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Uncertainty Analysis of Structural Strength of Stiffened Panels

ABSTRACT A prototype computational methodology for reliability assessment of continuum structures using finite element analysis with instability failure modes is described in this paper. Examples were used to illustrate and test the methodology. Geometric and material uncertainties are considered in the finite element model. A Computer program was developed to implement this methodology by integrating uncertainty formulations to create a finite element input file, and to conduct the reliability assessment on a machine level. A commercial finite element package was used as a basis for the strength assessment in the presented procedure. A parametric study for a stiffened panel strength was also carried out.

The finite element model for the stiffened panel is based on the 8-node doubly curved shell element, which can provide the non-linear behavior prediction of the stiffened panel. The mesh is designed to ensure the convergence of eigenvalue estimates. Failure modes are predicted on the basis of elastic non-linear analysis using the finite element model.

Reliability assessment is performed using Monte Carlo simulation with variance reduction techniques that consisted of the conditional expectation method. Conditional expectation estimates the failure probability in each simulation cycle, then the average failure probability and its statistical error are computed. The developed method is expected to have significant impact on the reliability assessment of structural components and systems. This impact can extend beyond structural reliability into the generalized field of engineering mechanics.

Introduction

The main objective of engineering design is to ensure the safety and performance of an engineering system for a given period of time and for specified loading conditions. The absolute safety of a system cannot be guaranteed due to the uncertainties involved in structures, their loading and behavior. However, through reliability methods the probability of failure of the system can be limited to an acceptable level. In order to have control on the reliability of structural systems, it needs first to be estimated and assessed. Reliability assessment of structures have been significantly developed primary for discrete systems. However, dealing with a structure as a continuum can provide a generalized basis of analysis. Such a modeling approach can lead to a better understanding of its behavior and analysis of its performance under different conditions. Also, it can deal with uncertainties in geometric and material properties for conventional and innovative materials, as well as uncertainties in failure definition and failure modes. Therefore, a need exists to adapt and use state-of-the art reliability assessment methods to deal with continuum structures.

Reliability assessment requires the knowledge of structural strength. Structural strength in its turn is associated with different types of geometric and material uncertainties. In general, uncertainties in structural behavior and performance may be associated with physical phenomena that are inherently random or with predictions of reality performed under conditions of incomplete or inadequate information. Therefore, uncertainty may be associated with inherent variability of a physical process or with imperfection in the modeling of a physical process. Moreover, prediction or modeling error may contain two components, the systematic component and the random component. In measurement theory, these are known as the systematic error and random error, respectively. From a practical standpoint, inherent variability is essentially a state of nature and the resulting uncertainty may not be controlled or reduced. The uncertainty associated with prediction or modeling error may be reduced through the use of more accurate models or acquisition of additional data. Uncertainties associated with inherent variability as well as random error can be expressed in terms of coefficients of variation. These uncertainty measures are needed to accurately assess probabilities of interest. Methods for evaluating uncertainty measures including biasedness depend on the form of the available data and information (Ang and Tang 1984 [1]). Failure or survival of real structures according to the different serviceability or strength criteria are continuous and gradual rather than crisp and abrupt. Consequently, failure or survival definitions are accompanied with uncertainty that can be considered to be of the vagueness type. Fuzzy set theory can be used to deal with this type as demonstrated by Ayyub and Lai (1992 [2]).

Simulation methods can be used for structural reliability assessment. In their fundamental form, applied loads to a structure can be randomly generated, then finite element analysis can be used to predict the response of the structure un-

der a combined state of the generated loads. The response can be in the form of a stress field, strain field, or deformation field, or their combinations. Using a crude simulation procedure, the response needs to be compared with a specified failure definition, and failures counted. By repeating the simulation procedure several times, the failure probability according to the specified failure definition can be estimated as the failure fraction of simulation repetitions.

The objectives of this study are: a) to develop a prototype computational procedure for reliability assessment of a continuum structure using finite element analysis with instability failures modes, and b) to test the procedure. A state-of-the art Finite Element (FE) commercial package was used in this study to assess the strength of a continuum structure. In addition, parametric analyses for the strength and reliability of a stiffened panel due to changes in the uncertainties in geometric and material properties were performed.

Methodology

GENERAL DESCRIPTION

The development of a methodology for the reliability assessment of continuum ship structural components or systems requires the consideration of the following three components: 1) loads, 2) structural strength, and 3) methods of reliability analysis. Also, the reliability analysis requires knowing the probabilistic characteristics of the operational-sea profile of a ship, failure modes, and failure definitions. A reliability assessment methodology can be developed in the form of the following modules: operational-sea profile and loads; nonlinear structural analysis; extreme analysis and stochastic load combination; failure modes, their load effects, load combinations, and structural strength; library of probability distributions; reliability assessment methods; uncertainty modeling and analysis; failure definitions; and system analysis. Each module is independently investigated and developed, although some knowledge about the details of the other modules is needed for the development of a particular module. These modules are described by Ayyub, Beach and Packard (1995 [3]).

Prediction of structural failure modes of continuum ship structural components or systems requires the use of nonlinear structural analysis. Therefore, failure definitions should be expressed using deformations rather than forces or stresses. Also, the recognition and proper classification of failures based on a structural response within the simulation process should be performed based on deformations. The process of failure classification and recognition needs to be automated in order to facilitate its use in a simulation algorithm for structural reliability assessment. Figure 1 (Ayyub, et al. 1995 [3]) shows a procedure for an automated failure classification that can be implemented in a simulation algorithm for reliability assessment.

NOTATION

B	=	Predicted Strength
CDF	=	Cumulative Distribution Function
COV	=	Coefficient of Variation
F	=	Cumulative Distribution Function
L0i	=	Plate Side Dimensions
L1i	=	Web Side Dimensions
L2i	=	Flange Side Dimensions
L11i	=	Web Height, Side 1
L13i	=	Web Height, Side 3
L21i	=	Flange Width, Side 1
L23i	=	Flange Width, Side 3
N	=	Number of Simulation Cycles
PDF	=	Probability Density Function
Q_s	=	Still Water Load
Q_w	=	Wave Load
P_f	=	Failure Probability
P_f	=	Sample Mean of Failure Probability
P_s	=	Survival Probability
t0	=	Plate Thickness
t1	=	Web Thickness
t2	=	Flange Thickness
Var	=	Variance
X21i	=	Web Tilting, Side 1
X23i	=	Web Tilting, Side 3
Y	=	Performance Function
zP0i	=	Plate Corners Coordinates
Z2i0	=	Flange Tilting, Side y = 0
Z2iL	=	Flange Tilting, Side y = L
XBOi	=	Web Bowing

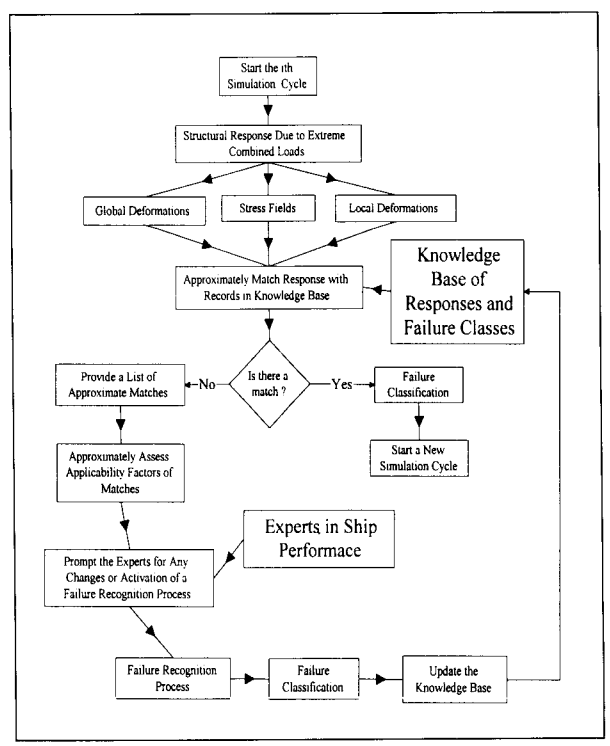


FIGURE 1. Failure Classification (Ayyub, et al. 1995)[3]

The failure classification is based on matching a deformation or stress field with a record within a knowledge base of response and failure classes. In cases of no match, a list of approximate matches is provided, with assessed applicability factors. The user can then be prompted for any changes to the approximate matches and their applicability factors. In the case of a poor match, the user can have the option of activating the failure recognition algorithm shown in Figure 2 (Ayyub, et al. 1995 [3]) to establish a new record in the knowledge base. The adaptive or neural nature of this algorithm allows the updating of the knowledge base of responses and failure classes. The failure recognition and classification algorithm shown in Figure 2 evaluates the impact of the computed deformation or stress field on several systems of a ship. The impact assessment includes evaluating the remaining strength, stability, repair criticality, propulsion and power systems, combat systems, and hydrodynamic performance. The input of experts in ship performance is needed to make these evaluations using either numeric or linguistic measures. Then, the assessed impacts should be aggregated and combined to obtain an overall failure recognition and classification within the established failure classes. The result of this process is then used to update the knowledge base.

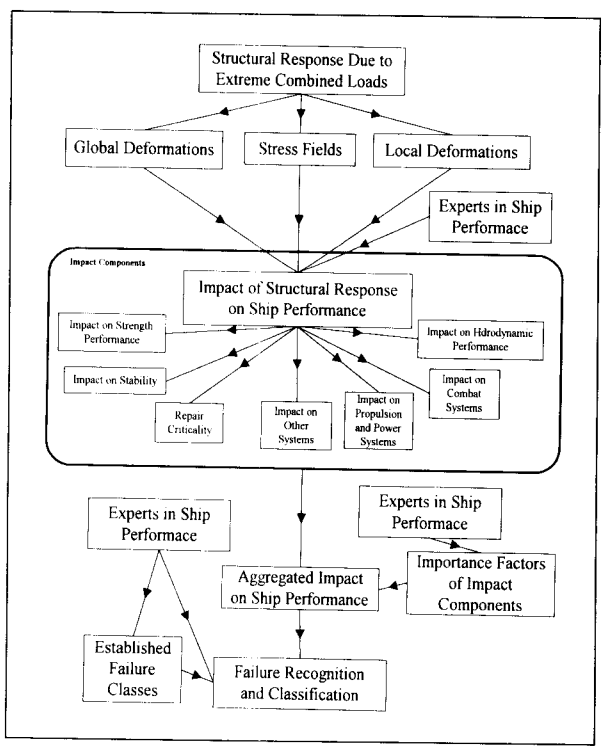


FIGURE 2. Failure Recognition and Classification (Ayyub, et al. 1995)[3]

FINITE ELEMENT METHOD

The development of a general Finite Element (FE) computational approach for continuum mechanics has dominated the efforts of researchers over the past three decades. The efforts have followed three main courses: 1) the so-called degenerated solid approach, which finds its point of departure in the work of Ahmad, Irons and Zienkiewicz (1970 [4]); 2) the natural approach of Argyris et al. (1968 and 1969 [5,6,7]), which illustrated their development of the SHEBA family of shell elements; and 3) recently the shell theory, phrased as one-director Cosserant surface, which was initially presented in the paper of Simo and Fox (1989 [8]).

