Reliability-Based Load and Resistance Factor Design (LRFD) Guidelines for Stiffened Panels and Grillages of Ship Structures

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ABSTRACT

Stiffened panels and grillages are very important components in ship and offshore structures, and therefore they should be designed for a set of failure modes that govern their strength. They form the backbone of most ship's structure, and they are by far the most commonly used element in a ship. They can be found in bottom structures, decks, side shell, and superstructures. To evaluate the strength of a stiffened panels or grillages, it is necessary to review various strength predicting models and to study their, biases, applicability, and limitations for different loading conditions acting on the element. In this paper, strength limit states for various failure modes of ship panels are presented. For each limit state, commonly used strength models were collected from many sources for evaluating their limitations and applicability and to study their biases and uncertainties. Wherever possible, the different types of biases resulting from these models were computed. The bias and uncertainty analyses for these strength models are needed for the development of load and resistance factor design (LRFD) guidelines for stiffened panels and grillages of ship structures. The uncertainty and biases of these models were assessed and evaluated by comparing their predictions with ones that are more accurate or real values.

The objective of this paper is to develop load and resistance factor design (LRFD) for stiffened panels and grillages of ship structures. Monte Carlo simulation was used to assess the biases and uncertainties for these models. Recommendations for the use of the models and their biases in LRFD development are provided. The first-order reliability method (FORM) was used to develop the partial safety factors (PSFs) for selected limit states.

Introduction

The main type of framing system found in ships nowadays is a longitudinal one, which has stiffeners running in two orthogonal directions (**Figure 1**). Deck and bottom structure panels are reinforced mainly in the longitudinal direction with widely spaced heavier transverse stiffeners. The main purpose of the transverse stiffeners is to provide resistance to the loads induced on bottom and side shell by water pressure. The types of stiffeners used in the longitudinal direction are the T-beams, angles, bulbs, and flat bars, while the transverse stiffeners are typically T-beam sections. This type of structural configuration is commonly called gross stiffened panel or grillage (Vroman 1995). Besides

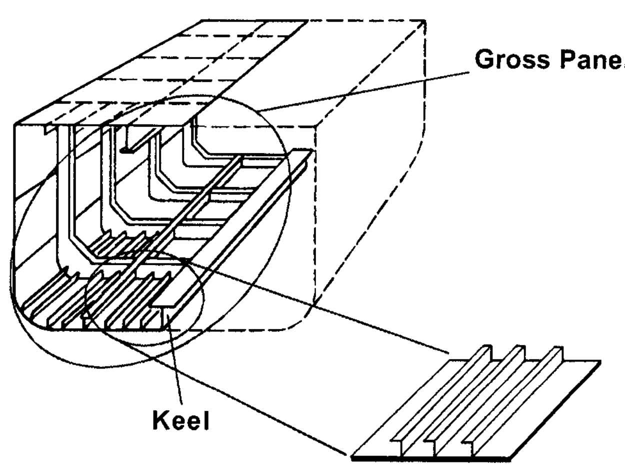


FIGURE 1: *Portion of the Hull Girder Showing the Gross Panel (i.e., Grillage) and a Longitudinally Stiffened Sub-Panel (Hughes 1988).*

their use in ship structures, these grillages are also widely used in land-based structures such as box and plate girders. A typical longitudinal stiffened sub-panel, as shown in **Figure 1**, is bounded on each end by a transverse structure, which has significantly greater stiffness in the plane of the lateral load. The sides of the panel are defined by the presence of a large structural member that has greater stiffness in bending and much greater stiffness in axial loading.

In ship structures, there are three types of loading that can effect the strength of a plate-stiffener panel: (1) negative bending moment, (2) positive bending moment, and (3) in-plane compression or tension. Negative bending loads are the lateral loads because of lateral pressure. They cause the plate to be in tension and the stiffener flange to be in compression. Positive bending loads are those loads that put the plating in compression and the stiffener flange in tension. The third type of loading is uniform in-plane compression. This type of loading arises from hull girder bending and will be considered to be positive when the panel is in compression. The three types of loading can act individually or in combination with one another.

To evaluate the strength of a stiffened panels and grillages element, it is necessary to review various strength prediction models and to study their applicability and limitations for different loading conditions acting on the element. The uncertainties that are associated with a numerical analysis are generally a result of experimental approximation or numerical inaccuracies that can be reduced by some procedures. However, the uncertainties that are associated with a strength design model are different and cannot be eliminated because they result from not accounting for some variables, which can have a strong influence on strength. For this reason, the uncertainty and the bias of a design equation should be assessed and evaluated by comparing its predictions with more accurate ones. Wherever possible, the different types of biases resulting from these models were computed. In doing so, these prediction models were classified as follows (Atua and Ayyub 1996): (1) prediction models that can be used by the LRFD guidelines, (2) advanced prediction models that can be used for various analytical purposes, (3) some experimental results from model testing, and (4) some real measurements based on field data during the service life of a ship. Furthermore, the relationships and uncertainty analyses for these models are required. The relationships can be defined in terms of biases (bias factors). In this paper, only selected strength models that are deemed suitable for LRFD design format are highlighted and presented.

Design Loads and Load Combinations

Primary structural loads on a ship are due to its own weight, cargo, buoyancy, and operation in a random environment, i.e., the sea. The loads acting on the ship's hull girder can be categorized into three main types that are used in this paper: (1) stillwater loads, (2) wave loads, and (3) dynamic loads. The load effect of concern herein is bending moment exerted on the ship hull girder. Hydrostatic lateral pressure on stiffened plates (panels) is due to several sources that include: stillwater, wave and dynamic effects, green seas, and liquids in tanks. Only the first two types are considered in this paper. Mansour et al. (1996) assumed coefficients of variation (*COVs*) of 0.2 and 0.1 for stillwater and wave-induced pressures. In this paper, the *COV* for stillwater pressure is assumed to be 0.15, the *COV* for wave-induced pressure is 0.15, the *COV* for dynamic-induced pressure is 0.25, and the *COV* for the combined wave and dynamic-induced pressure is 0.25. These values were selected based on judgment.

Stillwater loads can be predicted and evaluated with a proper consideration of variability in weight distribution along the ship's length, variability in its cargo loading conditions, and buoyancy. Both wave loads and dynamic loads are related and affected by many factors such as ship characteristics, speed, heading of ship at sea, and sea state (wave heights). Wave height is a random variable that requires statistical and extreme analyses of ship response data collected over a period of time in order to estimate maximum wave-induced and dynamic bending moments that the ship might encounter during its life. The statistical representation of sea waves allows the use of statistical models to predict the maximum wave loads in the ship's life.

Procedures for computing design wave loads for a ship's hull girder based on spectral analysis can be found in numerous references pertaining to ship structures such as Hughes (1988) and Ayyub et al. (2002b).

DESIGN LOADS

The design loads that are of concern in this study for developing reliability-based design for stiffened panels and grillages of ship structures are those loads resulting from ship hull girder bending and their combinations. As indicated earlier, the loads acting on the ship's hull girder can be categorized into three main types: (1) stillwater loads, (2) wave loads, and (3) dynamic loads. Each of these types of load is described in detail in Assakkaf et al. (2002). These are the same types of loads used for the development of LRFD guidelines for unstiffened panels in Assakkaf et al. (2002).

LOAD COMBINATIONS AND RATIOS

Reliability-based structural design of stiffened panels and grillages as presented in this paper is based on two load combinations that are associated with correlation factors as presented in the subsequent sections (Mansour et al. 1984).

Stillwater and Vertical Wave-induced Bending Moments

The load effect (stress) on a stiffened panel element because of combinations of stillwater and vertical wave-induced bending moments is given by

$$f_c = f_{SW} + k_{WD} f_{WD} \tag{1}$$

where f_{SW} = stress because of stillwater bending moment, f_{WD} = stress because of wave-induced bending moment, f_c = un-factored combined stress, and k_{WD} = correlation factor for wave-induced bending moment and can be set equal to one (Mansour et al. 1984).

Stillwater, Vertical Wave-induced, and Dynamic Bending Moments

The load effect on a stiffened panel element because of combinations of stillwater, vertical wave-induced, and dynamic bending moments is given by

$$f_c = f_{SW} + k_W (f_W + k_D f_D) \tag{2}$$

where f_{SW} = stress because of stillwater bending moment, f_W = stress because of wave bending moment, f_D = stress because of dynamic bending moment, f_c = unfactored combined load, and k_D = correlation factor between wave-induced and dynamic bending moments. The correlation factor k_D is given by the following two cases of hogging and sagging conditions (Mansour et al. 1984 and Atua 1998):

a. Hogging Condition:

$$k_D = Exp\left[\frac{53080}{\left(158LBP^{-0.2} + 14.2LBP^{0.3}\right)LBP}\right] \tag{3}$$

b. Sagging Condition:

$$k_D = Exp\left[\frac{21200}{\left(158LBP^{-0.2} + 14.2LBP^{0.3}\right)LBP}\right] \tag{4}$$

where LBP = length between perpendiculars for a ship (ft). Values of k_D for LBP ranging from 300 to 1000 ft can be obtained either from **Table 1** or from the graphical chart provided in **Figure 2**.

