribution function of life $F(t) = 1 - R(t)$.
- Estimate the statistical uncertainty of the estimated reliability by its COV as follows:
The COV values of Eq. 31 treat the simulation cycles as Bernoulli trials [16], and they approach 0 as \( N \) approaches infinity. The COV can be used as a criterion to determine the required number of simulation cycles (\( N \)) to accurately estimate \( R(t) \).

3 System Reliability Analysis

The system reliability can be analyzed using a model of discrete components. The proposed method is based on the assumption of a discretized system in a series composed of \( n \) components, i.e., the system is considered to perform adequately, if and only if all of its \( n \) components are performing adequately. Figure 4 depicts a reliability block diagram representation of a series system consisting of three components.

With the reliability (\( R \)) defined as \( 1-P_f \), the time-dependent reliability function of a system composed of \( n \) components, \( R_i(t) \), is given by

\[
R_i(t) = \prod_{j=1}^{n} R_i(t) \quad \text{for independent events} \tag{32}
\]

The component reliability values are obtained using the methods described in previous sections. The system reliability can be evaluated for any group of components that represent a subsystem or a system of interest, such as a bulkhead, a deck, a station, or the entire vessel. Equation 32 is based on the assumption that the failure events of the components are independent. For the case of perfect, positive dependence among failure events of components, the minimum value of all the component reliabilities is the value that should be assigned to \( R_i(t) \) as follows:

\[
R_i(t) = \min_{i=1}^{n} R_i(t) \quad \text{for perfect dependence} \tag{33}
\]

For the purposes of system reliability evaluation of stiffened panels, the condition of independence is closer to reality than perfect dependence, and is assumed in the system reliability computations for stiffened panels, although the real level of dependence falls in between these two extreme cases. As for computing system reliability of the hull structure including both cases of stiffened panels and fatigue details of interest, the respective system reliability results for stiffened panels and fatigue details as systems can be combined based on the assumption of a system in series with independent events. Bounding methods for system reliability assessment are used to examine the effects of independence and perfect, positive dependence [17].

The computational procedure proposed for aggregating component reliability function for the purpose of estimating the system reliability function requires defining the following notations:

\[
R \quad \text{reliability} \\
\quad \text{time} \\
R_{ss} \quad \text{reliability of a group of stations or an entire vessel as a system} \\
R_{s} \quad \text{reliability of station } i \text{ as a system} \\
R_{sp} \quad \text{reliability of several panels as a subsystem in station } j \\
R_{fr} \quad \text{reliability of several fatigue details as a subsystem in station } j \\
R_{frj} \quad \text{reliability of several fracture details as a subsystem in station } j \\
R_{j} \quad \text{reliability of panel } j \text{ as a component} \\
R_{ij} \quad \text{reliability of fatigue detail } j \text{ as a component} \\
R_{frj} \quad \text{reliability of fracture detail } j \text{ as a component} \\
R_{hi} \quad \text{reliability of hull girder station } i
\]

The models and associated assumptions are summarized in Table 5 where the asterisk (*) means arithmetic multiplication at corresponding times for \( R(t) \).

While the method for the systems analyses combine the reliability results at the component level, their individual contributions should be compared along with their respective consequences in a risk assessment framework based on the user’s risk metrics and risk tolerance. The development of a comparative approach for risk assessment is the subject of further work.

4 Examples

In this section, three examples are used to illustrate the computations for panel buckling failure, fatigue, and fracture.

\[\text{Fig. 4 Series system composed of three components}\]

\[\text{Fig. 5 Effects of corrosion rate on panel reliability based on buckling strength}\]

<table>
<thead>
<tr>
<th>Item</th>
<th>Computation order</th>
<th>Independent</th>
<th>Perfectly positively correlated</th>
<th>System in series</th>
<th>System in parallel</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panels within a vessel’s station</td>
<td>1</td>
<td>( x )</td>
<td>( x )</td>
<td>( R_{ss} )</td>
<td>( R_{ss} \times R_{ss} \times \ldots )</td>
<td>( R_{ss} )</td>
</tr>
<tr>
<td>Fatigue</td>
<td>2</td>
<td>( x )</td>
<td>( x )</td>
<td>( R_{fr} )</td>
<td>( R_{fr} \times R_{fr} \times \ldots )</td>
<td>( R_{fr} )</td>
</tr>
<tr>
<td>Fracture</td>
<td>3</td>
<td>( x )</td>
<td>( x )</td>
<td>( R_{frj} )</td>
<td>( R_{frj} \times R_{frj} \times \ldots )</td>
<td>( R_{frj} )</td>
</tr>
<tr>
<td>Hull girder station ( i )</td>
<td>4</td>
<td>( x )</td>
<td>( x )</td>
<td>( R_{j} )</td>
<td>( R_{j} \times R_{j} \times \ldots )</td>
<td>( R_{j} )</td>
</tr>
<tr>
<td>Within station ( i )</td>
<td>5</td>
<td>( x )</td>
<td>( x )</td>
<td>( R_{ij} )</td>
<td>( R_{ij} \times R_{ij} \times \ldots )</td>
<td>( R_{ij} )</td>
</tr>
<tr>
<td>Several stations or an entire vessel</td>
<td>6</td>
<td>( x )</td>
<td>( x )</td>
<td>( R_{hi} )</td>
<td>( R_{hi} \times R_{hi} \times \ldots )</td>
<td>( R_{hi} )</td>
</tr>
</tbody>
</table>

\( x \) = assumption employed.
4.1 Example 1: Panel Buckling and Corrosion. Figure 5 provides the results for the panel buckling with the average corrosion shown in Table 2 and parameters shown in Table 3. In the figure, the aggressive corrosion is ten times the average value and occurs in isolated areas in the structure due to standing water that are difficult to inspect. The results in Fig. 5 indicate the coating systems are generally effective. The areas with the highest probability of failure are associated with aggressive corrosion. It can be inferred from Fig. 5 that aggressive corrosion can be managed in a five-year inspection schedule to reduce the risk of significant failure. However, class-wide, targeted inspections and maintenance are required to reduce the risk associated with aggressive local corrosion.