Stochastic finite element methods were also developed to calculate the stochastic response and reliability of linear and non-linear engineering systems by first-order or second-order reliability methods (FORM and SORM). In these methods geometric and materials property randomness are incorporated into a finite element formulation using approximate moment-based formulations. For example, the works of Contreras (1980[9]), Hisada and Nakagiri (1985 [10]), Liu, et al. (1986 [11]), Liu, et al. (1988 [12]), Mochio, et al. (1985 [13]), Sarras, et al. (1993 [14]), Spanos, et al. (1989 [15]), Vanmarcke and Grigoriu (1984 [16]), and Vanmarcke, et al. (1986 [17]) provide such formulations and examples. Faravelli (1988 [18]) describes the use of finite element methods for reliability assessment based on response surface techniques.

Finite element methods have been used extensively in many areas such as vibration and dynamic response, buckling and postbuckling analysis with or without geometrical and material nonlinearities, thermal effect, fluid-structural interaction, aeroelasticity, structure-acoustics interaction, fracture, laminated composites, wave propagation, structural dynamics and control interaction of aircraft and space structures, random dynamic response, and others. Therefore, any development in the field of reliability assessment of continuum structures may be transferred to other areas.

Selecting an FE-package

The purpose of a finite element analysis is to predict the response of a model to some form of external loading, or to some nonequilibrium conditions. Reliability assessment of a structure requires the prediction of the structural response due to extreme loading. Generally, this response reflects both linear and/or non-linear behavior in geometry and material of the structure. The structural behavior could be expressed by local or global deformations, stress fields, strain fields, static or dynamic criteria. Therefore, any selected FE-package for estimating the structural response should be capable of calculating all relevant values with acceptable accuracy levels. In addition, the output files of the selected FE-package should be in a suitable format to facilitate finding and reading strength prediction

values for failure recognition and ultimately reliability assessment.

RANDOM NUMBER GENERATION

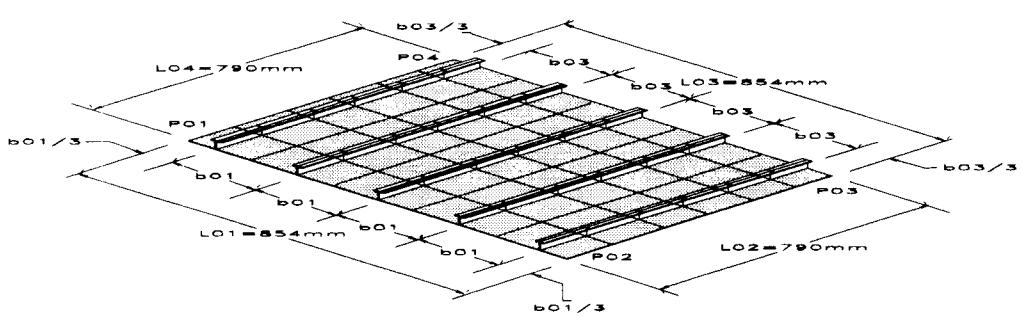
Numerical simulation is associated with random number generation of random variables in accordance with their respective or prescribed probability distributions. The random generation of each random variable can be accomplished systematically by first generating uniformly distributed random numbers between 0 and 1, and through appropriate transformations obtaining the corresponding random values with the specified probability distribution. The random generation efforts in a simulation process include generating random numbers (e.g., by a pseudo random number generator) and generating random variates through transformation (e.g., by substituting the generated random numbers into the inverse cumulative distribution functions of the respective random variables). A random number generator (Press et al. 1992 [19]) is then used to produce random numbers for all random variables.

Ayyub and Chao (1994 [20]) developed a library of subroutines for generating random variables with any of the following distribution types: uniform, triangular, normal, lognormal, Poisson, binomial, geometric, exponential, Rayleigh, extreme value type I—largest, extreme value type II—largest, extreme value type III—smallest (Weibull), and Gamma. For each distribution, the following computations are needed: probability density values, or probability mass value; cumulative distribution value; inverse of cumulative distribution; random generation; and relationship between its parameters and moments. This library of subroutines is used in this study.

RANDOM VARIABLES IN FEM

Finite element methods have the flexibility to represent different types of geometric and material uncertainties. In this study and as an illustrative example, a stiffened panel as shown in Figure 3 was analyzed for different types of variability. The panel consists of a rectangular thin plate with parallel stiffeners spaced at equal distances. The finite element model was based on the 8-node doubly curved shell element, which can give the non-linear behavior prediction of the stiffened panel. A systematic variability analysis of the panel required the hypothetical division of the panel into three levels: the plate level (level-0), the web level (level-1) and the flange level (level-2). The dimensions of the plate, the web, and the flange sides were denoted as L0i, L1i and L2i (i = 1,. . . ., 4), respectively. Geometric and material uncertainties in the form of random variables are accounted for in the three levels, i.e., plate, web, and flange levels. The underlying geometric and strength random variables are defined based on reported values, practical aspects, and judgment (Ellingwood, et al. 1980 [21]).

The plate level as shown in Figures 4-a and 4-b included three types of variability, its sides' dimensions, out-of plane distortion, and thickness. The variability in the sides' dimensions for the plate (Figure. 4-a) was modeled using random variables for the first and third sides of the plate, i.e., sides L01 and L03. Such a variability subjects the stiffeners to possible out of plane moments. The second variability as shown in Figure 4-b is the out-of-plane distortion of the plate that was modeled using three random variables for the z coordinates of its three corners, namely, zP02, zP03 and zP04. The third variability is the plate thickness and it is represented by a single random variable t0.