Limit States and Design Strength

The stiffened panel of ship structure for all stations should meet one of the following conditions, where the selection of the appropriate equation depends on the availability of information as required by these two equations:

Table 1

Correlation Coefficient of Whipping Bending Moment (k_D) for LBP between 300 and 1000 ft (Mansour at al. 1984 and Atua 1998)

LENGTH (FT)	300	400	500	600	700	800	900	1000
$k_{D(sag)}$	0.5779	0.672	0.734	0.778	0.810	0.835	0.854	0.870
$k_{D(hog)}$	0.2539	0.369	0.461	0.533	0.591	0.637	0.675	0.706

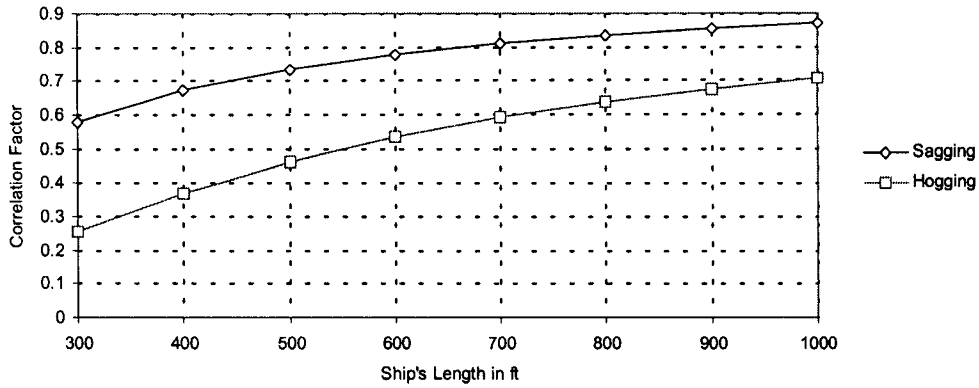


FIGURE 2: *Correlation Coefficient of Whipping Bending Moment (k_D) for 300 < LBP < 1000 ft (Mansour et al. 1984 and Atua. 1998)*

$$\phi F_u \geq \gamma_{SW} f_{SW} + \gamma_{WD} k_{WD} f_{WD} \qquad (5)$$

$$\phi F_u \geq \gamma_{SW} f_{SW} + k_W (\gamma_W f_W + \gamma_D k_D f_D) \qquad (6)$$

where

F_u = ultimate strength (stress) for the stiffened panel

ϕ = strength reduction factors for ultimate strength capacity of stiffened panels and grillages

γ_{SW} = load factor for the stress because of stillwater bending moment

f_{SW} = stress because of stillwater bending moment

k_{WD} = combined wave-induced and dynamic bending moment factor

γ_{WD} = load factor for the stress because of combined wave-induced and dynamic bending moment

f_{WD} = stress because of combined wave-induced and dynamic bending moments

k_W = load combination factor, which can be taken as 1.0

f_W = load factor for the stress because of wave bending moment

k_D = stress because of wave bending moment load combination factor, which can be taken as 0.7

γ_D = load factor for the stress because of dynamic bending moment

f_W = stress because of dynamic bending moment

The nominal (i.e., design) values of the strength and load components should satisfy these formats in order to achieve specified target reliability levels. The nominal strength for stiffened panels can be determined as described in subsequent sections.

DESIGN STRENGTH FOR STIFFENED PANELS AND GRILLAGES

Stiffeners

Stiffeners are very important structural components that are used to strengthen plates and to increase their load carrying capacity. In ship structures, most of grillage failures are due to the collapse of one or more of the longitudinal and transverse stiffeners. Thus, the first and most basic principle with regard to stiffeners is that they should be designed at least as strong as the plating. Also, they should be sufficiently rigid and stable so that neither local stiffener buckling nor overall buckling occurs before local plate buckling. A plate stiffener can be subjected to a variety of primary and secondary loads and load combinations that cause the stiffened plate to fail in one of the following types of buckling: (1) column buckling, (2) beam-column buckling, and (3) flexural-torsional buckling. Numerous strength models for stiffeners are available according to the type of stiffener buckling involved, and can be found in API (1993), Assakkaf (1998), and Atua (1998).

Longitudinal Strength of Stiffened Panels

In this section, a summary of selected strength models that are deemed suitable for *LRFD* design formats is presented. These strength models are for longitudinally stiffened panels subjected to various types of loading. They are presented herein in a concise manner, and they were evaluated in terms of their applicability, limitations, and biases with regard to ship structures. A complete review of the models used by different classification agencies such as the AISC (1994), ASSHTO (1994), and the API (1993) is provided in Atua (1998) and Assakkaf (1998).

Herzog's Model

Based on reevaluation of 215 tests by various researchers and on empirical formulation, Herzog (1987) developed a simple model (formula) for the ultimate strength of stiffened panels that are subjected to uniaxial compression without lateral loads. The ultimate strength F_u of a longitudinally stiffened plate is given by the following empirical formula (Herzog 1987):

$$F_u = \begin{cases} m\overline{F}_y\left[0.5+0.5\left(1-\dfrac{ka}{r\pi}\sqrt{\dfrac{\overline{F}_y}{E}}\right)\right] & \text{for } \dfrac{b}{t} \le 45 \\ m\overline{F}_y\left[0.5+0.5\left(1-\dfrac{ka}{r\pi}\sqrt{\dfrac{\overline{F}_y}{E}}\right)\right]\left[1-0.007\left(\dfrac{b}{t}-45\right)\right] & \text{for } \dfrac{b}{t} > 45 \end{cases} \tag{7}$$

where

$\overline{F}_y = \dfrac{F_{ys}A_s + F_{yp}A_p}{A_s + A_p}$, mean yield strength for the entire plate-stiffener cross section

f_{yp} = yield strength of the plating

f_{ys} = yield strength of the stiffener

E = modulus of elasticity of thestiffened panel

A_p = bt, cross sectional area of the plating

A_s = $t_f f_w + t_w d_w$, cross sectional area of the stiffener

A = $A_s + A_p$, cross sectional area of the plate-stiffener

t_f = stiffener flange thickness

f_w = stiffener flange width or breadth

t_w = stiffener web thickness

d_w = stiffener web depth

a = length or span of a longitudinally stiffened panel

b = distance between longitudinal stiffeners

t = plate thickness

I = moment of inertia of the entire cross section

r = $\sqrt{\dfrac{I}{A}}$ radius of gyration of entire cross section

m = corrective factor, which accounts for initial deformation and residual stresses

k = buckling coefficient, which depends on the panel end constraints

Values for m and k for use in Equations (6)-(7) can be obtained from **Tables 2** and **3**, respectively.

The 215 tests evaluated by Herzog belong to three distinct groups. Group I (75 tests) consisted of small values for imperfection and residual stress, Group II (64 tests) had average values for imperfection and residual stress, while the third group (Group III, 76 tests) consisted of higher values for imperfection and residual stress. The statistical uncertainty (*COV*) associated with the Herzog model of Equation (7) is 0.218. The mean value μ, standard deviation σ, and *COV* of the measurement to prediction are given in **Table 4**.

Hughes's Model

According to Hughes (1988), there are three types of loading that must be considered for determining the ultimate strength of longitudinally stiffened panels. These types of loading are: (1) lateral load causing negative bending moment of the plate-stiffener combination (the panel), (2) later-

Table 2

Recommended m Values (Herzog 1987)

DEGREE OF IMPERFECTION AND RESIDUAL STRESS	*m*
No or average imperfection and no residual stress	1.2
Average imperfection and average residual stress	1.0
Average or large imperfection and high value for residual stress	0.8

Table 3

Recommended k Values (Herzog 1987)

END CONDITION	*k*
Both ends are simply-supported	1.0
One end is simply-supported and the other is clamped	0.8
Clamped ends	0.65

al load causing positive bending moment of the panel, and (3) in-plane compression resulting from hull girder bending. The sign convention to be used throughout this section is that of Hughes (1988). Bending moment in the panel is considered positive when it causes compression in the plating and tension in the stiffener flange, and in-plane loads are positive when in compression (**Figure 3**). The deflection, w_0, because of the lateral load (i.e., lateral pressure) M_0 and initial eccentricity, δ_0, are considered positive when they are toward the stiffener as shown in Figure 3. In beam-column theory, the expressions for the moment M_0 and the corresponding deflection w_0 are based upon an ideal column, which is assumed to be simply supported.

Disregarding plate failure in tension, there can be three distinct modes of collapse (see Figure 3) according to Hughes (1988):

1. Compression failure of the stiffener (mode I collapse),
2. Compression failure of the plating (mode II collapse), and
3. Combined failure of stiffener and plating (mode III collapse).

The ultimate axial strength (stress) F_u for a longitudinally stiffened panel under a combination of in-plane compression and lateral loads (including initial eccentricities), therefore can be defined as the minimum of the collapse (ultimate) values of applied axial stress computed from the expressions for the three types (modes) of failure. Mathematically, it can be given as

$$F_u = \min(F_{a,u\mathrm{I}}, F_{a,u\mathrm{II}}, F_{a,u\mathrm{III}}) \qquad (8)$$

where $F_{a,uI}$, $F_{a,uII}$, and $F_{a,uIII}$ correspond to the ultimate collapse value of the applied axial stress for mode I, mode II, and mode III, respectively. The mathematical expressions for the collapse stress for each mode of failure are provided in Hughes (1988).

Adamchak's Model

Adamchak developed this model in 1979 to estimate the ultimate strength of conventional surface ship hulls or hull components under longitudinal bending or axial compression. The model itself is very complex for hand calculation and therefore it is not recommended for use in a design code without some computational tools or a computer program. To overcome the computational task for this model, Adamchak developed a computer program (ULTSTR) based on this model to estimate the ductile collapse strength of conventional surface ship hulls under longitudinal bending. The recent version of the ultimate strength (ULTSTR) program is intended for preliminary design and

Table 4

Statistics of 215 Tests Conducted on Longitudinally Stiffened Plates in Uniaxial Compression (Herzog 1987)

GROUP	NUMBER OF TESTS	MEAN VALUE (μ)	STANDARD DEVIATION (σ)	*COV*
I	75	1.033	0.134	0.130
II	64	0.999	0.100	0.100
III	76	0.981	0.162	0.169
All	215	1.004	0.136	0.135

based on a variety of empirically based strength of material solutions for the most probable ductile failure modes for stiffened and unstiffened plate structures. The probable ductile failure modes include section yielding or rupture, inter-frame Euler beam-column buckling, and inter-frame stiffener tripping (lateral-torsional buckling). The program also accounts for the effects of materials having different yield strength in plating and stiffeners, for initial out-of-plane distortion because of fabrication, and for lateral pressure loading.