4.2 Example 2: Fatigue Life Prediction With Corrosion. Table 6 and Fig. 6 illustrate the computations of the probability of failure of a structural detail with corrosion. The analysis is performed for an assumed sea load condition and S-N curve AASHTO-D. The probability function parameters for fatigue damage limit and stress uncertainty are present in Fig. 5. The corrosion model parameter \( a_1 \) was arbitrarily set equal to 0.005 for demonstration purposes of the proposed model that accounts for corrosion effects in fatigue reliability and life. Clearly, the life of coating threshold time has a strong influence in fatigue reliability.

4.3 Example 3: Trend Prediction of Crack Size Distribution. An illustrative example was prepared for the purposes of this report using an example sea load condition and crack growth curve proposed by Lassen [25,26]. The stress uncertainty is modeled by a normal distribution with a mean equal to 1.0 and COV of 0.1. The crack is modeled as semi-elliptical (in a structural panel) with the relation \( (a/c) = 0.1 \) that leads to \( a \) equal to 1.10 [25] in fracture mechanics analysis. The initial crack depth distribution is assumed to follow an exponential probability distribution with a mean value of 0.025 in. The structural detail thickness is assumed to be 0.25 in. The analysis considers the depth growth of the crack along the ship operational life.

Figure 7 shows a prediction of the mean value of crack depth as a function of the ship’s operational use in years using Eq. 30. A model was fitted to the mean crack depth as shown on the same figure. Figure 8 shows the histograms of final crack depth for different operational times based on the simulation results. The simulation limits the maximum crack depth to the structural detail thickness. The longer the operational life, the greater the likelihood that the crack becomes a through thickness crack, i.e., the probability of exceeding the 0.25-in. limit increases with the increase in the operational time as shown in Fig. 9 in terms of a reliability function \( R(t) \).

5 Structural Life Assessment of Ship Hulls

The Fatigue Life Assessment of Ship Structures (FLASH) methodology was implemented using a web-enabled computer code and database to compute the time-dependent reliability functions at the component and system levels. The tool calculates the structural reliability function based on time to first failure at the component and system levels for hull girders, stiffened panels, and fatigue by accounting for degraded strength due to corrosion, and fracture. SLASH requires specification of the basic random variables for the corresponding performance functions for ship structure failure modes.

SLASH includes a database to manage users, ships, and associated stations, components, details, and results. Navigation is handled through tabs and highlighted links. Each tab or link corresponds to a specific action required for analysis of a ship. A tab for ship system definition provides tools for defining a ship through stations. Each station can be further expanded into a hull girder,
stiffened panels, and fatigue profiles for each panel. Tabs are also present to examine each station’s components individually as well as an additional tab to define fracture characteristics. Figures 10 and 11 illustrate the screens used for database access and management.

An analyze option for created ship elements allows the user to mathematically define the appropriate characteristics of the model. Input is handled through drop-down menus and text fields. User-defined quantities include deterministic factors for defining corrosion and fatigue models. A library of predefined load histograms and S-N curves are available for fatigue calculations. Values for probabilistic analysis including distribution type, corresponding parameters, and analysis types can also be set as shown in Fig. 12. An option to update quantities for existing elements is available.

Fig. 8 Crack depth histograms as a function of years of operation: (a) After one year of operation, (b) after five years of operation, (c) after ten years of operation, and (d) after 15 years of operation

Fig. 9 Fatigue reliability as a function of years of operation

Fig. 10 SLASH ship database
Fig. 11 SLASH ship definition tab

Fig. 12 Example SLASH input screen
through the *Analyze* function or alternatively through specific *Update* links for each element. The execution of the SLASH program corresponding to a ship element of interest is available under the *Analyze* menu. Results of the most recent execution of the SLASH program are stored in a Results tab to avoid the necessity to re-execute SLASH to view results.

The individual results (see Fig. 13) for each ship component are tied together for each station through the ship definition. Stations are analyzed by aggregating the effects of the individual components defined for each station. A total ship reliability result is calculated by combining the reliabilities of each station. Results are also presented as reliability plots by year for each station defined for the ship. For additional information on this tool, the authors may be contacted.

6 Summary and Conclusions

This paper provides the technical background on the approach developed for assessing the time-dependent reliability functions for marine vessels within the context of the strength of hulls, stiffened panels, fatigue, and fracture. This analysis supports decision making for maintenance, operation and repair as part of fleet support, and hull structural life evaluations. Moreover, the assessment can be used as a basis for making statements relating to the fatigue life to probability of failure for a critical location, groupings of similar details, and the entire vessel. It is envisioned that such methods will be used to perform probabilistic life assessments of vessels in support of designing new vessels and the prediction of remaining life of vessels in operation.

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References