STIFFENED PANEL

Assumptions: b01=L01 / 4.666667 b03=L03 / 4.666667

FIGURE 3. Typical Stiffened Panel

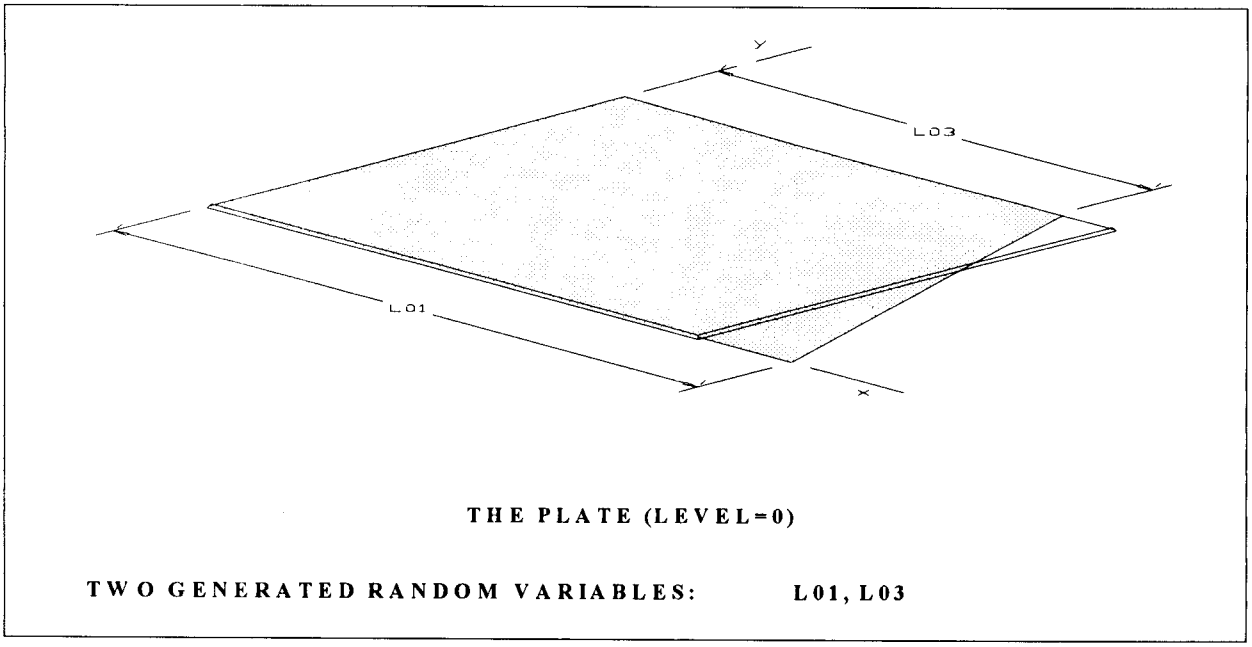


FIGURE 4A. Plate Width Variability

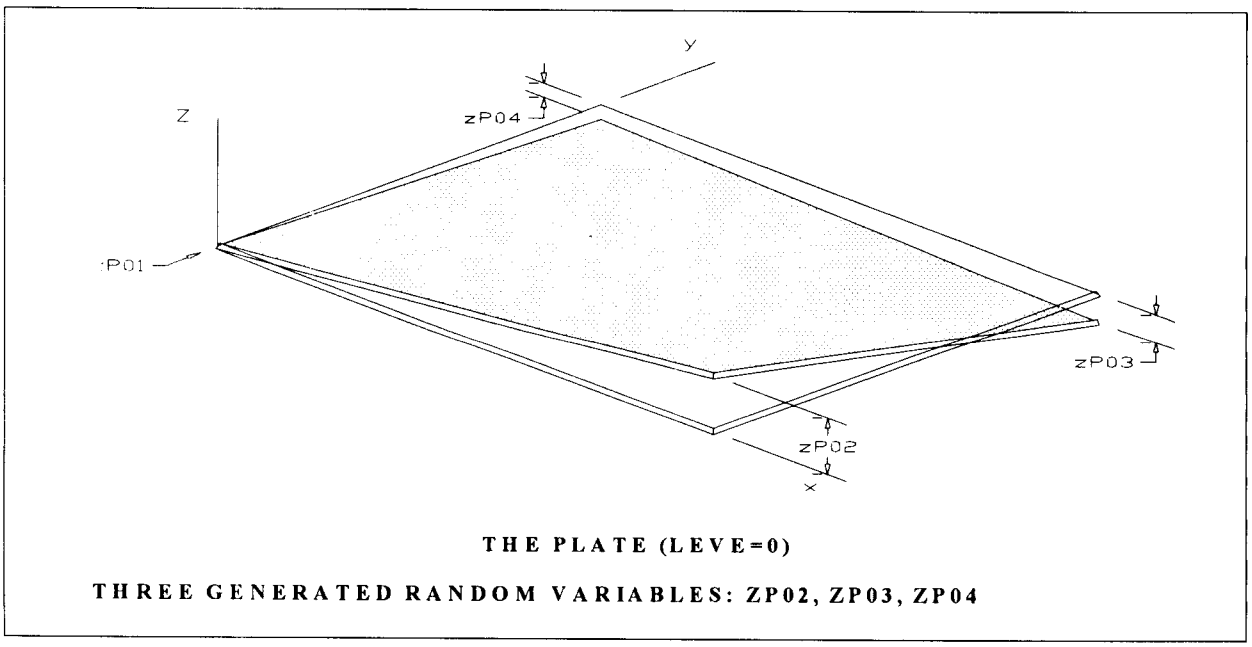


FIGURE 4B. Plate Distortion

The variability at the web level is shown in Figures 5-a, b and c to consist of four types, i.e., web tilting, web height, web bowing, and web thickness. The web tilting of Figure 5-a was represented by two random variables X21i and X23i per web, where (i = 1,. . . ., number of stiffeners). These random variables model the horizontal deviation of the web top from the vertical position. The variability in the web height as shown in Figure 5-b was represented by two random variables L11i and L13i per web, where (i = 1,. . . ., number of stiffeners). The web

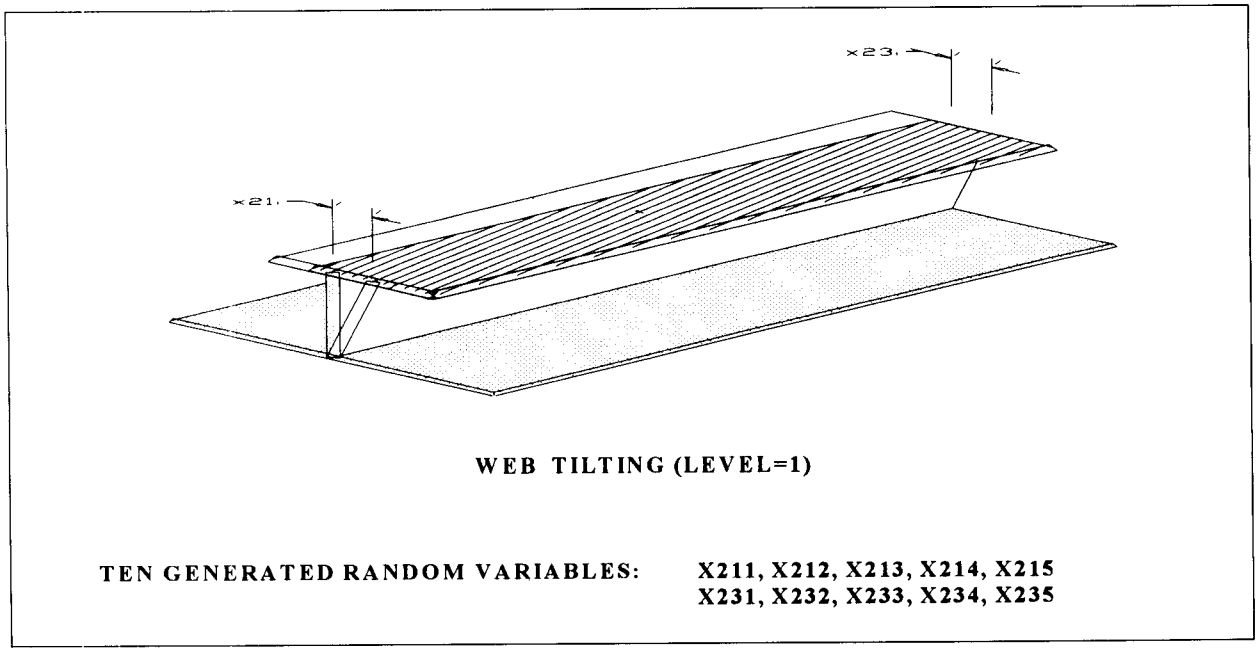


FIGURE 5A. Web Tilting

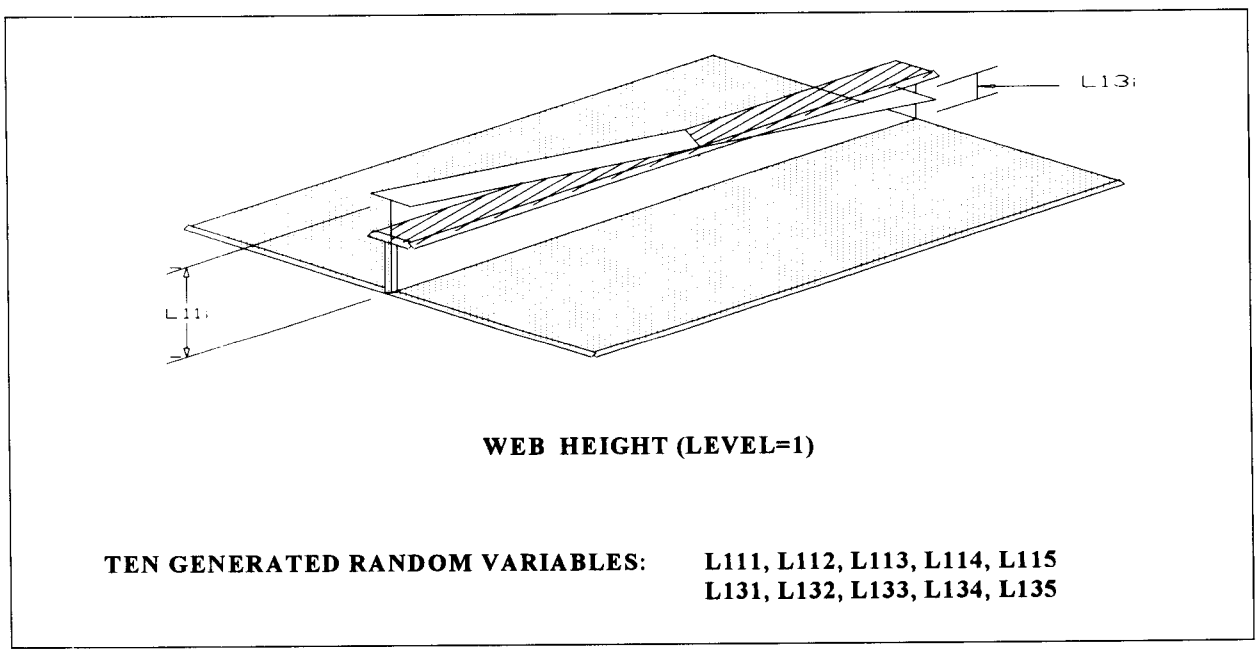


FIGURE 5B. Web Height Variability

bowing is shown in Figure 5-c, and was represented by the random variable XB0i, where (i = 1,. . . ., number of stiffeners). This variable represents the possible out-of-plane deviation of the mid-web from the original plane position. The fourth variability is the web thickness that is represented by the random variable t1.

The variability at the flange level is shown in Figure 6-a and b with its three types, i.e., flange tilting, flange height, and flange thickness. The flange tilting as shown in Figure 6-a was represented by two random variables Z2i0 and Z2iL per flange, where (i = 1,. . . ., number of stiffeners). These random variables represent the vertical