The basic theory behind this model (or ULT-STR) originated preliminary in a joint project on ship structural design concepts involving representatives of the Massachusetts Institute of Technology (MIT), the Ship Structure Committee (SSC), and Navy practices in general. Longitudinally stiffened panel elements can fail either by material yielding, material rupture (tension only), or by some form of structural stability. The instability failure modes for this model include Euler beam-column buckling and stiffener lateral torsional buckling (tripping). These modes of failure are illustrated in **Figure 4**. Euler beam-column buckling is actually treated in this model as having two distinct types of failure patterns as shown in **Figure 5**. Type I is characterized by all lateral deformation occurring in the same direction. Although this type of failure is dependent on all geometrical and material properties that define the structural element, it is basically yield strength dependent. Type I failure is assumed to occur only when either lateral pressure or initial distortion, or both, are present. On the other hand, Type II failure is modulus (E) dependent, as far as initial buckling is concerned. This type of failure can be initiated whether or not initial distortion or lateral pressure, or both, are present. Type III failure is a stiffener tripping or lateral-torsional buckling. Therefore, the ultimate axial strength (stress) for a longitudinally stiffened panel under various types of loading (including material fabrication distortion) is the minimum value of the axial compressive stress computed from the expressions for the three types (modes) of failures, that is

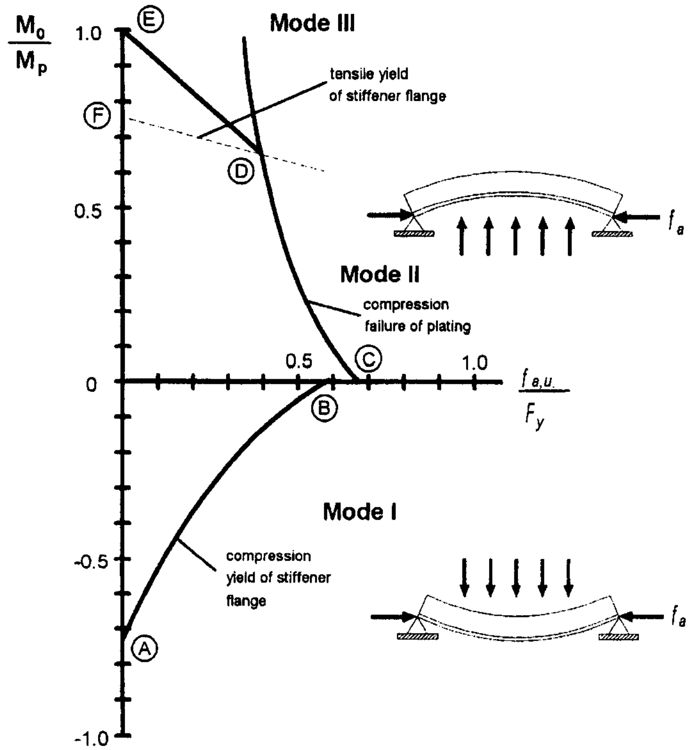


FIGURE 3: *Interaction Diagram for Collapse Mechanism of a Stiffened Panel Under Lateral and In-plane Loads (Hughes 1988)*

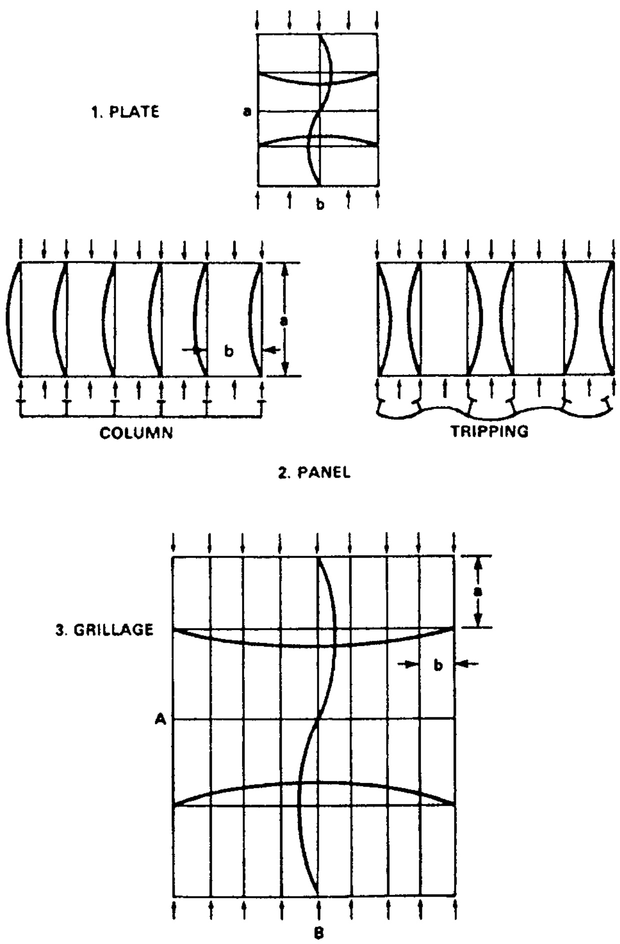
$$F_u = \min(F_{uI}, F_{uII}, F_{uIII}) \qquad (9)$$

Detailed mathematical expressions for the three modes of failures as implemented in the program ULTSTR can be found in Adamchak (1979).

Paik and Lee's Model

An empirical formula, developed by Paik and Lee (1996), for predicting the ultimate strength of longitudinal stiffened sub-panels based on 130-collapse test data for stiffened plates with initial imperfections is presented. The formula expresses the ratio of ultimate strength of the sub-panels to their yield strength in terms of the plate slenderness ratio, β, and the stiffener slenderness ratio, λ, as follows:

$$F_u = F_{y(panel)}\left[0.95 + 0.936\lambda^2 + 0.170B^2\right. \qquad (10)$$



$$+0.188\lambda^2 B^2 - 0.067\lambda^4\Big]^{-0.5}$$

where $F_{y(panel)}$ = yield strength of the whole panel and is given by

$$F_{y(panel)} = \frac{F_{yp}\zeta F_{ys}}{1+\zeta} \tag{11}$$

where

$$\zeta = \frac{d_w t_w + f_w t_f}{bt} \tag{12}$$

The plate slenderness ratio, B, is given by

$$B = \frac{b}{t}\sqrt{\frac{F_y}{E}} \tag{13}$$

The stiffener slenderness ratio, λ, is given by

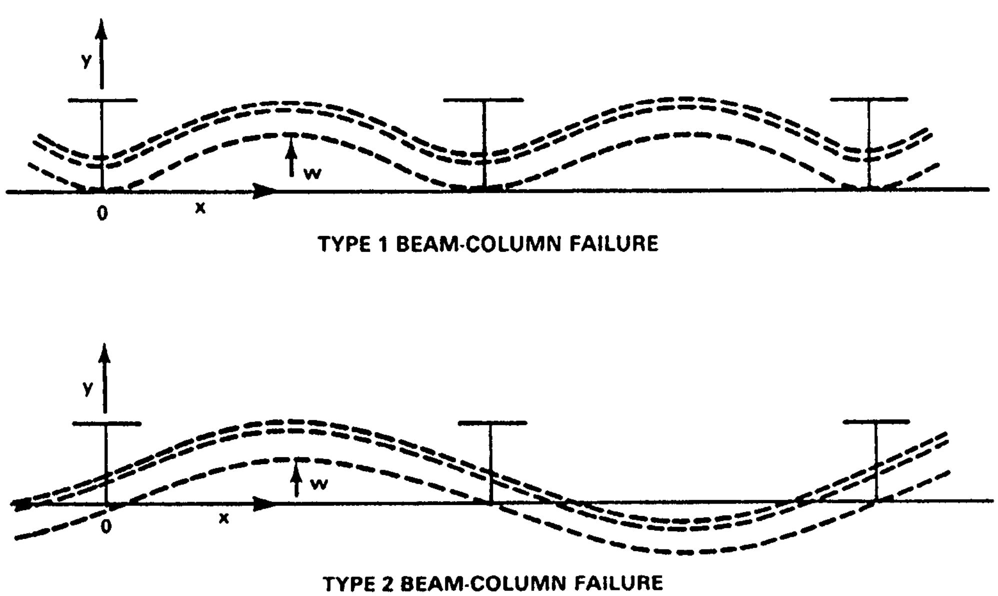
$$\lambda = \frac{a}{\pi r}\sqrt{\frac{F_y}{E}} \tag{14}$$

in which, a = span (length) of stiffener, r = radius of gyration of one stiffener with fully effective plating and is given by

$$r = \sqrt{\frac{I}{A}} \tag{15}$$

FIGURE 4 (above): *Instability Failure Modes (Adamchak 1979)*

FIGURE 5 (right): *Types of Beam-column Failure (Adamchak 1979)*



where A = sectional area of the plate and the stiffener and is given by

$$A = bt + d_w t_w + f_w t_f \tag{16}$$

The moment of inertia of one stiffener with fully effective plating (I) is given by

$$I = \frac{bt^3}{12} + bt\left(z_0 - \frac{t}{2}\right)^2 + \frac{d_w^3 t_w}{12} + d_w t_w\left(z_0 - t - \frac{d_w}{2}\right)^2 + \frac{f_w t_f^3}{12} + f_w t_f\left(z_0 - t - d_w - \frac{t_f}{2}\right)^2 \tag{17}$$

where z_0 = distance of the neutral axis from the base line of the plate, t = thickness of the plate, t_w = thickness of the stiffener web, t_f = thickness of stiffener flange, d_w = stiffener web height, b = spacing between stiffeners, and f_w = stiffener flange width. The formula was compared with experimental and numerical data (Paik and Lee 1996 and Paik 1997) and proved to predict the strength value reasonably.

AASHTO

The ultimate strength of a stiffened panel subjected to uniaxial compressive strength is given by AASHTO in 1994

$$F_u = \begin{cases} F_y (0.66)^\lambda & \text{for } \lambda \le 2.25 \\ \dfrac{0.88F_y}{\lambda} & \text{for } \lambda > 2.25 \end{cases} \tag{18}$$

where

$$\lambda = \left(\frac{ak}{r\pi}\right)^2 \frac{F_y}{E} \tag{19}$$

The limiting width/thickness ratios for axial compression is to satisfy

$$\frac{b}{t} \le k\sqrt{\frac{E}{Fy}} \tag{20}$$

where b = spacing between stiffeners, a = length of panel, E = Young's Modulus, F_y = weighted yield strength, and k = plate buckling coefficient as specified in **Table 5**.

Table 5a

Limiting Width/Thickness Ratios for Plates Supported along One Edge (Unstiffened Plates) as given by AASHTO LRFD Specifications (1994)

CASE	B	b
Flanges and Projecting Legs or Plates	0.56	– Half-flange width of I-sections – Full-flange width of channels – Distance between free edge and first line of bolts or welds in plates – Full-width of an outstanding leg for pairs of angles in continuous contact
Stems of Rolled Tees	0.75	– Full-depth of tee
Other Projecting Elements	0.45	– Full-width of outstanding leg for single angle strut or double angle strut with separator – Full projecting width for others

Table 5b

Limiting Width/Thickness Ratios for Plates Supported along Two Edges (Stiffened Plates) as given by AASHTO LRFD Specifications (1994)

CASE	B	b
Box Flanges	1.40	– Clear distance between webs minus inside corner radius on each side for box flanges and Cover Plates – Distance between lines of welds or bolts for flange cover plates
Webs and other Plate Element	1.49	– Clear distance between flanges minus fillet radii for webs of rolled beams – Clear distance between edge supports for all others
Perforated Cover Plates	1.86	– Clear distance between edge supports

Gross Panels and Grillages

To perform a reliability (safety) check on the design of grillages, the ratio of the stiffness of the transverse and longitudinal stiffeners should at least equal the load-effect term given by the geometrical parameters shown as the right-hand term of the following expression:

$$\frac{I_y}{I_x} = \frac{(n+1)^5}{n\pi^2\left(0.25 + \dfrac{2}{N^3}\right)}\left(\frac{b}{a}\right)^5 \tag{21}$$

where I_x = moment of inertia of the longitudinal plate-stiffener, I_y = moment of inertia of the transverse plate-stiffener, a = length or span of the panel between transverse webs, b = distance between longitudinal stiffeners,

n = number of longitudinal stiffeners, and N = number of longitudinal sub-panels in the overall (or gross) panel. A target reliability level can be selected based on the ship type and usage. Then, the corresponding safety factor can be looked up from **Table 6.**

Comparison and Evaluation of Existing Models for Stiffened Panels

In this section, a comparison between real and predicted values of ultimate strength was performed based on real test specimens from various sources. Some of the strength models used in this comparison are adopted in the current design codes such as the AISC LRFD (1994), AASHTO LRFD (1994), and API (1993). Other models that are also used in this comparison, are those developed by different researchers such as Hughes (1988), Adamchak (1997), Herzog (1987), Paik and Lee (1996), and Mikami and Niwa (1996).

The purpose of this comparison is to select the most appropriate model (models) for use in LRFD design format. The level of complexity associated with the above-mentioned strength models ranges from highly complex models to simple ones. The more complex theoretical models, such as that of Hughes, Equation (8), and Adamchak, Equation (9), do not necessarily lead to less uncertainty. Although they can be accurate and rigorous models, they can lead to more uncertainty because they involve a larger number of variables, some of which may be very uncertain. On the other hand, simple empirical formulations based on real test data, such as that of Herzog, Equation (7) and Paik and Lee, Equation (10), can lead to fairly good results. Although theoretically less rigorous, they can be of practical use because they were derived from real world stiffened plate tests. In formulating a design model, a balance must be achieved between the model accuracy, bias, applicability, and simplicity, all of which are desired features.

Uncertainty and bias of a strength model can be assessed by comparing its strength prediction with a model that has a more accurate result, or real value. In the subsequent sections, bias assessments for uniaxial strength of longitudinally stiffened panels under axial and lateral (pressure) loads are presented.

BIAS ASSESSMENT FOR UNIAXIAL STRENGTH MODELS WITHOUT LATERAL PRESSURE

This section summarizes the results of comparisons that were performed by Assakkaf and Atua (1997) on nearly 80 test specimens under uniaxial load alone. The failure axial stress and the mode of failure for each test were reported. **Table 7** gives the mean, standard deviation, and the *COV* of the bias (real/predicted) for Hughes (1988), Herzog (1987), Adamchak (1997), and Paik and Lee (1996) models. It is apparent from the results in this table that these models have the least bias values for the predicted strength (stress). **Table 8** gives the mean, standard deviation, and the *COV* of the bias (real/predicted) for the strength models used in the current design codes for stiffened panels. Variations in the bias as a function of column slenderness ratio for Hughes (1988), Herzog (1987), Adamchak (1997), and Paik and Lee (1996) models are shown in **Figure 6. Figure 7** gives the variation in the bias for the current design codes.

BIAS ASSESSMENT FOR UNIAXIAL STRENGTH MODELS WITH LATERAL PRESSURE

Assakkaf (1998) and Atua (1998) performed comparison analyses on 14 test specimens subjected to a combination of uniaxial stress and uniform lateral pressure. For each test, they reported the failure axial stress and the mode of failure. **Table 9** gives the mean, stan-

Table 6

Computed Partial Safety Factor for the Stiffness Ratio I_y/I_x

TARGET RELIABILITY INDEX (β)	GRILLAGE STRENGTH REDUCTION FACTOR (ϕ_g)
2.0	0.82
2.5	0.78
3.0	0.75

Table 7

Statistics of the Bias (real/predicted) for Strength Models Under Uniaxial Stress, Without Lateral Pressure (Assakkaf 1998 and Atua 1998)

BIAS	HUGHES	PAIK AND LEE	HERZOG (*m = 1.2*)	HERZOG (*m = 1.0*)	HERZOG (*m = 0.8*)	ADAMCHAK
Mean	1.085	1.030	0.837	1.004	1.255	0.844
Standard Deviation	0.187	0.188	0.154	0.185	0.231	0.245
COV	0.173	0.182	0.184	0.184	0.184	0.291

m = correction factor accounts for initial deformation and residual stresses dard deviation, and the *COV* of the bias (real/predicted) for Hughes (1988), Herzog (1987), Adamchak (1997), and Paik and Lee (1996) models. The results in this table suggest that these models have the least bias values for the predicted ultimate strength (stress) as compared to the values predicted by the codes. **Table 10** gives the mean, standard deviation, and the *COV* of the bias (real/predicted) for the strength models used

Table 8

Statistics of the Bias (real/predicted) of the Current Design Codes for Stiffened Plates Under Uniaxial Stress, Without Lateral Pressure (Assakkaf 1998 and Atua 1998)

BIAS	API (1993)	AISC (1994)	AASHTO (1994)	NAVY PRACTICES
Mean	0.794	0.819	0.818	0.784
St. Dev.	0.203	0.168	0.167	0.160
COV	0.255	0.205	0.205	0.204

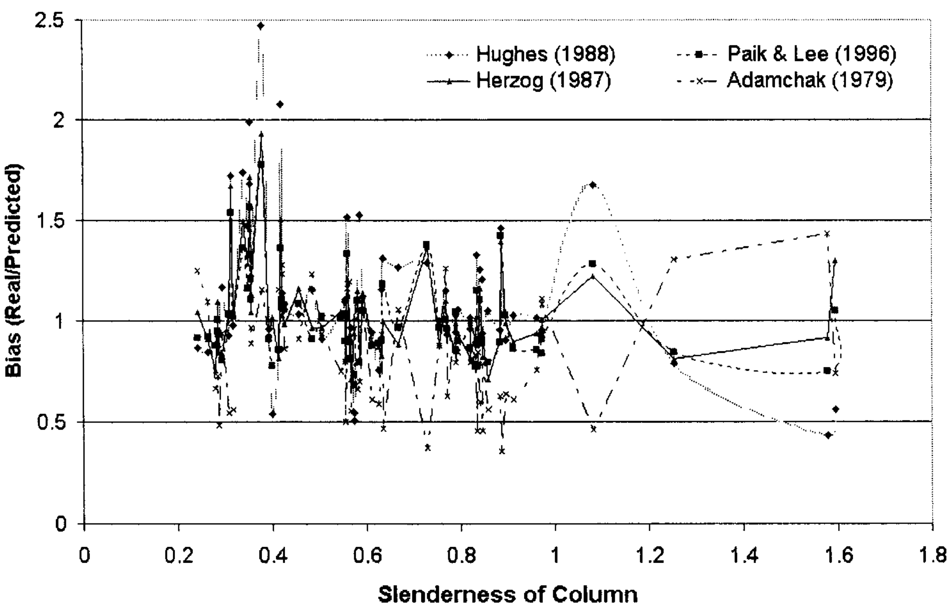


FIGURE 6:
Variation of Bias of Strength Models as a Function of Slenderness Ratio of Column Under Uniaxial Load Only (Assakkaf 1998 and Atua 1998)

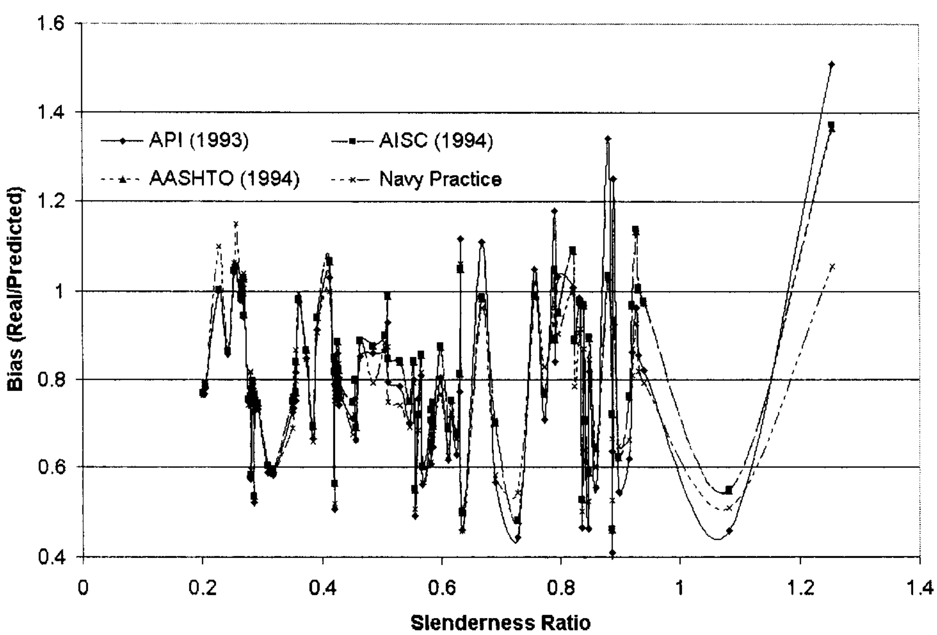


FIGURE 7
Variation of Bias of Current Design Codes as a Function of Slenderness Ratio of Column Under Uniaxial Load Only (Assakkaf 1998 and Atua 1998)

Table 9

Statistics of the Bias (real / predicted) for Strength Models Under Uniaxial Stress with Lateral Pressure (Assakkaf 1998 and Atua 1998)

BIAS	HUGHES	PAIK AND LEE	HERZOG (m = 1.2)	HERZOG (m = 1.0)	HERZOG (m = 0.8)	ADAMCHAK
Mean	1.316	1.061	0.828	0.994	1.242	1.08
Standard Deviation	0.303	0.160	0.152	0.182	0.228	.25
COV	0.230	0.151	0.183	0.183	0.183	.232

m = correction factor accounts for initial deformation and residual stresses

in the current design codes for stiffened panels. Variations in the bias as a function of the ratio of applied moment to plastic moment for stiffened panels with simply supported ends are shown in **Figure 8**. **Figure 9** gives the variations in the bias for the clamped case.