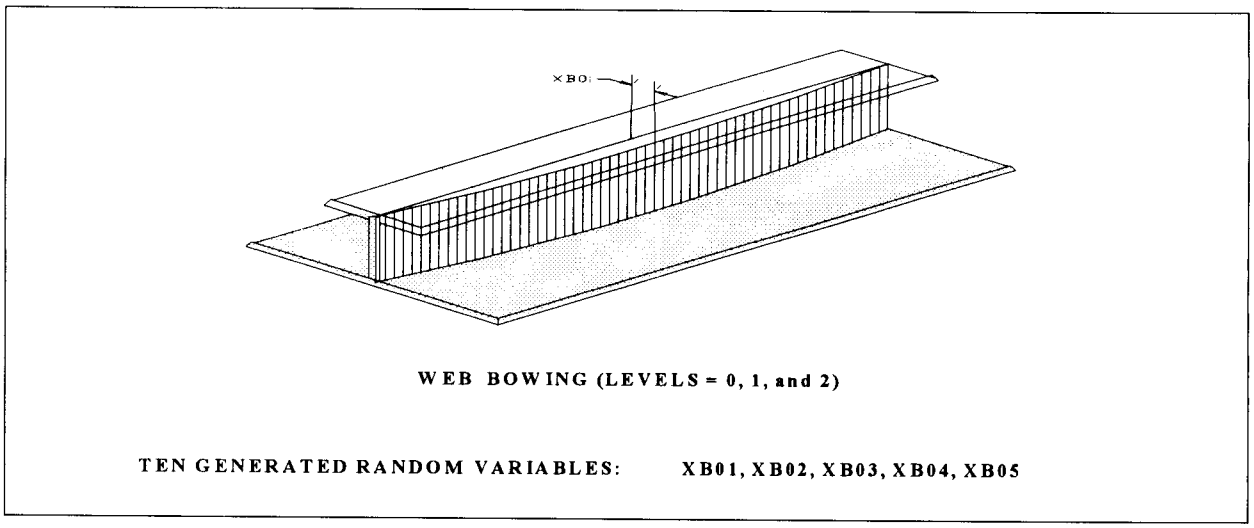


FIGURE 5C. Web Bowing

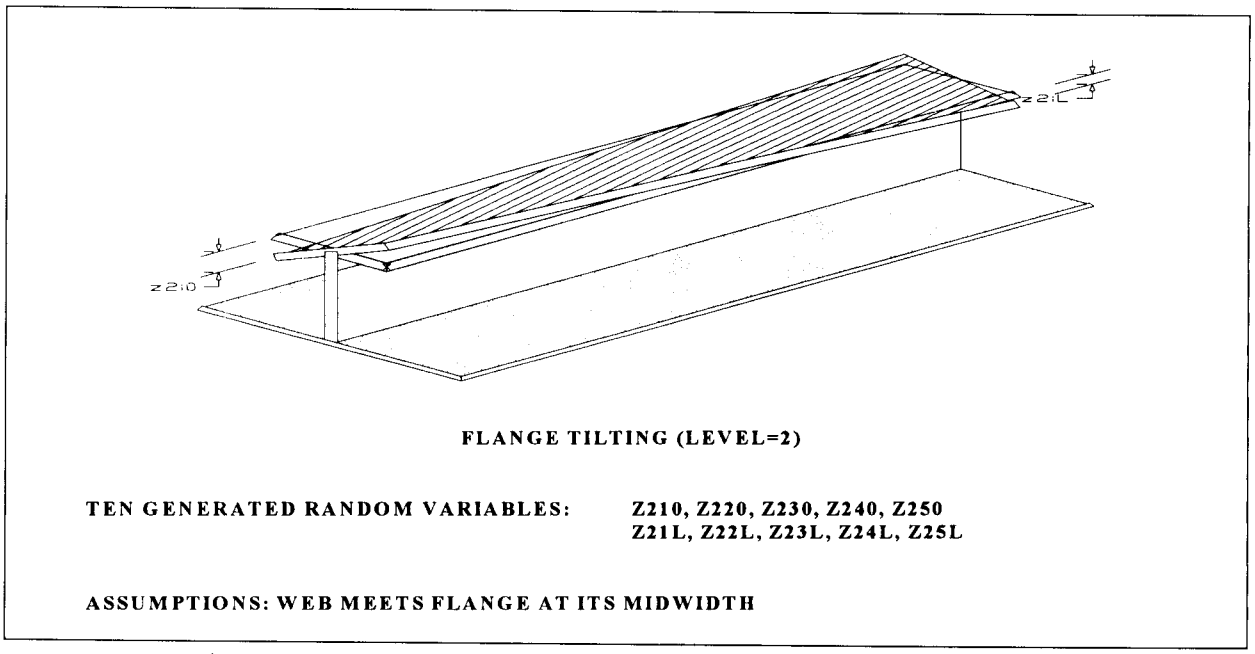


FIGURE 6A. Flange Tilting

deviation of the flange front and back sides from the horizontal position assuming that the web meets the flange at mid-width. The variability in the flange width as shown in Figure 6-b was represented by two random variables L21i and L23i per flange, where (i = 1,. . . ., number of stiffeners). The third variability is the flange thickness that is represented by the random variable t2.

REPRODUCTION OF FE INPUT FILES

Reliability assessment based on finite element analysis requires an automated access to the input and output of a finite element package. Most general purpose finite element packages restrict automated changes in their input files in a sequence of runs as needed especially in simulation-based reliability assessment methods. Usually, input

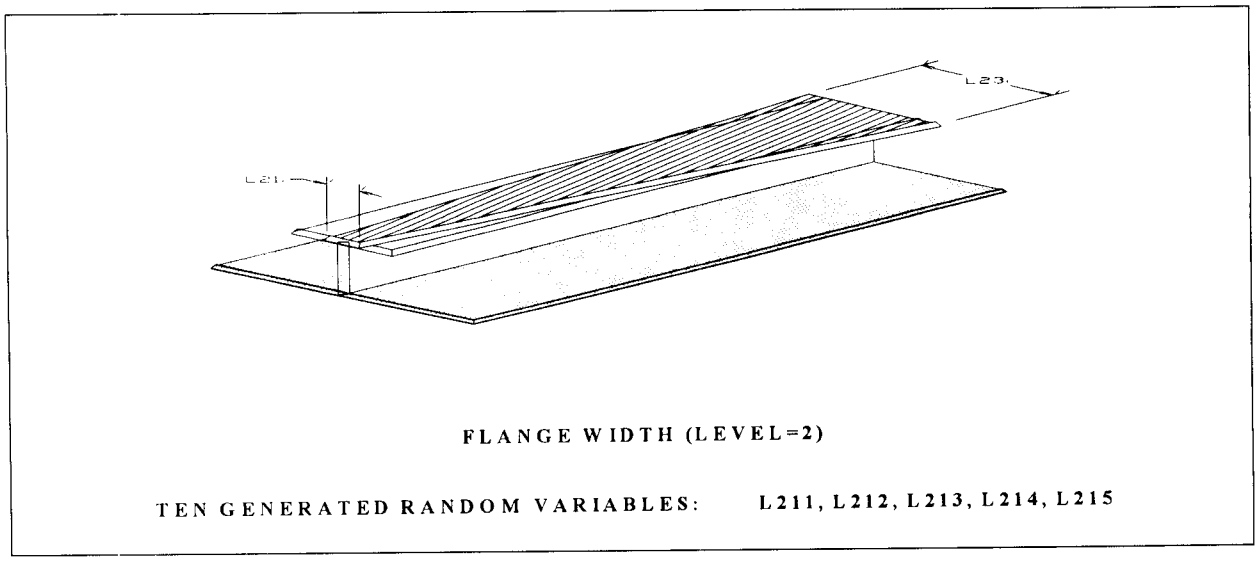


FIGURE 6B. Flange Width Variability

files are prepared for specific models with deterministic values for the variables of the models. The use of a FE package in a simulation process requires the reproduction of the FE input file that corresponds to the generated random variable in a simulation cycle. Figure 7 shows a developed procedure for FE input file reproduction in the *i*th simulation cycle. The block corresponding to FE input file reproduction consists of a main program that calls a random number generator (Press, et al. 1992 [19]), a distributions library, and a subroutine for creating FE input file and calculating the related FE model parameters. At the start, a seed input value is fed manually to the random number generator, where a random number is generated, then the distribution library is called to calculate a value for a random variable based on a prescribed distribution type. Inverse transformation of the cumulative distribution function (CDF) of the random variable is used to obtain the random value for the variable based on the generated random number. The resulting random value for the variable is then saved. The seed input value is updated automatically based on the internally produced new seed in the random number generator. The previous operation is repeated to generate all random variables under consideration. The second step is to call the FE input file subroutine that is designed specifically for a selected FE package. The subroutine takes the generated random variables from the previous step for the structural geometry, loads, and strength parameters to reproduce the FE input file according to the format required by the FE package.

POST PROCESSING OF OUTPUT

The post processing of the FE output files is the step that follows running the FE package in each simulation cycle as shown in Figure 8. Subroutines were developed to conduct the post processing on a machine level. The first step is to read the FE output files and extract the values of interest. The extracted values are manipulated and used

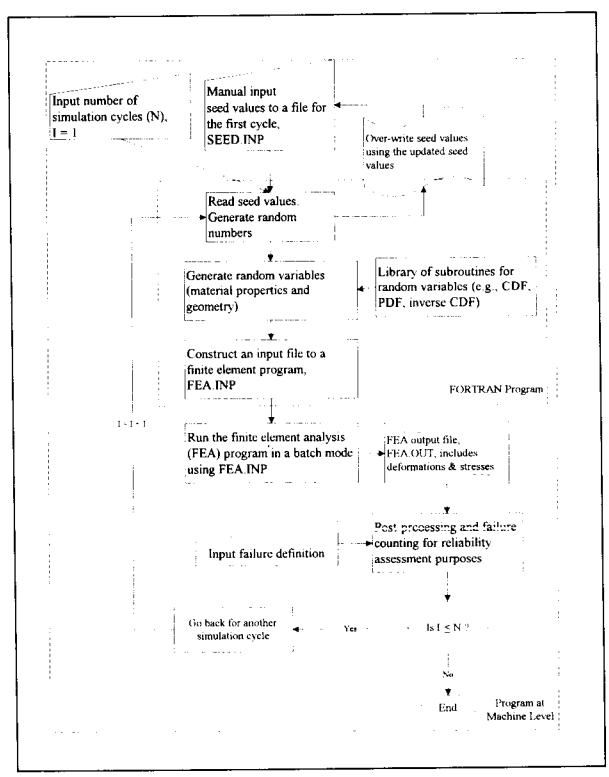


FIGURE 7. Computational Procedure for *ith* Simulation Cycle

in failure recognition, then structural strength is evaluated. The probability of failure is then estimated, for example based on the conditional expectation variance reduction technique as described below. This technique requires load definition in the form of a probability distribution in order to use the strength-load reliability model as shown in Figure 9. Finally, relevant statistical measures for the failure probability are calculated. The statistical measures include the average failure probability, the coefficient of variation of failure probability, and the coefficient of variation of the sample average failure probability.

Reliability Assessment

Commonly used structural reliability assessment methods can be classified into the following two types: moment and simulation methods.

The moment methods have been studied and described by many researchers (e.g., Ang and Tang 1984 [1]; Ayyub and Haldar 1984[22]; Grigoriu 1982 [23]; Hasofer and Lind 1974 [24]; Melchers 1987 [25]; Shinozuka 1983 [26]; Thoft-Christensen and Baker 1982 [27]; and White and Ayyub 1985 [28]). In these methods, approximations are made about the distribution types, linearity of the failure surface, design or failure points, statistical characteristics

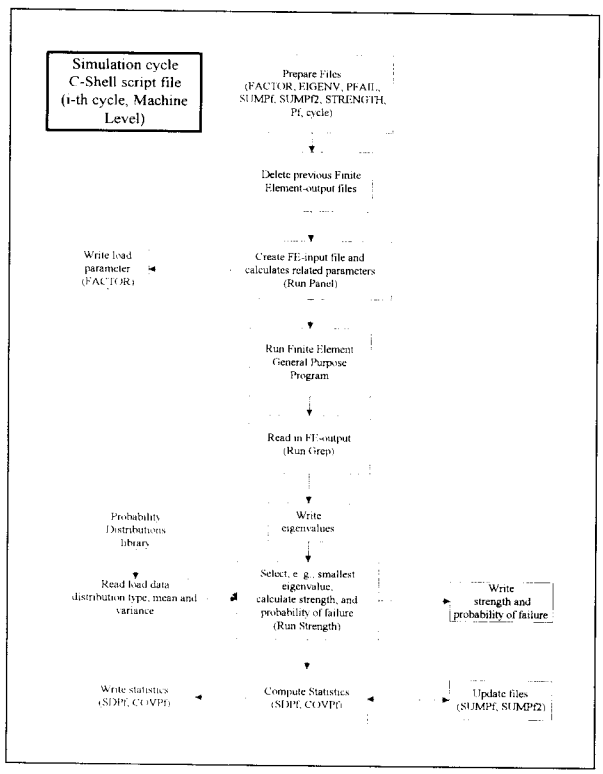


FIGURE 8. C-Shell Script Flow Chart for *ith* Simulation Cycle-Machine Level

of the basic random variables, and other parameters. Some of these methods are based on step-by-step approximations of the previous parameters in an optimization scheme and, consequently, lead to an improved estimate of the reliability or probability of failure of the structure. However, such methods can have problems in convergence due to limitations in the level of nonlinearity of the failure surface that can be considered by the methods, the number of random variables that can be considered in the performance functions defining the potential failure modes of the structure, and the level of skewness of the probability distributions of the basic random variables.

In structural reliability, the simulation-based methods determine the probability of failure of a structural component or system according to a specified performance function. They require complete information about the probabilistic characteristics of the basic random variables. In the classical use of the simulation-based methods, all the basic random variables are randomly generated and a performance equation for a failure mode is evaluated. Failures are then counted depending on the outcome of the evaluation. The probability of failure is estimated as the ratio of the number of failures to the total number of simulation cycles. Therefore, the smaller the probability of failure, the larger the needed number of simulation cycles to estimate the probability of failure with an acceptable level of statistical error. The efficiency of simulation can largely be improved by using variance reduction techniques. The result is a reduced level of computational effort and an increased analytical level. For example, Ayyub and Haldar (1984 [22]) and White and Ayyub (1985 [28]) suggested using conditional expectation and antithetic variates variance reduction techniques for structural reliability assessment. This method was determined to be efficient, and converges to the correct probability of failure in a relatively small number of simulation cycles. Other methods are based on common random numbers, importance sampling and antithetic variates variance reduction techniques. Simulation methods are commonly used by researchers to validate other (non-simulation) methods and are generally considered accurate. The main shortcoming of simulation methods is a large computational effort requirement for its use. This shortcoming is valid for the direct (hit or miss) method, but not necessarily true for simulation with variance reduction techniques or selective sampling algorithms.

In this study, simulation with conditional expectation as a variance reduction technique is used for reliability assessment (Ayyub and Haldar 1984 [22]). The non-closed nature of the predicted strength based on finite element (B) does not allow for choosing any basic variable X_i in B as the conditioned random variable even if X_i is the one with the highest variability level. The reason is because B has to be computed using an a non-closed form based on FE methods. The only remaining choices are the load variables, for example, stillwater lead (Q_s) or wave load

(Q_w). If Q_w has a higher variability level than Q_s, then the survival probability P_s is given by

$$\begin{aligned} P_s &= P(Y > 0) \\ &= P(B - Q_s - Q_w > 0) \\ &= P(Q_w < B - Q_s) \\ &= E[F_{Qw}(B - Q_s)] \end{aligned} \tag{1}$$

where Y = performance function; $B = B(X_1, X_2, \ldots, X_{n-2})$ as the strength variable that is a function of n-2 basic random variables, F_{Qw} = the cumulative distribution function of Q_w, and $E[\ldots]$ is the expected value of $[\ldots]$. Therefore, the failure probability P_f can be determined as

$$P_f = 1 - P_s = 1 - E[F_{Qw}(B - Q_s)] \tag{2}$$

Therefore, each simulation cycle (ith cycle) is expected to produce a failure probability P_{fi} based on evaluating the cumulative distribution function of Q_w at generated values of B and Q_s (i.e., B_i and Q_{si}). The sample mean of the probability of failure ($\overline{P}_f$) is then computed as

$$\overline{P}_f = \frac{1}{N}\left(\sum_{i=1}^{N} P_{fi}\right) \tag{3}$$

in which N = the number of simulation cycles. This estimate of P_f can be considered an unbiased estimator of the population value. The variance associated with this estimated value is (Ang and Tang 1975 [29])

$$\begin{aligned} Var(\overline{P}_f) &= \frac{Var(P_f)}{N} \\ &= \frac{1}{N}\left(\frac{1}{N-1}\sum_{i=1}^{N}(P_{fi} - \overline{P}_f)\right) \end{aligned} \tag{4a}$$

where $Var(\overline{P}_f)$ indicates the accuracy in estimating $\overline{P}_f$. A smaller value of $Var(\overline{P}_f)$ is always preferred. The coefficient of variation (COV) for the estimated failure probability is given by

$$\mathrm{COV}(\overline{P}_f) = \frac{\sqrt{\frac{1}{N}\left(\frac{1}{N-1}\sum_{i=1}^{N}(P_{fi} - \overline{P}_f)\right)}}{\overline{P}_f} \tag{4b}$$

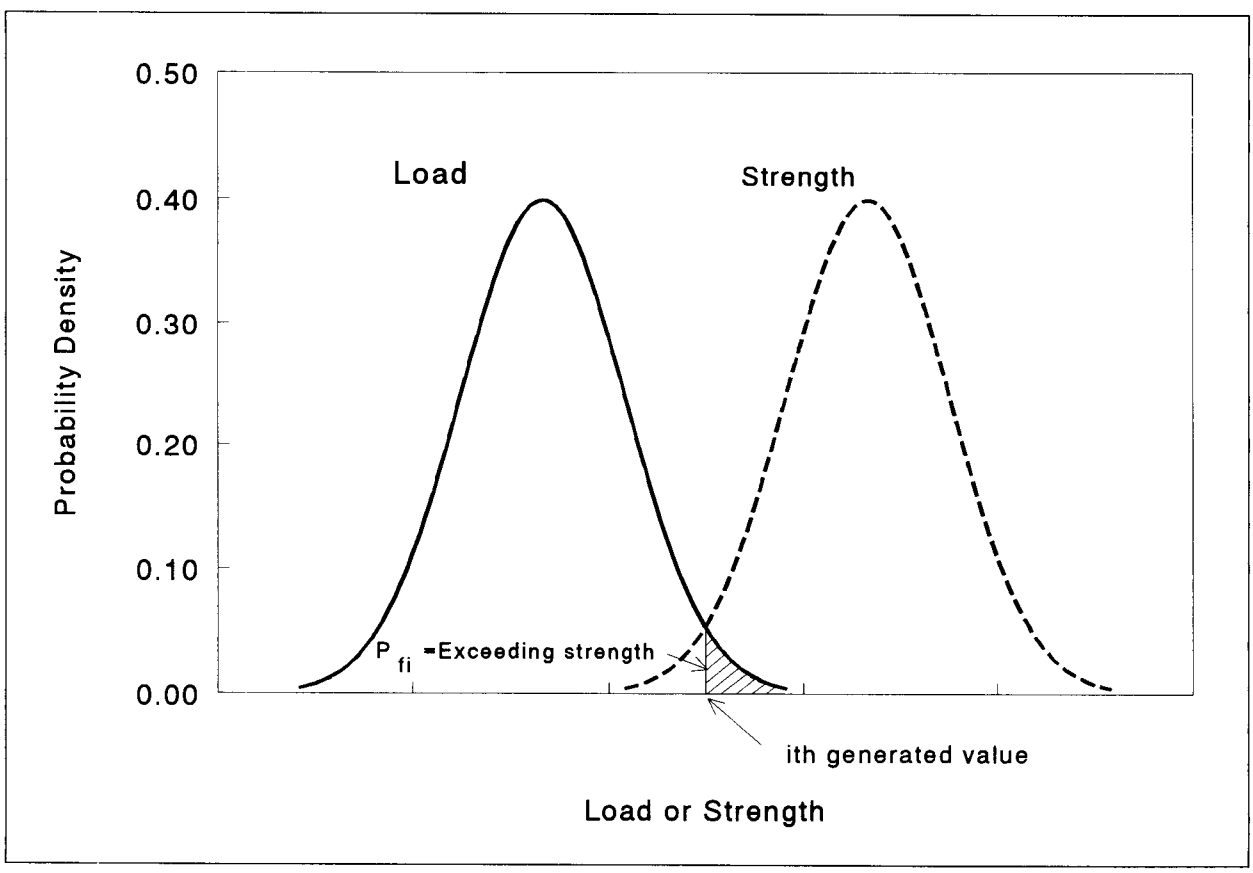


FIGURE 9. Conditional Expectation for Probability of Failure

Example

The example that is presented in this section illustrates the application of the methodology to a stiffened shell-type structure. A small-scale stiffened panel of a ship structure that was tested by Faulkner (1977 [30]) is used in this example. The panel was also investigated by Hess, et al. (1994 [31]). The panel was modeled as a simply-supported structure with two states of loading: 1) concentric axial compression only, and 2) concentric axial compression with lateral pressure. The primary failure mode for this panel is an elastic non-linear instability. A commercially available FE package was selected based on its ability to deal with this failure mode. The selected package was ABAQUS, which was used to predict the buckling eigenvalues.

GEOMETRY AND MATERIAL

The panel consists of a rectangular thin plate stiffened with equally spaced parallel T-shaped stiffeners as shown in Figure 3. The overall plate dimensions are 854 × 790 mm with a plate thickness of 3.0 mm. The web height is 26.66 mm with a web thickness of 4.9 mm. The flange width is 25.4 mm with a flange thickness of 5.84 mm. The five equally spaced stiffeners are 183 mm apart, they are parallel to the short side of the plate and the plate extends equally beyond the external stiffeners a distance equal to 61 mm each way. The material is assumed to be isotropic elastic with a modulus of elasticity E = 208000 MPa, and Poisson's ratio v = 0.3.

LOADING AND BOUNDARY CONDITIONS

Two loading conditions were used: 1) the panel is loaded in the short direction by a concentric uniformly distributed edge loads, and 2) the panel is simultaneously loaded in the short direction by concentric uniformly distributed edge loads and a lateral pressure on the plate surface. The lateral pressure values were taken −0.07, −0.035, 0.0, 0.035, and 0.07 MPa (where a positive pressure is normal to the plate creating compressive bending stress in the plate). The concentric uniformly distributed edge loads were taken thickness dependent to account for the possible variability in the thicknesses. The panel was simply supported in the plate level on two opposite edge, namely, L01 and L03.

FE MODEL

For the purpose of FE modeling, the middle planes of the plate, webs, and flanges were considered as reference planes for dimensioning. Consequently, the considered dimensions of the FE model were 854 × 970 × 3 mm for the plate, 31.08 × 4.9 mm for the web and 25.4 × 5.84 mm for the flange. The model was based on the 8-node doubly curved shell element S8R5, which can provide the eigenvalue buckling estimates. In general, shell buckling stability requires two types of analysis. First, eigenvalue analysis is used to obtain estimates of the buckling loads and modes. This type of analysis can provide guidance in mesh design to ensure the convergence of eigenvalue estimates of buckling loads. The mesh needs to be adequate to model the buckling modes, which are usually more complex than the prebuckling deformation mode. The key aspect of the eigenvalue analysis is the mesh design. For the example under study, the buckling could be an overall buckling of the whole panel or a local buckling in either the plate, the web or the flange. In this demonstration, the whole panel was modeled without accounting to any possible symmetry. The following meshes were used: 6 × 10 for the plate, 1 × 6 for each web, and 2 × 6 for each flange as shown in Figure 10. The second phase of the study is to perform load-displacement analyses.