LRFD Guidelines for Stiffened Panels and Grillages

TARGET RELIABILITY LEVELS

Selecting a target reliability level is required in order to establish reliability-based design guidelines for ship structures such as stiffened panels and grillages. The selected reliability level determines the probability of failure of the stiffened panels and grillages. The following three methods can be used to select a target reliability value:

1. Agreeing upon a reasonable value in cases of novel structures without prior history.
2. Calibrating reliability levels implied in currently used design codes.
3. Choosing a target reliability level that minimizes total expected costs over the service life of the structure for dealing with design for which failures result in only economic losses and consequences.

The recommended range of target reliability indices for stiffened panel can be set to range from three to four (Mansour et al. 1996), while for grillage it ranges from two to three.

STATISTICAL CHARACTERISTICS OF BASIC RANDOM VARIABLES

The statistical characteristics of random variables of strength and load models are needed for reliability-based LRFD and assessment of ship structures including stiffened panels. The moment method for calculating partial safety factors (Ang and Tang 1990; Ayyub and McCuen 1997; and Ayyub and White 1978) require full probabilistic characteristics of both strength and load variables in the limit state equation. For example, the relevant strength variables for stiffened panel elements are the material's yield strength (stress) F_y, length of a panel a, thickness t of the plating, and dimensions of the stiffener. While the relevant load variables are the external pressures because of stillwater bending moment, wave bending moment, and dynamic loads.

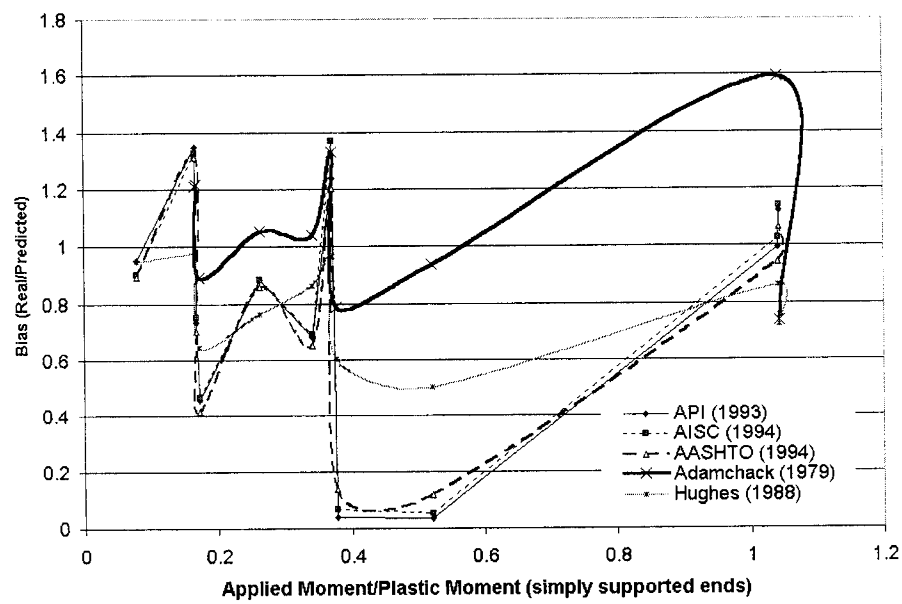


FIGURE 8: *Variation of Bias of the Current Design Codes as a Function of Applied Moment to Plastic Moment, Simply Supported Ends (Assakkaf 1998 and Atua 1998)*

The definition of these random variables requires the investigation of their uncertainties and variability. In reliability assessment of any structural system, these uncertainties must be quantified. Furthermore, partial safety factor (PSF) evaluation for both the strengths and loads in any design equation also requires the characterization of these variables. For example, the first-order reliability method (FORM) as outlined earlier requires the quantification of mean values, standard deviations (or the coefficient of variation (*COV*), and distribution types of all relevant random variables. They are needed to compute the safety index β or the PSFs. Therefore, complete information on the probability distributions of the basic random variables under consideration must be developed. Quantification of random variables of loads and strength in terms of their means, standard deviations or *COVs*, and probability distributions can be achieved in two steps: (1) data collection and (2) data analysis. The first step is the task of collecting as many sets of data deemed to be appropriate for representing the random variables under study. The second is concerned with statistically analyzing the collected data to determine the probabilistic characteristics of these variables.

The objective herein is to compile statistical information and data based on literature review on both strength and load random variables for quantifying the probabilistic characteristics of these variables. The quantification of the probabilistic characteristics of these variables is needed for reliability analysis and design of hull structural components. **Tables 11, 12, 13,** and **14** provide all the recommended values of information required to establish a reliability-based design code for ship structures. This information includes limit state functions for different load combination, probabilistic characteristics (mean values, *COV*, and distribution type) of random variables involved in these limit state functions, mean to nominal ratios of these random variables, deterministic values of the probabilistic load-combination factors, mean ratios between different load components, ranges of target reliability index, the

Table 10

Statistics of the Bias (real/predicted) of the Current Design Codes for Stiffened Plates under Uniaxial Stress with Lateral Pressure (Assakkaf 1998 and Atua 1998)

BIAS	API (1993)	AISC (1994)	AASHTO (1994)
Mean	0.758	0.777	0.709
Standard Deviation	0.468	0.432	0.484
COV	0.617	0.556	0.683

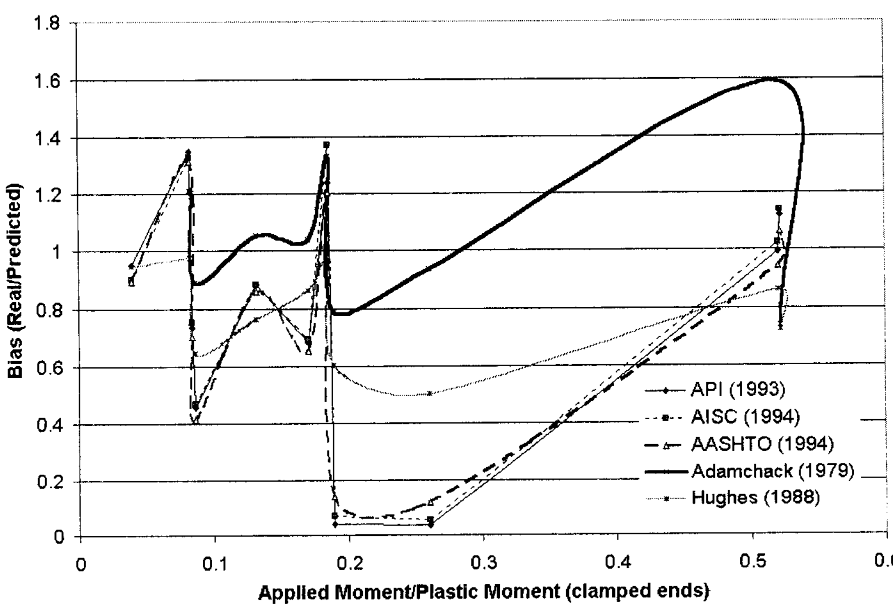


FIGURE 9
Variation of Bias of the Current Design Codes as a Function of Applied /Plastic Moments Ratio, Clamped Ends (Assakkaf 1998 and Atua 1998)

Table 11

Recommended Probabilistic Characteristics of Random Variables

RANDOM VARIABLE	MEAN/NOMINAL	COV	DISTRIBUTION TYPE	BIASES OR ERROR
F_u	1.1	0.15	Normal	1.0
f_{SW}	1.0 (Cruisers)	0.15	Normal	1.0
f_W	1.0	0.1 to 0.2	Type I (EVD) - largest	1.0
f_D	0.83 to 1.11	0.2 to 0.3	Type I (EVD) - largest	1.0
f_{WD}	0.971	0.222 to 0.287	Weibull - smallest	0.971

EVD = extreme value distribution

Table 12

Recommended Combination Factors for Load Components

FACTOR	DETERMINISTIC VALUE	REFERENCES AND COMMENTS
k_W	1.0	– Sikora (1983) and Mansour et al (1995)
k_D	$EXP\left[\dfrac{53080}{\left(158LBP^{-0.2}+14.2LBP^{0.3}\right)LBP}\right]$ (Hogging) $EXP\left[\dfrac{21200}{\left(158LBP^{-0.2}+14.2LBP^{0.3}\right)LBP}\right]$ (Sagging)	– Sikora (1983) – Ranging from 0.35 to 0.65 for LBP = (400 to 800) ft – Ranging from 0.65 to 0.85 for LBP = (400 to 800) ft
	1.0	– Assumption and Sikora (1983)

Table 13

Recommended Ratios of Different Load Components

RATIO	RECOMMENDED VALUE	REFERENCE
$f_{SW}/\overline{f_W}$	0.25 to 0.35	Mansour et al (1995)
$\overline{f_D}/\overline{f_W}$	0.25 to 0.35	Mansour et al (1995)
$\overline{M_{WD}}/\overline{M_W}$	1.0 to 1.35	Assumption

Table 14

Recommended Ranges of Target Reliability Index

RANGE	REFERENCE
3.5 to 4.5	Mansour et al (1995)

biases between different values of each of the random variables, and probabilistic characteristics of modeling uncertainty.

CALCULATION OF PARTIAL SAFETY FACTORS

In this section, calculation of partial safety factors (PSFs) for both the strength and load components in the limit state functions for stiffened panels are presented for demonstration purposes. The first-order reliability method (FORM) as outlined in Ayyub et al. (2002a) was used to develop the partial safety factors. The partial safety factors are defined as the ratio of the value of a variable in a limit state at its most probable failure point to the nominal value. The subsequent sections summarize the methods for calculating partial safety factors. They also give a brief review of recommended load and load combinations and their probabilistic characteristics used in computing the partial safety factors. The final section presents the development of reliability checking for gross panels (grillages) based on stiffness of the transverse and longitudinal stiffeners.

Performance Functions for Calculating Partial Safety Factors for Stiffened Panels

Reliability-based design LRFD format involves the ultimate strength capacity of a stiffened plate element and the load random variable of stillwater, wave-induced, and dynamic bending moments. The partial safety factors format allows transforming the desired reliability index into separate safety factors for each of the design variables in the recommended format. Two recommended limit state formats for stiffened panels are provided as follows:

Limit State I:

$$g(F_u, f_{SW}, f_{WD}) = F_u - f_{SW} - k_{WD} f_{WD} \quad (22)$$

Limit State II:

$$g(F_u, f_{SW}, f_W, f_D) = F_u - f_{SW} - k_W (f_W + k_D f_D) \quad (23)$$

where g = the limit state or performance function, f_{SW} = stress because of stillwater bending moment, f_{WD} = stress because of combined wave-induced and dynamic bending moments, f_W = stress because of wave bending moment, k_{WD} = combined wave-induced and dynamic bending moment factor (equals unity), k_W = load combination factor equals unity, k_D = load combination factor (equals 0.7), and F_u = ultimate strength capacity of a stiffened plate. The ultimate strength capacity F_u depends on the loading conditions for the stiffened panel and is given by the design strength models as described earlier. The two limit states given by Equations (22) and (23) are referred to as limit states 1 and 2, respectively.

Partial Safety Factors for Uniaxial Compression without Lateral Pressure

The calculations of the partial safety factors for both limit states given in Equations (22) and (23) are performed to provide values for the PSFs for all cases, such as different target reliability levels (3.5, 4.0, and 4.5) and sagging and hogging conditions. These values are rounded to some level deemed to be practical for engineering use. For each case, the values of the PSFs before rounding are denoted by the subscript (1) as follows for an example case:

$$\bar{\phi}_{F_{u_1}} \bar{F}_{u_1} = \bar{\gamma}_{f_{SW_1}} \bar{f}_{SW_1} + k_W \left(\bar{\gamma}_{f_{W_1}} \bar{f}_{W_1} + k_d \bar{\gamma}_{f_{D_1}} \bar{f}_{D_1} \right) \quad (24)$$

and

$$\bar{\phi}_{F_{u_1}} \bar{F}_{u_1} = \bar{\gamma}_{f_{SW_1}} \bar{f}_{SW_1} + k_{WD} \bar{\gamma}_{f_{WD_1}} \bar{f}_{WD_1} \quad (25)$$

where the above partial safety factors are used as multipliers to the corresponding mean values of the random variables. The ultimate strength capacity of a stiffened panel in this case is based on the strength calculated using the Herzog (1987) empirical formula as discussed earlier.

Table 15 provides the recommended load factors applied to the corresponding mean load values based on previously developed LRFD guidelines for hull girder bending (Atua 1998) and unstiffened panels (Assakkaf 1998). They are denoted by the subscript (2) after rounding. These recommended factors are referred to as the mean values of the load PSFs. The recommended mean values of the load PSFs are used to compute the recommended values of the strength factors (applied to the corresponding mean strength values) as follows for an example case:

$$\phi_{F_{u_2}} = \phi_{F_{u_1}} \left[\frac{\gamma_{f_{SW_1}} f_{SW} + k_W \left(\gamma_{f_{W_1}} f_W + k_d \gamma_{f_{D_1}} f_D \right)}{\gamma_{f_{SW_2}} f_{SW} + k_W \left(\gamma_{f_{W_2}} f_W + k_d \gamma_{f_{D_2}} f_D \right)} \right] \quad (26)$$

and

$$\phi_{F_{u_2}} = \phi_{F_{u_1}} \left[\frac{\gamma_{f_{SW_2}} f_{SW} + k_{WD} \gamma_{f_{WD_1}} f_{WD}}{\gamma_{f_{SW_2}} f_{SW} + k_{WD} \gamma_{f_{WD_2}} f_{WD}} \right] \quad (27)$$

Table 16 provides the recommended mean values of stiffened panel strength factors that are denoted by the subscript (2).

Table 17 provides a summary of the bias factors (mean to nominal ratios) of all random variables involved in the limit states. Based on these bias factors, nominal PSF's that can be applied to the corresponding nominal values of the variables, are given by

Table 15

Recommended Mean Load Factors Without Lateral Pressure

β	γ_{SW}	γ_W	γ_D	γ_{WD}
3.5	1.05	1.55	1.10	1.50
4.0	1.05	1.70	1.10	1.55
4.5	1.05	1.90	1.10	1.60

Table 16

Recommended Mean Strength Factors Without Lateral Pressure

LIMIT STATE	ϕ_{F_u} $\beta = 3.5$	$\beta = 4.0$	$\beta = 4.5$
$g = F_u - f_{sw} - k_w(f_w + k_D f_D)$	0.56	0.53	0.50
$g = F_u - f_{sw} - k_{WD} f_{WD}$	0.52	0.47	0.42

Table 17

Summary of Bias Factors (Mean/Nominal Ratios)

LIMIT STATE	F_u	f_{SW}	f_W	f_D	f_{WD}
$g = F_u - f_{sw} - k_w(f_w + k_D f_D)$	1.1	1.0	1.0	1.0	na
$g = F_u - f_{sw} - k_{WD} f_{WD}$	1.1	1.0	na	na	1.0

Table 18

Recommended Nominal Load Factors With and Without Lateral Pressure

β	γ_{SW}	γ_W	γ_D	γ_{WD}
3.5	1.05	1.55	1.10	1.50
4.0	1.05	1.70	1.10	1.55
4.5	1.05	1.90	1.10	1.60

$$\phi_{F_{u_n}} = \phi_{F_{u_2}} B_{ij} \tag{28}$$

where B_{ij} = bias factor (mean to nominal ratio).

Table 18 provides the recommended nominal values of the load factors, and **Table 19** provides nominal values of strength factors. It is to be noted that the values shown in Tables 18 and 19 are rounded, which causes a slight change in the implied reliability index, β, according to the LRFD guidelines. Therefore, the reliability level calculated for different ratios of load components will be slightly greater than the target reliability level for each case, which means that the rounded values of the PSFs produce slightly safer designs for stiffened panel bending and meet the target reliability level.

Partial Safety Factors for Uniaxial Compression with Lateral Pressure

The ultimate strength capacity of a stiffened panel in this case is based on the strength calculated using the strength model proposed by Adamchak (1997). The procedure for computing the partial safety factor is the same as that used with Herzog's (1987) model except that the value of mean and *COV* of the bias of the ultimate strength should be revised to account for the variability because of lateral pressure. These values are 1.080 for the mean and 0.23 for the *COV*.

The general form of the limit state in this case will be the same as that in Equations (24) and (25) except that the lateral pressure effect will be included in the value of the strength reduction factor, ϕ_R. This means that the lateral pressure existence is represented by both the higher value of the *COV* of the ultimate strength of the stiffened panel and the resultant smaller value of ϕ_R (strength value is further reduced to count for the lateral load).

The partial safety factor calculations in this case will be based on the recommended mean load factors (Table 15) and the mean and *COV* values of the ultimate strength based on Adamchak (1997) model analysis.

The resulting recommended mean strength factors in this case are provided in **Table 20.** The mean/nominal ratio of the strength model used (Adamchak 1997) based on the test results was found to be 1.15. The recommended nominal load factors will be the same as those given in Table 18. The resulting recommended nominal strength reduction factors are provided in **Table 21.**

Reliability (Safety) Checking for a Grillage

As indicated earlier, the problem of the overall grillage will be reduced to the failure of the longitudinally stiffened sub-panels by preventing the grillage from buckling as a whole. This is achieved by insuring that the transverse stiffeners do not deflect beyond a certain limit that, in turn, will cause the longitudinal stiffeners to buckle between the transverse stiffeners. To perform reliability checking on the design, the ratio of the stiffness of the transverse and longitudinal stiffeners should not be less than the load effect given by the geometrical parameters shown in the right hand term of the following formula (Hughes 1988):

$$\frac{I_y}{I_x} \geq \frac{(n+1)^5}{n\pi^2\left(0.25+\frac{2}{N^3}\right)}\left(\frac{b}{a}\right)^5 \qquad (29)$$

The safety or reliability checking limit state will be reduced to the form:

$$g(x)=\frac{I_y}{I_x}-C_1\left(\frac{b}{a}\right)^5 \geq 0.0 \qquad (30)$$

where C_l= panel stiffness parameter which depends on the number of bays and stiffeners. The first term represents the stiffness ratio, and the second term represents the load effect. The goal here is to develop partial safety factors so that the value of Equation (30) is not less than zero. The designer in this case will look for the value of the partial safety factors according to his design case (number of bays, number of longitudinal stiffeners, and the *b/a* ratio). The minimum required value of the moment of inertia of the transverse stiffener should satisfy Equation (30).

The partial safety calculations were performed for different design parameters (number of bays, number of longitudinal stiffeners, and the *b/a* ratio). However, by examining Equation (30), it is clear that changing any of these parameters will result in only changing the mean value of the load effect, $c_1\left(\frac{b}{a}\right)$.

This means that the distribution type and the *COV* of both stiffness ratio and the load effect will remain the same, i.e., for the same target reliability index, β_0, the same partial safety factors will result for any design case. The difference will happen only when the *COV* and distribution type of the stiffness ratio change, when the *COV* and distribution type of the load effect change, or when the target reliability index changes.