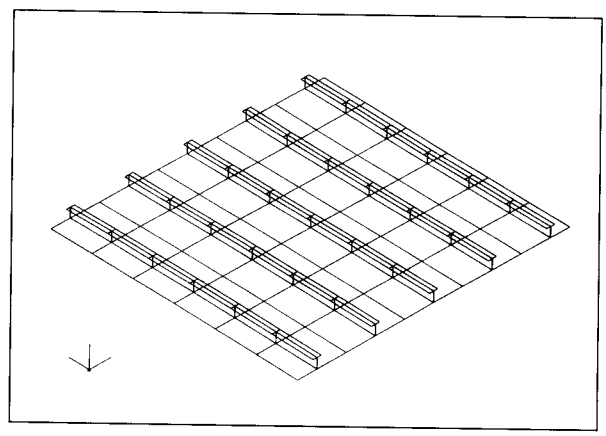


FIGURE 10. Finite Element Mesh of the Stiffened Panel

RANDOM VARIABLES

The basic geometric and material random variables were defined based on reported values, practical aspects, and judgment (Ellingwood, et al. 1980 [21]). The adopted values are summarized in Table 1. The overall number of random variables that were considered in this example is fifty-five. They included random variables at the three levels, i.e., the plate, web, and flange, that are described above in the section entitled, "Random Variables in FEM." Six geometric variables were defined at the plate level, twenty five variables at the web level, and twenty variables at the flange level. In addition, three variables were defined for thicknesses, one for each level, and the modulus of elasticity was assumed to be the same for the three levels and was expressed by one random variable. The random variables were divided into two categories. A random variable in the first category was defined by its mean values and standard deviations, whereas a random variable in the second category was defined by its mean values and coefficients of variation (COV). The standard deviation for the

TABLE 1

Geometric and Material Random Variables for the Stiffened Panel

Variable no.	Geometrical Variables	Notation	Mean value	Coefficient of Variation (COV)[1] (%)	Standard deviation[1]	Variation of Coefficient of Variation (%)	Variation of Standard deviation
1	Plate size (mm)	L0i	854		4.0		4.0 to 8
2	Plate thickness (mm)	t_0	3.0	4		4 to 8	
3	Web thickness (mm)	t_1	4.9	4		4 to 8	
4	Flange thickness (mm)	t_2	5.84	4		4 to 8	
5	Plate-out of plane distortion (mm)	zP0i	0.0		1.0		1.0 to 3.0
6	Web height (mm)	L1ji	31.08	2.5		2.5 to 5	
7	Web tilting	X2ji	0.0		0.2		0.2 to 0.5
8	Web bowing (mm)	XB0i	0.0		0.1		0.1 to 0.2
9	Flange width (mm)	L2ji	25.4	2.5		2.5 to 5	
10	Flange tilting (mm)	Z2i0, Z2iL	0.0		0.2		0.2 to 0.5
11	Modulus of elasticity (MPa)	E	208000	4		4 to 8	

[1]Reference case

overall size of the plate was assumed to be 4.0 mm, and for its out-of-plane distortion was assumed to be 1.0 mm. The web tilting, bowing and flange tilting have standard deviations of 0.5 mm, 0.1 mm, and 0.2 mm respectively. The variability of the thicknesses at all levels was expressed by four percent COV, and the web height and the flange width by 2.5 percent COV. The COV of the modulus of elasticity was taken as four percent. A normal probability distribution function (PDF) was used for all basic random variables.

STRENGTH AND RELIABILITY ASSESSMENT

The strength prediction and reliability assessment of the panel were based on generated values of the basic random variables and those of the applied load. The axial strength (buckling load) for the panel was evaluated by manipulating the estimated eigenvalues obtained by the FE analysis. The load was assumed to have a lognormal distribution with an assumed mean value that is two thirds of the nominal buckling strength and an COV of ten percent. Reliability assessment was conducted using simulation with conditional expectation variance reduction technique as described above in the section entitled, "Reliability Assessment." For the purpose of demonstration, conditional expectation as defined in Equations 1 to 4 was used with one loading type. The cumulative distribution function of the load (lognormal distribution) was used to determine the failure probability in each simulation cycle.

Concentric Axial Compression Loading

Two cases are discussed in this section, the nominal case which does not account for any type of variability and the reference case which is defined according to the data in Table 1.

Nominal Case. Prediction of the panel strength for this case was based on the nominal values of all variables without accounting for any type of variability. The panel was loaded in the short direction (parallel to the stiffeners) by a uniformly distributed concentric compressive load acting on both edges, and simply supported at the plate level in the short direction. The estimated axial strength (buckling load) was determined to be 273.3 MPa, the failure probability based on random loading was assessed to be 1.51×10^{-5}, and the predicted failure mode was a local buckling in the plate as shown in Figure 11 for an example case. The same panel was investigated by Hess, et al. (1994 [31]) using several combinations of concentric axial and transverse in-plane loads, and lateral pressure. The model was simply supported at the elastic centroid. The statistics of the strength were estimated using Monte Carlo simulation. The panel axial strength was predicted using an algorithm as defined by Hughes (1988 [32]). The presented examples by Hess et al. (1994 [31]) were analyzed with an assumed eccentricity. The predicted mean axial strengths for the case of zero lateral pressure, and zero transverse loads, which corresponds to the nominal

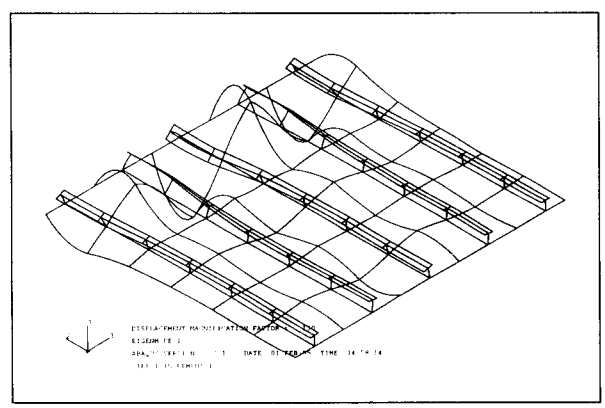


FIGURE 11. Buckling Shape of the Stiffened Panel

case, but with an eccentricities of -0.5273 mm and $+0.5273$ mm were 247.5 MPa and 156.5 MPa, respectively. The differences in the axial strength values between the presented work herein and that of Hess et al. (1994 [31]) can be attributed to the presence of the eccentricity and the difference in boundary conditions.

Reference Case. The reference case accounts for geometric and material variability as defined in Table 1. The results for the axial strength and failure probability are shown in Figures 12 and 13 respectively. Figure 12 shows the histograms and statistical measures for both axial strength and a normalized axial strength with respect to the nominal case based on 500 simulation cycles. The average axial strength value was predicted to be 273.9 MPa. Figure 13 shows the convergence of the average failure probability as the number of simulation cycles is increased, and the statistical measures of the failure probability with its histogram using 500 simulation cycles.

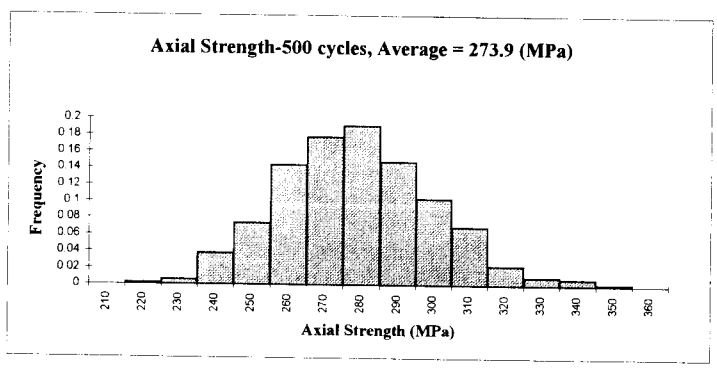
Concentric Axial Compression Loading with Lateral Pressure

The axial strength of the panel under concentric axial compression loading and lateral pressure was predicted for both the nominal and reference cases. The lateral pressure was varied from 0.07 MPa to -0.07 Mpa (a positive pressure is in the positive direction of a normal vector to the plate, i.e., its direction from the plate to the stiffeners). The mean strength, nominal strength, and the mean strength plus and minus one and two standard deviations are shown in Figure 14 for the whole range of the lateral pressure from -0.07 to $+0.07$ MPa. The strength decreases with the increase in pressure, which is in agreement with the predicted failure mode. For a local buckling failure mode for the plate, applying a positive pressure means an increase in the compressive stresses in the plate, consequently a decrease in the axial strength of the panel.

PARAMETRIC ANALYSIS

A parametric analysis was conducted for the axial strength and failure probability of the panel. The analysis was carried out by individually varying the coefficients of variation or standard deviations of the basic random variables. The notations, mean values, and ranges of COV and standard deviations of the random variables are given in Table 1. Full discussion of the results of the parametric study is

Statistical Measures	Axial Strength
Mean	273.9064
Standard Error	0.933275
Median	272.1663
Standard Deviation	20.86865
Sample Variance	435.5007
Kurtosis	0.031126
Skewness	0.278253
Range	121.3818
Minimum	219.5003
Maximum	340.8821
Sum	136953.2
Count	500
Confidence Level(95%)	1.829182



Statistical Measures	Normalized Axial Strength
Mean	1.00221868
Standard Error	0.00341484
Median	0.99585163
Standard Deviation	0.07635806
Sample Variance	0.00583055
Kurtosis	0.03112581
Skewness	0.27825334
Range	0.44413392
Minimum	0.80314782
Maximum	1.24728174
Sum	501.109341
Count	500
Confidence Level(95%)	0.006692946