Table 19

Recommended Nominal Strength Factors without Lateral Pressure

		ϕ_{F_u}		
LIMIT STATE	TYPE OF STEEL	$\beta = 3.5$	$\beta = 4.0$	$\beta = 4.5$
$g = F_u - f_{SW} - k_W(f_W + k_D f_D)$	All	0.61	0.57	0.54
$g = F_u - f_{SW} - k_{WD} f_{WD}$	All	0.56	0.51	0.46

Table 20

Recommended Mean Strength Factors with Lateral Pressure

	ϕ_{F_u}		
LIMIT STATE	$\beta = 3.5$	$\beta = 4.0$	$\beta = 4.5$
$g = F_u - f_{SW} - k_W(f_W + k_D f_D)$	0.53	0.49	0.46
$g = F_u - f_{SW} - k_{WD} f_{WD}$	0.48	0.43	0.40

Table 21

Recommended Nominal Strength Factors with Lateral Pressure

		ϕ_{F_u}		
LIMIT STATE	TYPE OF STEEL	$\beta = 3.5$	$\beta = 4.0$	$\beta = 4.5$
$g = F_u - f_{SW} - k_W(f_W + k_D f_D)$	All	0.66	0.61	0.58
$g = F_u - f_{SW} - k_{WD} f_{WD}$	All	0.61	0.54	0.50

Different design cases were tested to demonstrate the effect of *COV* and the target reliability index on the PSFs. The results are provided in **Table 22**, which represents the computed partial safety factor that should be multiplied by the stiffness ratio to assure the safety criteria of the design concept proposed earlier. However, regardless of the *COV*'s of the *b/a* ratio or the stiffness ratio, the recommended partial safety factors for target reliability levels of 2.5, 3.0, and 3.5, are 0.82, 0.78, and 0.75, respectively, as shown in Table 6.

SAMPLE LRFD GUIDELINES

This section provides sample reliability-based LRFD guidelines for stiffened panels and grillages of ship structures. The guidelines, as demonstrated herein, consist of limit state expressions, partial safety factors for both the strength and the loads, and a range of target reliability levels. Stiffened plate elements of ship structure for all stations should meet one of the limit states as given by Equations (22) and (23).

Table 22

Computed Partial Safety Factor for the Stiffness Ratio

COV(Iy/Ix)	TARGET RELIABILITY INDEX β								
	2.0			2.5				3.0	
	COV (b/a)								
0.07	0.002	0.005	0.008	0.002	0.005	0.008	0.002	0.005	0.008
0.07	0.8662	0.8599	0.8493	0.8361	0.8286	0.8158	0.8071	0.7984	0.7837
0.09	0.8313	0.8266	0.8183	0.7946	0.7899	0.7791	0.7595	0.7530	0.7417
0.11	0.7975	0.7938	0.7871	0.7548	0.7504	0.7425	0.7143	0.7093	0.7004

Table 23

Given Dimensions of the Stiffened Panel
Value (in)Variable

Width of plating, *b*	24.0
Stiffener web depth, d_w	4.50
Stiffener flange breadth, d_w	1.75
Stiffener web thickness, t_w	0.205
Stiffener flange thickness, t_f	0.375

The ultimate strength capacity F_u depends on the loading conditions for the stiffened panel (i.e., uniaxial, edge shear, etc.) and the strength model that is used. The two limit states given by Equations (22) and (23) are referred to as limit state 1 and 2, respectively.

The nominal (i.e., design) values of the strength and load components should satisfy these limit states in order to achieve specified target reliability levels. The strength factors are provided in Table 14 in accordance with the following parameters: (1) target reliability level ranging from three to four, (2) the type of load combinations as shown in the table, and (3) ultimate strength prediction for stiffened panel as provided by Herzog (1987). The target reliability should be selected based on the ship type and usage. Then, the corresponding strength factor can be looked up from Table 19 based on the strength model under consideration. The load factors that can be used in conjunction with strength factors are provided in Table 18.

For reliability checking on a grillage, Equation (31) should be used in conjunction with Table 6 to insure that the ratio of the stiffness of the transverse and longitudinal stiffeners is met according to

$$\phi_g \frac{I_y}{I_x} \geq \frac{(n+1)^5}{n\pi^2\left(0.25 + \frac{2}{N^3}\right)}\left(\frac{b}{a}\right)^5 \qquad (31)$$

where

I_x = moment of inertia of a longitudinal plate-stiffener

I_y = moment of inertia of a transverse plate-stiffener

a = length or span of the panel between transverse webs

b = distance between longitudinal stiffeners

n = number of longitudinal stiffeners

N = number of longitudinal sub-panels in overall (or gross) panel or grillage

ϕ_g = grillages strength reduction factor

In using the above equation for safety checking for a grillage, a target reliability level should be selected based on the ship type and usage. Then, the corresponding safety factor can be looked up from Table 6.

Design Examples

The following two examples demonstrate the use of LRFD-based partial safety in the limit state equation for designing and checking the adequacy of stiffened panels of a ship:

EXAMPLE 1. STIFFENED PANEL DESIGN

Given:

A stiffened panel, pinned at the ends, whose dimensions are shown in **Figure 10** is to be designed at the bottom deck of a ship to withstand a uniaxial compression stress because of environmental bending moment loads acting on the ship. The stresses because of the environmental loads are estimated to have the following values: 0.15 ksi because of stillwater bending, 4.5 ksi because of waves bending, and 2.2 ksi because of dynamic bending. If the yield strength of steel is 34 ksi for the plating and 36 ksi for the stiffener (i.e., web & flange), and the dimensions of the panel are as shown in **Table 23**, design the thickness t and length a of the plating assuming a target reliability level of four. Note that the length of the plating is not to exceed 80 in, and not to be less than 48 in.

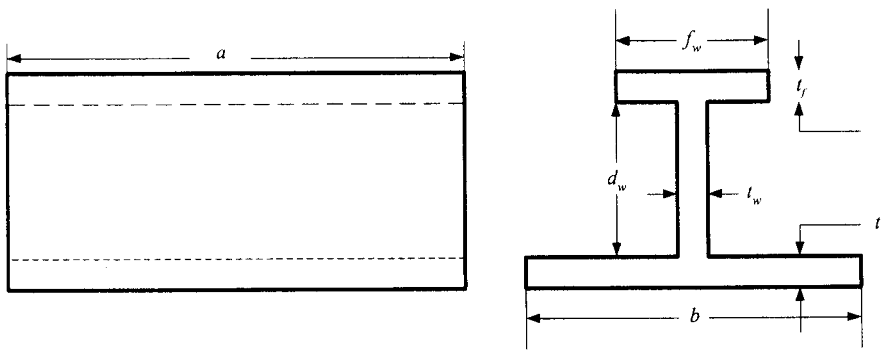


FIGURE 10: *Stiffened Panel Design*

Solution:

For a stiffened panel under uniaxial compression without lateral pressure, the strength model as given by Equation (7) (Herzog) applies

$$F_u = \begin{cases} m\bar{F}_y\left[0.5+0.5\left(1-\dfrac{ka}{r\pi}\sqrt{\dfrac{\bar{F}_y}{E}}\right)\right] & \text{for } \dfrac{b}{t} \le 45 \\ m\bar{F}_y\left[0.5+0.5\left(1-\dfrac{ka}{r\pi}\sqrt{\dfrac{\bar{F}_y}{E}}\right)\right]\left[1-0.007\left(\dfrac{b}{t}-45\right)\right] & \text{for } \dfrac{b}{t} > 45 \end{cases}$$

Assume an initial value for t = 0.2 in, and for a = 80 in, hence

$$A_p = bt = 24(0.2) = 4.8\,\text{in}^2$$

$$A_s = t_f f_w + t_w d_w = 0.375(1.75) + 0.205(4.5) = 1.579\,\text{in}^2$$

$$\bar{F}_y = \frac{F_{ys}A_s + F_{yp}A_p}{A_s + A_p} = \frac{36(1.579)+34(4.8)}{1.579+4.8} = 34.50\,\text{ksi}$$

Check the slenderness ratio b/t:

$\dfrac{b}{t} = \dfrac{24}{0.2} = 120 > 45$, therefore, the following equation applies

$$F_u = m\overline{F}_y\left[0.5 + 0.5\left(1 - \frac{ka}{r\pi}\sqrt{\frac{\overline{F}_y}{E}}\right)\right]\left[1 - 0.007\left(\frac{b}{t} - 45\right)\right]$$

The radius of gyration r for the cross section can be found when the moment of inertia I has been established. To compute I, the location of neutral axis must be calculated:

$$\bar{y} = \frac{\frac{0.2}{2}(24)(0.2) + \left(0.2 + \frac{4.5}{2}\right)(4.5)(0.205) + \left(0.2 + 4.5 + \frac{0.375}{2}\right)(0.375)(1.75)}{1.579 + 4.8}$$

= 0.932 inches from base of the plating.

Therefore, I = 17.23 in^4, and $r = \sqrt{\frac{I}{A}} = \sqrt{\frac{17.23}{1.579 + 4.8}} = 1.65$ in

Assuming m and k both equal to one (see Tables 2 and 3),

$$F_u = (1)(34.50)\left[0.5 + 0.5\left(1 - \frac{80}{(1.65)\pi}\sqrt{\frac{34.50}{29{,}000}}\right)\right]\left[1 - 0.007\left(\frac{24}{0.2} - 45\right)\right] = 12.03 \text{ ksi}$$

In reference to Tables 18 and 19, and for a target reliability index $\beta_0 = 4.0$ as given, the following partial safety factors are obtained for use in the design equation:

$\phi = 0.57$, $\gamma_{SW} = 1.05$, $\gamma_W = 1.7$, and $\gamma_D = 1.1$, therefore,

$\phi F_u = 0.57(12.03) = 6.86$ ksi and

$\gamma_{SW} f_{SW} + k_W(\gamma_W f_W + \gamma_D k_D f_D) = (1.05)(0.15) + (1)[1.7(4.5) + (1.1)(0.7)(2.2)]$

$= 9.50$ ksi

$(\phi F_u = 6.86 \text{ ksi}) < 9.84$ ksi ***Not Acceptable***

Now try t = 0.25 in and a = 80 in, hence:

$$A_p = bt = 24(0.25) = 6 \text{ in}^2$$

$$A_s = t_f f_w + t_w d_w = 0.375(1.75) + 0.205(4.5) = 1.579 \text{ in}^2$$

$$\overline{F}_y = \frac{F_{ys}A_s + F_{yp}A_p}{A_s + A_p} = \frac{36(1.579) + 34(6)}{1.579 + 6} = 34.42 \text{ ksi}$$

Check the slenderness ratio b/t:

$\frac{b}{t} = \frac{24}{0.25} = 96 > 45$, therefore, the following equation applies:

$$F_u = m\overline{F}_y\left[0.5 + 0.5\left(1 - \frac{ka}{r\pi}\sqrt{\frac{\overline{F}_y}{E}}\right)\right]\left[1 - 0.007\left(\frac{b}{t} - 45\right)\right]$$

Again, the radius of gyration r for the cross section can be found when the moment of inertia I is established. To compute I, the location of neutral axis must be calculated:

$$\bar{y} = \frac{\frac{0.25}{2}(24)(0.25) + \left(0.25 + \frac{4.5}{2}\right)(4.5)(0.205) + \left(0.25 + 4.5 + \frac{0.375}{2}\right)(0.375)(1.75)}{1.579 + 6}$$

= 0.831 inches from the base of the plating

Therefore, I = 18.22 in^4, and $\sqrt{\frac{I}{A}} = \sqrt{\frac{18.22}{1.579 + 6}} = 1.55$ in

Assuming m and k both equal to one (see Tables 2 and 3),

$$F_u = (1)(34.42)\left[0.5 + 0.5\left(1 - \frac{80}{(1.55)\pi}\sqrt{\frac{34.42}{29{,}000}}\right)\right]\left[1 - 0.007\left(\frac{24}{0.25} - 45\right)\right] = 15.87 \text{ ksi}$$

$\phi F_u = 0.57(15.87) = 9.05$ ksi and

$\gamma_{SW} f_{SW} + k_W(\gamma_W f_W + \gamma_D k_D f_D) = (1.05)(0.15) + (1)[1.7(4.5) + (1.1)(0.7)(2.2)]$

$= 9.50$ ksi

$(\phi F_u = 9.05 \text{ ksi}) < 9.50$ ksi ***Not Acceptable***

Now try t = 25 in and a = 60 in.

Therefore, in this case, the section properties calculations (i.e., $\bar{y}$, I, and r) will be the same. However, the strength will change because of a new value of a = 60 in:

$$F_u = (1)(34.42)\left[0.5 + 0.5\left(1 - \frac{60}{(1.55)\pi}\sqrt{\frac{34.42}{29{,}000}}\right)\right]\left[1 - 0.007\left(\frac{24}{0.25} - 45\right)\right] = 17.43 \text{ ksi}$$

$\phi F_u = 0.57(17.43) = 9.94$ ksi and

$\gamma_{SW} f_{SW} + k_W(\gamma_W f_W + \gamma_D k_D f_D) = (1.05)(0.15) + (1)[1.7(4.5) + (1.1)(0.7)(2.2)]$

$= 9.50$ ksi

$(\phi F_u = 9.94 \text{ ksi}) > 9.50$ ksi ***Acceptable***

Hence, select ***t = 0.25 in, and a = 60 in***

EXAMPLE 2. ADEQUACY CHECKING FOR GRILLAGE

Given:

Assume a target reliability level of 2.5, check the adequacy of the following grillage:

$I_x = 16$ in^4, $I_y = 26.5$ in^4, $N = 5$, $n = 3$, $a = 60$ in, $b = 24$ in

Solution:
For a grillage, the strength is given by Equation (31) as

$$\phi_g \frac{I_y}{I_x} \geq \frac{(n+1)^5}{n\pi^2\left(0.25+\frac{2}{N^3}\right)}\left(\frac{b}{a}\right)^5$$

For a target reliability index of 2.5, Table 6 gives $\phi_g = 0.78$, therefore,

$$\phi_g \frac{I_y}{I_x} = 0.78\frac{26.5}{16} = 1.29 \quad \text{and}$$

$$\frac{(n+1)^5}{n\pi^2\left(0.25+\frac{2}{N^3}\right)}\left(\frac{b}{a}\right)^5 = \frac{(3+1)^5}{(3\pi^2)\left(0.25+\frac{2}{(5)^3}\right)}\left(\frac{24}{60}\right)^5 = 1.33$$

Since $1.29 < 1.33$, the grillage will be ***inadequate***.

Summary and Conclusions

Future design guidelines for stiffened panels and grillages of ship structures will be developed using reliability methods and they will be expressed in a special and practical formats such as the Load and Resistance Factor Design (LRFD). The LRFD guidelines for stiffened panels, which are based on structural reliability theory, can be built on previously and currently used specifications for ships, buildings, bridges, and offshore structures. This paper provides methods for and demonstrates the development of LRFD guidelines for ship stiffened panels and grillages elements subjected to uniaxial loading. These design methods were developed according to the following requirements: (1) spectral analysis of wave loads, (2) building on conventional codes, (3) nominal strength and load values, and (4) achieving target reliability levels.

The First-Order Reliability Method (FORM) was used to develop the LRFD-based partial safety factors (PSF's) for selected limit states and for various types of loading acting on unstiffened panel element. These factors were determined to account for the uncertainties in strength and load effects. FORM was used to determine these factors based on prescribed probabilistic characteristic of strength and load effects. Also, strength factors were computed for a set of load factors to meet selected target reliability levels for demonstration purposes. The resulting LRFD guidelines are demonstrated in this paper using examples design.

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REFERENCES

Ang, A. H.-S., and W. H. Tang, 1990. "Probability Concepts in Engineering Planning and Design," Vol. II Decision, *Risk, and Reliability*, John Wiley & Sons, NY.

Assakaf, I., B. M. Ayyub, P. Hess, and K. Atua, 2002. "Reliability-Based Load and Resistance Factor Design (LRFD) Guidelines for Unstiffened Panels," *Naval Engineers Journal*, Vol. 114, No.2.

Assakkaf, I. A and B. M. Ayyub, 1995. "An Excel Spreadsheet for Reliability Assessment and Partial Safety Factor Calculations," the Center for Technology and Systems Management, Department of Civil and Environmental Engineering, University of Maryland, College Park, Md..

Assakkaf, I. A. and K. I. Atua, 1997. "A Spreadsheet Calculations for Evaluating the Strength of Stiffened Panels of Naval Ships." the Center for Technology and Systems Management, Department of Civil and Environmental Engineering, University of Maryland, College Park, Md.

Assakkaf, I. A., 1998. "Reliability-based Design of Panels and Fatigue Details of Ship Structures," A dissertation submitted to the faculty of the graduate school of the University of Maryland, College Park in partial fulfillment of the requirements for the degree of doctor of philosophy.

Atua, K. I., 1998. "Reliability-Based Structural Design of Ship Hull Girders and Stiffened Panels," A dissertation submitted to the faculty of the graduate school of the University of Maryland, College Park in partial fulfillment of the requirements for the degree of doctor of philosophy.

Ayyub, B. M. and R. H. McCuen, 1997. *Probability, Statistics And Reliability For Engineers*, CRC Press, Fla..

Ayyub, B. M. and A. M.. White, 1987. "Reliability-Conditioned Partial Safety Factors", *Journal of Structural Engineering*, Vol. 113, No. 2, February, pp. 280-283.

Ayyub, B. M., I. Assakkaf, J. E Beach, W. Melton, and J. A Conley, 2002a, "Methodology for Developing Reliability-Based Load and Resistance Factor Design (LRFD) Guidelines for Ship Structures," *Naval Engineers Journal*, Vol. 114, No.2.

Ayyub, B. M., I. Assakkaf, J. Sikora. J. Adamchack, and K. Atua, 2002b, "Reliability-Based Load and Resistance Factor Design (LRFD) Guidelines for Hull Girder Bending," *Naval Engineers Journal*, Vol. 114, No. 2.

Bleich, F., 1952. *Buckling Strength of Metal Structures*, McGraw-Hill, NY.

Bruchman, D. and A. Dinsenbacher, 1991. "Permanent Set of Laterally Loaded Plating: New and Previous Methods," SSPD-91-173-58, David Taylor Research Center, Bethesda , Maryland.

Faulkner, D., 1975. "A Review of Effective Plating for Use in the Analysis of Stiffened Plating in Bending and Compression," *Journal of Ship Research*, 19(1), 1-17.

Faulkner, D., 1975. "Compression Strength of Welded Grillages," Chapter 21 in *Ship Structural Design Concepts*, editor J. N. Evans, Cornell Marine Press.

Frieze, P. A., P. J. Dowling and R. W. Hobbs, 1977. "Ultimate Load Behavior of Plates in Compression," International Symposium on Steel Plated Structures, Crosby Lockwood Staples, London.

Hughes, O. F., 1988. "Ship Structural Design, A rationally-Based, Computer-Aided Optimization Approach," The Society of Naval Architects and Marine Engineers, Jersey City, New Jersey.

Mansour, A.E., H. Y. Jan,. C., I. Zigeman, Y. N. Chen, S. J. Harding, 1984. "Implementation of Reliability Methods to Marine Structures," Report, The Society of Naval Architects and Marine Engineering, 5-10.

Mansour, A.E., P.H. Wirsching, and G.J. White, and B. M.Ayyub, 1996. "Probability-Based Ship Design: Implementation of Design Guidelines," SSC 392, NTIS, Washington, D.C., 200 pages.

Sikora, J. P., A. Dinsenbacher and J. A.Beach, 1983. "A Method for Estimating Lifetime Loads and Fatigue Lives for Swath and Conventional Monohull Ships," *Naval Engineers Journal*, Vol. 95, No. 3, pp.63-85.

Soares, C. G., 1988. "Uncertainty Modeling in Plate Buckling," Shipbuilding Engineering Program, Department of Mechanical Engineering, Technical University of Lisbon, Elsevier Science Publishers B. V., Amsterdam, Printed in The Netherlands.

Valsgard, S., 1980. "Numerical Design Prediction of the Capacity of Plates in Biaxial In-Plane Compression," *Computers and Structures*, 12, 729-939.