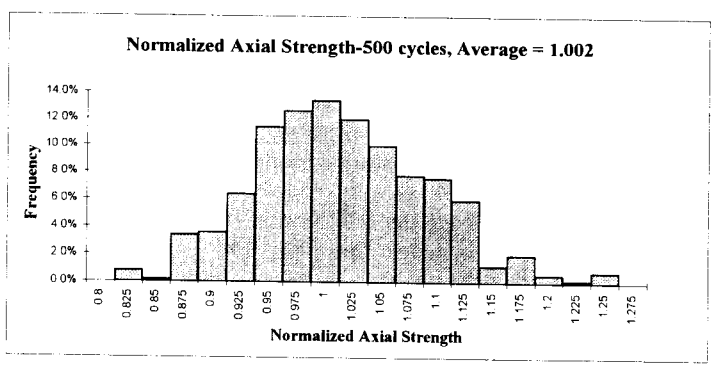
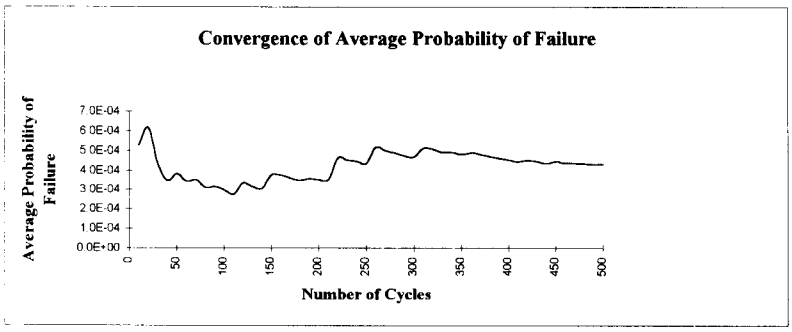


FIGURE 12. Axial Strength Statistics of the Stiffened Panel Using 500 Simulation Cycles



Statistical Measures	Probability of Failure
Mean	0.000431
Standard Error	8.83E-05
Median	1.83E-05
Standard Deviation	0.001973
Sample Variance	3.89E-06
Kurtosis	104.3945
Skewness	9.528373
Range	0.025693
Minimum	7.28E-11
Maximum	0.025693
Sum	0.215415
Count	500
Confidence Level (95.0%)	0.000173

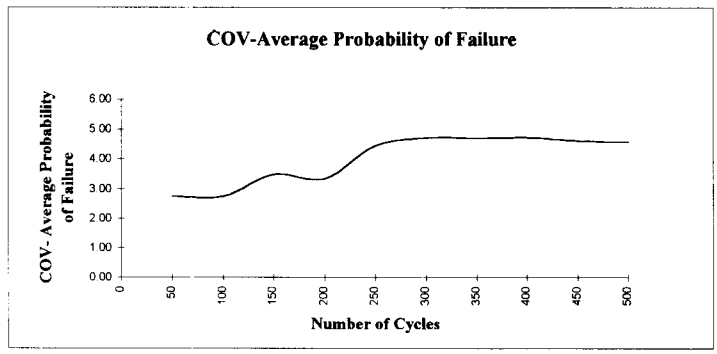


FIGURE 13. Probability of Failure Statistics of the Stiffened Panel Using 500 Simulation Cycles

given by Ayyub, Muhanna and Bruchman (1995 [33]). The following is a summary of these results that were based on 100 simulation cycles:

- For the plate width, increasing the COV from 0.47% to 0.94%, the normalized strength decreases from 1.007 to 0.988, the COV of the axial strength decreases from 8.87% to 7.79%, and the average of probability of failure decreases from 8.20×10^{-4} to 6.45×10^{-4}.
- For the plate out-of-plane distortion, increasing the standard deviation from 1.0 to 3.0, the normalized strength increases from 1.007 to 1.009, the COV of the axial strength decreases from 8.87% to 7.42%, and the average of probability of failure decreases from 8.20×10^{-4} to 6.45×10^{-4}.
- For the web height, increasing the COV from 2.5% to 5.0%, the normalized strength increases from 1.007 to 1.011, the COV of the axial strength decreases from 8.87% to 7.37%, and the average of probability of failure decreases from 8.20×10^{-4} to 3.94×10^{-4}
- For the web tilting, increasing the standard deviation from 0.2 mm to 0.5 mm, the normalized strength increases from 1.005 to 1.007, the COV of the axial strength increases from 7.17% to 9%, and the average of probability of failure increases from 5.0×10^{-4} to 8.45×10^{-4}.
- For the web bowing, increasing the standard deviation from 0.1 mm to 0.2 mm, the normalized strength decreases from 1.007 to 0.99, the COV of the axial strength decreases from 9.0% to 7.8%, and the average of probability of failure decreases from 8.2×10^{-4} to 4.84×10^{-4}.
- For the flange width, increasing the COV from 2.5% to 5.0%, the normalized strength decreases from 1.007 to 1.004, the COV of the axial strength decreases from 9.0% to 7.26%, and the average of probability of failure decreases from 8.2×10^{-4} to 2.23×10^{-4}.
- For the flange tilting, increasing the standard deviation from 0.2 mm to 0.5 mm, the normalized strength decreases from 1.007 to 0.995, the COV of the axial strength decreases from 9.0% to 8.0%, and the average of failure probability increases from 8.2×10^{-4} to 1.77×10^{-3}.
- For the thicknesses, increasing the COV from 4.0% to 8.0%, the normalized strength decreases from 1.007

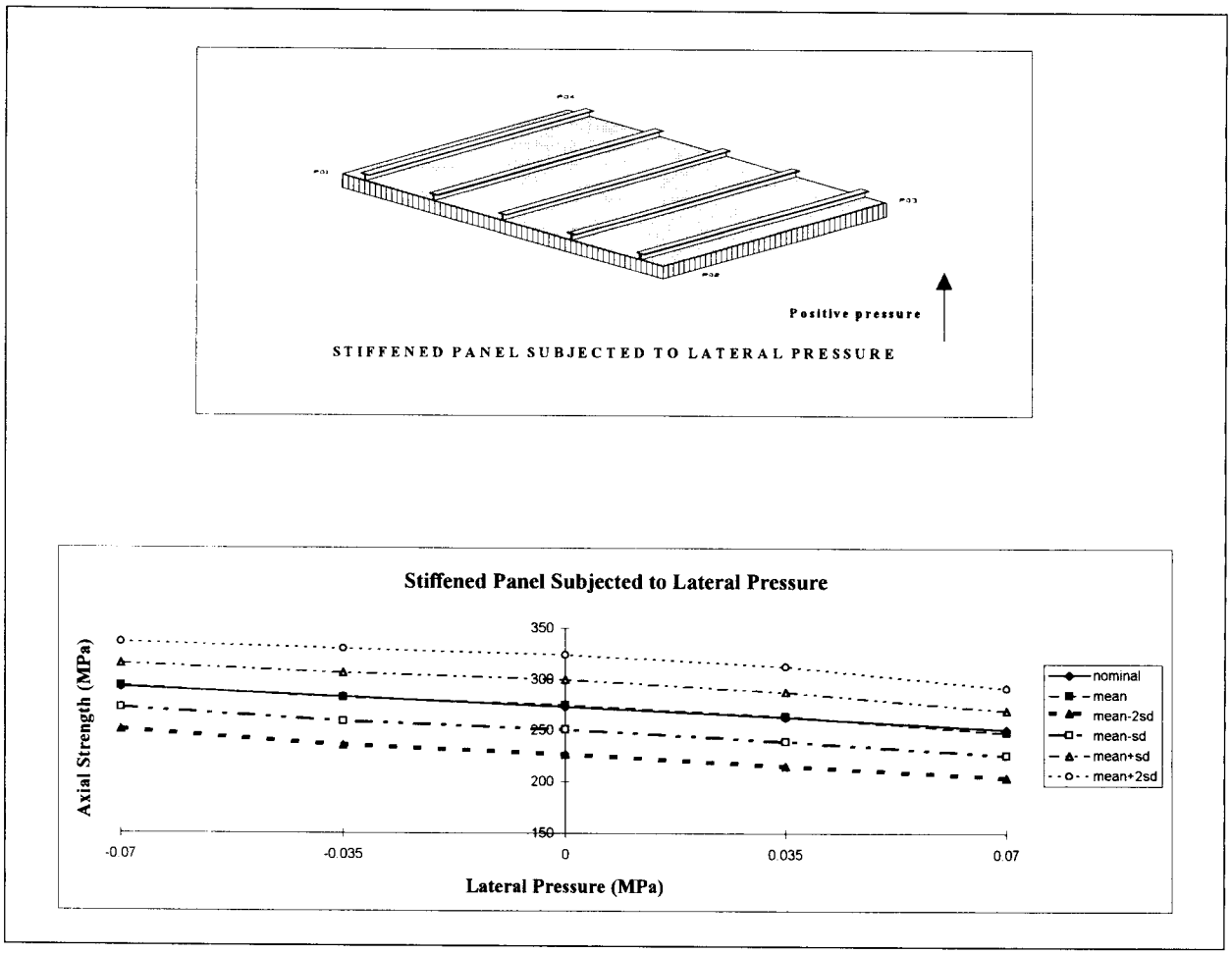


FIGURE 14. Stiffened Panel Subjected to Cincentric Axial Compression and Lateral Pressure

to 0.994, the COV of the axial strength increases from 8.87% to 13.0%, and the average of probability of failure decreases from 8.28×10^{-4} to 1.53×10^{-2}.

- For the modulus of elasticity, increasing the COV from 4.0% to 8.0%, the normalized strength decreases from 1.007 to 1.002, the COV of the axial strength remains constant at the value of 8.90%, and the average of probability of failure decreases from 8.2×10^{-4} to 1.49×10^{-3}.

The above failure probability observations were based on results from 100 simulation cycles. The number of simulation cycles might not be adequate for obtaining accurate failure probability results, but it is sufficient for determining the axial strength. The number of cycles was limited to 100 in order make the study feasible within the planned time frame of the project.

Table 2 shows a summary of the results of the parametric study. According to the table, variations in the variability of plate size and web bowing produced the largest effect on the mean axial strength ratio; whereas variations in the variability of thicknesses of the plate, webs, and flanges produced the largest effect on the coefficient of variation of the axial strength.

Conclusion

A prototype computational methodology for reliability assessment of continuum structures using finite element analysis with instability failure modes is presented in this work. Examples were used to illustrate and test the methodology. Geometric and material uncertainties were considered in the finite element model. A computer program was developed to implement this methodology by integrating uncertainty formulations to create a finite element input file, and to conduct the reliability assessment on a machine level. A commercial finite element package was used as a basis for the strength assessment in the presented procedure.

TABLE 2

Parametric Analysis Results

Variable no.	Geometrical Variables	Mean value	Variation of coefficient of variation (%)	Variation of standard deviation	Effect on axial strength ratio	Effect on coefficient of variation of strength
1	Plate size (mm)	854		4.0 to 8.0	High	Medium/Low
2	Plate thickness (mm)	3.0	4 to 8	0.12 to 0.24	Medium	High
3	Web thickness (mm)	4.9	4 to 8	0.196 to 0.392	Medium	High
4	Flange thickness (mm)	5.84	4 to 8	0.234 to 0.468	Medium	High
5	Plate-out of plane distortion (mm)	0.00		1.0 to 3.0	Low	Medium/Low
6	Web height (mm)	31.08	2.5 to 5	0.77 to 1.54	Low	Medium/Low
7	Web tilting (mm)	0.0		0.2 to 0.5	Low	Medium/Low
8	Web bowing (mm)	0.0		0.1 to 0.2	High	Medium/Low
9	Flange width (mm)	25.4	2.5 to 5	0.635 to 1.27	Low	Medium/Low
10	Flange tilting (mm)	0.00		0.2 to 0.5	Medium	Medium/Low
11	Modulus of elasticity (MPa)	208000	4 to 8	8320 to 16640	Low	None

A parametric study for a stiffened panel strength was also carried out. The following observations were made based on the parametric study:

- Increasing the COV of the plate width resulted in a slight decrease of the normalized strength, and its COV.
- Increasing the standard deviation of the plate out-of-plane distortion resulted in a slight increase of the normalized strength, and a moderate decrease in its COV.
- Increasing the COV of the web height resulted in a moderate increase of the normalized strength, and a moderate decrease of its COV.
- Increasing the standard deviation of the web tilting resulted in a slight increase of the normalized strength, and a significant increase of its COV.
- Increasing the standard deviation of the web bowing resulted in a moderate decrease of the normalized strength, and a moderate decrease of its COV.
- Increasing the COV of the flange width resulted in a slight decrease of the normalized strength, and a moderate decrease of the normalized strength, and a slight decrease of its COV.
- Increasing the COV of the thicknesses resulted in a moderate decrease of the normalized strength, and a significant increase of its COV.
- Increasing the COV of the modulus of elasticity resulted in a slight decrease of the normalized strength, and an unchanged value for its COV.

It can be concluded that variations in the variability of plate size and web bowing produced the largest effect on the mean axial strength ratio; whereas variations in the variability of thicknesses of the plate, webs, and flanges produced the largest effect on the coefficient of variation of the axial strength.

Based on this study, the following recommendations for future work are provided:

- The feasibility of using the developed method for complex structures with multiple failure modes should be investigated. The structures should be selected such that methods for failure recognition and classification as demonstrated in Figures 1 and 2 can be developed.
- The effects of failure recognition and classification for continuum structures on reliability estimates need to be studied.

The developed method is expected to have significant impact on the reliability assessment of structural components and systems; more specifically, the safety and reliability evaluation of continuum structures, the formulation of associated design criteria, evaluation of important variables that influence failures, the possibility of revising some codes of practice, reducing the number of required costly experiments in structural testing, and the safety evaluation of existing structures for the purpose of life extension. The approach developed in this study may be extend beyond structural reliability into the generalized field of engineering mechanics. ✤

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Any opinions, findings and conclusions or recommendations expressed in this paper are those of the authors and do not necessarily reflect the view of the Surface Ship Structures Department of the Carderock Division of the Naval Surface Warfare Center.

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COMMENTS BY

Robert A. Sielski

The authors have brought an important advancement to the quest for developing a reliability-based design procedure for ship structures. Finite element analysis is and essential part of structural design today, and a probabilistic analysis must address the subject. The authors have gone even further, and included an assessment of failure modes, so that the output of the analysis is more than vectors of displacements, strains, and stresses. This type of output from a process that receives random input is a sensible step.

Despite this advance, it would seem that only part of the entire ship structural design process is addressed. One of the difficulties that must be addressed when implementing reliability-based design procedures is the complexities of the load-derivation process. In the near future, load definition will come as part of the output of nonlinear seakeeping computer programs that are even more complex and time-consuming than structural finite element computations. How do the authors propose to integrate such computations of loads into the structural analysis procedure?

The analysis done for this paper was limited to nonlinear elastic analysis. However, realistic assessment of failure modes must account for plastic behavior. Is the limitation to elastic behavior a simplification taken because of limited project resources, or will the inclusion of plastic response make the process computationally prohibitive?

Determination of potential failure modes is important in structural design, but it is part of the analysis process and not the synthesis process. How do the authors see this procedure fitting into the entire design process?

Finally, consideration must be made for an automated ship design process. Future structural analysis, when conducted as part of the ship design process, will not be accomplished as a single task, but must be integrated in such a manner that the input to the structural design and analysis process comes for a product data base that defines the ship for the purpose of production. How do you see a reliability-based structural design process fitting into such a process?

COMMENTS BY

Gregory S. White

The authors are to be congratulated for presenting to this society an interesting and informative paper on a topic of current interest. Both the navy community and the marine industry in the United States are trying to move the current ship structural design approach into one based on the concepts of structural reliability. Work such as the efforts of these authors is needed in order to establish knowledge of the levels of uncertainty involved in the analytical modeling of structural elements. The stiffened panel is by far the most common structural element in a ship. Yet, because of the complex nature of the failure mechanisms of stiffened panels, not much quantitative information is known about the ability of the design algorithms to predict structural performance.

The combination of non-linear Finite Element (FE) analysis and Monte Carlo simulation seems to be a natural marriage. Both are procedures which are computationally intensive and are best carried out with computers. The simulation has been shown to be the most accurate way to determine the level of reliability on a wide variety of engineering problems. Finite Element analysis is well known for providing forces, stresses, displacements and reactions for geometries which are difficult to deal with analytically. Using the two methods together shows con-

siderable promise as a means to investigate the statistical nature of structural problems where random loading or uncertainty in geometry is present. Though the approach is computationally demanding, it is considerably easier and cheaper than running a series of physical tests.

The case of longitudinally-stiffened panels is an excellent one to demonstrate the use of these computational tools to increase our knowledge base. Vromen (1995) conducted an extensive literature source and found reports for 121 tests conducted over a period of forty years. Of those, only eighty-four tests had enough information reported to be useful for statistical analysis. All but eleven of those were single bay tests using either fixed or pinned end conditions. The eleven multi-bay tests were conducted by Smith (1975) and form the heart of best information on ship grillage behavior. Recently, a series of six multi-bay tests conducted at the U.S. Naval Academy (Vroman, 1975; White, Vroman & Kihl, 1996) have helped to increase the available information. The problem is that the USNA tests took a period of about eighteen months. In order to develop the amount of information needed for reliability-based design in a timely manner, a procedure like that proposed by the authors of this paper needs to be used.

I would like the authors to comment on one area of concern which I have regarding the comparison of the FE model results to those of Faulkner (1977). Figure 11 in this paper shows the buckling shape of the stiffened panel. The figure indicates that there is a considerable amount of stiffener rotation at the far end of the panel. Faulkner's tests were single panel tests which used heavy end plates to enforce a pin-end condition. Figure 2 from Faulkner shows the end fittings and a typical failure. Though in my copy of Faulkner's report the photograph is not very clear, I do not detect the type of stiffener rotation seen in the authors FE model. In fact, I would not expect that type of rotation at a pin-end boundary. Allowing the stiffener to rotate in the manner shown would very likely cause a much lower panel ultimate strength.

The rotation of the stiffener ends might also explain why the failure in the panel occurred near the end of the panel. In his research, Vroman (1995) found that in most single bay tests for which he had data (87 tests) the panel failure occurred near the panel center. I expect from looking at the deflection pattern in the author's Figure 11, the same would have happened here if the stiffener ends were prevented from rotating. The rotation of a stiffener end would also not be likely in a real ship's structure. The panel end would be a bulkhead which the stiffener might pass through, but there would be continuous welding of the stiffener to the bulkhead plate. If the panel end is a transverse web, the stiffener would pass through a cut-out but would usually have a welded clip connector or a flange weldment to prevent stiffener rotation.

Boundary conditions at the ends of a stiffened panel is a subject of much interest to this commentor. In the six grillage tests performed at the USNA Ship Structures Laboratory (White, Vroman & Kihl, 1996), we chose to use a multiple bay grillage so that a better representation of the boundary conditions at the ends of the center bay could be achieved. In general, the pin-ended tests give lower ultimate strengths, the fixed end tests give high ultimate strengths, and the multi-bay tests give results which are close to those predicted by theory. I would ask the authors of this paper to consider extending their present work to look at multiple bay configurations.

I would again like to thank the authors for their contribution to the literature on the subject of stiffened panels and say that the type of information presented in Table 2 will be of immense use to those of us working to make reliability-based structural design of ships a reality.

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AUTHOR'S REPLY

The authors would like to thank the reviewers, Dr. Robert A. Sielski and Professor Gregory J. White, for their valuable comments. In this work, the authors are presenting a new computational methodology for reliability assessment methods. Modeling continuum structures, their physical and geometric properties, boundary condition, and applied loads; incorporating the related uncertainties; and predicting their behavior are the primary objectives of this study. A stiffened panel as an important component of ship structures was used, under specific conditions, to demonstrate the proposed methodology. The comments of the reviewers are of actual importance and are discussed under the following headings:

Loads

The authors concur that the load prediction process is crucial in implementing reliability-based design procedures. The presented methodology was implemented us-

ing C-Shell script with an independent interactive module of computer source code that is fully flexible to include any new interactive modular components such as a load module. This modular structure for the methodology is an important attribute that will provide opportunities for collaborative efforts in the area of reliability analysis of structures by independent efforts on strength and load predictions.

Boundary Conditions

The FEM allows applying any type of boundary conditions. The analyzed stiffened panel in this study was selected from the works of Faulkner (1977) and Hess et al. (1994). A simply supported condition at only the plate level of the stiffened panel as tested by Faulkner is not a simple task due to lack of information on the precise nature and behavior of the boundary conditions as the tests were conducted and progressed. A comprehensive analysis of stiffened panels with more realistic boundary conditions could be conducted, and it is of real importance.

Inelastic Behavior

The presented nonlinear structural analysis for predicting the buckling failure mode of the stiffened panel was formulated in the form of an iterative procedure. A similar iterative procedure is required to predict the inelastic response of the panel. Either approach can be used to demonstrate the proposed method. The work was limited to the nonlinear elastic analysis due to limited project resources.

Producibility

This work, as a procedure for reliability assessment of continuum structures using finite element analysis, represents a module by itself, that can properly fit in an automated ship design modular process. However, this approach could have an effect on producibility of ship components by giving better evaluation and understanding of the effect of tolerances on the components' strength.

Design

This study can be used for developing future rules for ship structures; improving tolerance requirements, and identifying important structural variables. All these outcomes can affect the design process.